https://www2.physik.uni-muenchen.de/lehre/vorlesungen/wise_24_25/TA1_theoretical_condensed_matter/index.html

Problem set 6

Problem 1 Dynamic conductivity in the Drude model

We want to compute the Drude conductivity $\sigma_D(\omega)$ for the case of an oscillatory electric field $\vec{E} = \vec{E}_0 \mathrm{e}^{\mathrm{i}\omega t}$ with frequency ω. Show that it has the form

$$\sigma_D(\omega) = \frac{ne^2\tau}{m_e}\frac{1}{1 - \mathrm{i}\omega\tau} \; . \tag{1}$$

Why can $\sigma_D(\omega)$ be complex, what is the physical reason behind this fact? Show that $\sigma_D(\omega)$ becomes independent on τ in the limit $\omega \to \infty$ and give a physical explanation for this observation.

Problem 2 A handy theorem

Let $\hat{H}(\lambda)$ be the Hamiltonian of a system, which is parametrized by λ, and $|n(\lambda)\rangle$ its (non-degenerate) eigenstates. Show that

$$\frac{\partial E(\lambda)}{\partial \lambda} = \langle n(\lambda)|\frac{\partial \hat{H}}{\partial \lambda}|n(\lambda)\rangle \; , \tag{2}$$

with $E(\lambda) = \langle n(\lambda)|\hat{H}(\lambda)|n(\lambda)\rangle$.

Problem 3 Singularities everywhere

For a d-dimensional cubic lattice model with lattice constant a, we consider a dispersion relation of the form

$$\varepsilon_{\vec{k}} = -2t\sum_{\mu=1}^{d}\cos(k_\mu a) \; , \tag{3}$$

where k_μ are the spatial components of $\vec{k}$.

(3.a) Compute the density of states for $d = 1$ and determine the nature of the Van Hove singularities. Use eq. (2) to find those regions in momentum space, in which the velocity is (anti-)parallel to the momentum.

(3.b) For the case $d = 2$, determine the energy $\tilde{\varepsilon}$, which exhibits a Van Hove singularity. Expand the density of states in the vicinity of $\tilde{\varepsilon}$ to the leading order in $\varepsilon_{\vec{k}}$.