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Sheet 00: Differentiation and Integration

Solution Example Problem 1: Differentiation of polynomials [1]

$$\begin{aligned} \text{(a)} \quad f'(x) &= 9x^2 + 2, & f''(x) &= 18x. \\ \text{(b)} \quad f'(x) &= 4x^3 - 4x, & f''(x) &= 12x^2 - 4. \end{aligned}$$

Solution Example Problem 2: Derivatives involving powers, sine and cosine: product rule and chain rule [1]

Using $\sin' x = \cos x$, $\cos' x = -\sin x$, and the product and chain rules, we obtain:

$$\begin{aligned} \text{(a)} \quad f'(x) &= \sin x + x \cos x & \text{(b)} \quad f'(x) &= -\sin[\pi(x^2 + x)]\pi(2x + 1) \\ \text{(c)} \quad f'(x) &= \frac{2x}{(7 - x^2)^2} & \text{(d)} \quad f'(x) &= \frac{1}{x+1} - \frac{x-1}{(x+1)^2} = \frac{2}{(x+1)^2} \end{aligned}$$

Solution Example Problem 3: Differentiation of powers, exponentials, logarithms [2]

$$\begin{aligned} \text{(a)} \quad f'(x) &= \frac{1}{2\sqrt{2x^3}} & \text{(b)} \quad f'(x) &= \frac{1}{2} \frac{1}{x^{1/2}(x+1)^{1/2}} - \frac{1}{2} \frac{x^{1/2}}{(x+1)^{3/2}} = \frac{1}{2} \frac{1}{x^{1/2}(x+1)^{3/2}} \\ \text{(c)} \quad f'(x) &= e^x(2x - 1) & \text{(d)} \quad f'(x) &= \frac{d}{dx} e^{\ln 3^x} = \frac{d}{dx} e^{x \ln 3} = e^{x \ln 3} \ln 3 = 3^x \ln 3 \\ \text{(e)} \quad f'(x) &= \ln x + \frac{x}{x} = \ln x + 1 & \text{(f)} \quad f'(x) &= \ln(9x^2) + x \frac{1}{9x^2} 18x = \ln(9x^2) + 2 \end{aligned}$$

Solution Example Problem 4: Elementary integrals [1]

$$\begin{aligned} \text{(a)} \quad I(x) &= \int_1^x dy(2y^3 - 2y + 3) = \left[\frac{1}{2}y^4 - y^2 + 3y \right]_1^x = \boxed{\frac{1}{2}x^4 - x^2 + 3x - \frac{5}{2}}. \\ \text{(b)} \quad I(x) &= \int_0^x dy e^{3y} = \left[\frac{1}{3}e^{3y} \right]_0^x = \boxed{\frac{1}{3}(e^{3x} - 1)}. \end{aligned}$$

[Total Points for Example Problems: 5]
