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Sheet 00: Differentiation and Integration

Posted: Mi 01.10.25 Due: never

(b)[2](E/M/A) means: problem (b) counts 2 points and is easy/medium hard/advanced

Example Problem 1: Differentiation of polynomials [1]

Points: (a)[0,5](E); (b)[0,5](E).

Compute the first and second derivatives of the following polynomials. [Check your results against those in square brackets, where $[a; b, c]$ stands for $f'(a) = b$, $f''(a) = c$.]

(a) $f(x) = 3x^3 + 2x - 1$ $[2; 38, 36]$ (b) $f(x) = x^4 - 2x^2 + 2$ $[2; 24, 44]$

Example Problem 2: Derivatives involving powers, sine and cosine: product rule and chain rule [1]

Points: (a)[1](E); (b)[1](E)

Compute the first derivative of the following functions.

[Check your results against those in square brackets, where $[a, b]$ stands for $f'(a) = b$.]

(a) $f(x) = x \sin x$ $\left[\frac{\pi}{4}, \frac{1}{\sqrt{2}}\left(1 + \frac{\pi}{4}\right)\right]$ (b) $f(x) = \cos[\pi(x^2 + x)]$ $\left[\frac{1}{2}, -\pi\sqrt{2}\right]$
(c) $f(x) = \frac{1}{7-x^2}$ $\left[3, \frac{3}{2}\right]$ (d) $f(x) = \frac{x-1}{x+1}$ $\left[3, \frac{1}{8}\right]$

Example Problem 3: Differentiation of powers, exponentials, logarithms [2]

Points: [3](E).

Compute the first derivative of the following functions.

[Check your results against those in square brackets, where $[a, b]$ stands for $f'(a) = b$.]

(a) $f(x) = -\frac{1}{\sqrt{2x}}$ $\left[2, \frac{1}{8}\right]$ (b) $f(x) = \frac{x^{1/2}}{(x+1)^{1/2}}$ $\left[3, \frac{1}{16\sqrt{3}}\right]$
(c) $f(x) = e^x(2x-3)$ $[1, e]$ (d) $f(x) = 3^x$ $\left[-1, \frac{\ln 3}{3}\right]$
(e) $f(x) = x \ln x$ $[1, 1]$ (f) $f(x) = x \ln(9x^2)$ $\left[\frac{1}{3}, 2\right]$

Example Problem 4: Elementary integrals [1]

Points: (a)[0,5](E); (b)[0,5](E)

Compute the following integrals. [Check your results: (a) $I(2) = \frac{15}{2}$; (b) $I(\ln 2) = \frac{7}{3}$.]

(a) $I(x) = \int_1^x dy(2y^3 - 2y + 3),$ (b) $I(x) = \int_0^x dy e^{3y}.$

Homework Problem 1: Differentiation of polynomials [1]

Points: (a)[0,5](E); (b)[0,5](E).

Compute the first and second derivatives of the following polynomials. [Check your results against those in square brackets, where $[a; b, c]$ stands for $f'(a) = b$, $f''(a) = c$.]

(a) $f(x) = 4x^5 - x^3 + 2$ $[\frac{1}{2}; \frac{1}{2}, 7]$ (b) $f(x) = x^3 - 2x^2 - x + 9$ $[3; 14, 14]$

Homework Problem 2: Derivatives involving powers, sine and cosine: product rule and chain rule [2]

Points: (a)[1](E); (b)[1](E)

Compute the first derivative of the following functions.

[Check your results against those in square brackets, where $[a, b]$ stands for $f'(a) = b$.]

(a) $f(x) = (x + \frac{1}{\pi}) \sin[\pi(x + \frac{1}{4})]$ $[0, \sqrt{2}]$ (b) $f(x) = -x^2 \cos(\pi x)$ $[\frac{1}{3}, -\frac{1}{3} + \frac{\pi}{6\sqrt{3}}]$
(c) $f(x) = \cos[\pi \sin(x)]$ $[\frac{\pi}{6}, -\frac{\sqrt{3}}{2}\pi]$ (d) $f(x) = -\cos^4(\frac{3}{\pi}x^2 - x)$ $[\frac{\pi}{2}, 2]$
(e) $f(x) = \frac{1}{x^3 - 2x^2}$ $[3, -\frac{5}{27}]$ (f) $f(x) = \frac{x^2 - 2}{x^2 + 1}$ $[2, \frac{12}{25}]$

Homework Problem 3: Differentiation of powers, exponentials, logarithms [2]

Points: [2](E) (Solve any 4 subproblems; beyond that: 0.25 bonus per subproblem.)

Compute the first derivative of the following functions.

[Check your results against those in square brackets, where $[a, b]$ stands for $f'(a) = b$.]

(a) $f(x) = \sqrt[3]{x^2}$ $[8, \frac{1}{3}]$ (b) $f(x) = \frac{x}{(x^2 + 1)^{1/2}}$ $[1, \frac{1}{\sqrt{8}}]$
(c) $f(x) = -e^{(1-x^2)}$ $[1, 2]$ (d) $f(x) = 2^{x^2}$ $[1, 4 \ln 2]$
(e) $f(x) = 2 \frac{\sqrt{\ln x}}{x}$ $[e, -\frac{1}{e^2}]$ (f) $f(x) = \ln \sqrt{x^2 + 1}$ $[1, \frac{1}{2}]$

Homework Problem 4: Elementary integrals [1]

Points: (a)[0,5](E); (b)[0,5](E)

Compute the following integrals. [Check your results: (a) $I(6) = \ln 2$; (b) $I(\ln 9) = \frac{4}{3}$.]

(a) $I(x) = \int_0^x dy \frac{1}{2y+4},$ (b) $I(x) = \int_0^x dy \sinh(\frac{1}{2}y).$