https://www2.physik.uni-muenchen.de/lehre/vorlesungen/wise_24_25/TA1_theoretical_condensed_matter/index.html

Problem set 13

Problem 1 Let there be quasi particles

Consider a system of fermions with annihilation (creation) operators $\hat{c}_{\vec{k}}^{(\dagger)}$ where $\vec{k}$ labels single-particle eigenstates of a non-interacting Hamiltonian $\hat{h}_0$. Now an interaction $\hat{V}(t)$ is switched on adiabatically with $\hat{V}(-\infty) = 0$ and $\hat{V}(0) = \hat{V}$. We want to establish the notion of quasi particles, i.e., if $|\Psi_0\rangle$ denotes the ground state of the unperturbed N-particle ground state, the operators $\hat{a}_{\vec{k}}^{\dagger} = \hat{U}(0,-\infty)\hat{c}_{\vec{k}}^{\dagger}\hat{U}^{\dagger}(0,-\infty)$ and their adjoints describe fermionic quasi-particle excitations. Here, $\hat{U}(t_1, t_0)$ denotes the time-evolution operator evolving a state from t_0 to t_1.

(1.a) Show that for a time evolution satisfying the adiabatic theorem, the N-particle ground state of the interacting system described by $\hat{h}_0 + \hat{V}$ is given by $|\Phi\rangle = \hat{U}(0,-\infty)\,|\Psi_0\rangle$.

(1.b) Show that the $N \pm 1$ particle states $\hat{a}_{\vec{k}}^{(\dagger)}\,|\Phi\rangle$ describe single-particle excitations above $|\Psi_0\rangle$, after adiabatically switching on the interaction.

Problem 2 Landau parameters

Now we consider a weakly interacting system of electrons. We denote non-interacting electronic annihilation (creation) operators by $\hat{c}_{\vec{k},\sigma}^{(\dagger)}$ and the corresponding single-particle energies by $\varepsilon_{\vec{k},\sigma}^{0}$. Use first order perturbation theory to compute the Landau interaction parameters introduced in the lecture for the perturbed Hamiltonian

$$\hat{H} = \sum_{\vec{k},\sigma} \varepsilon_{\vec{k},\sigma}^{0}\hat{n}_{\vec{k},\sigma} + \frac{\lambda}{2}\sum_{\substack{\vec{k},\sigma\\ \vec{k}',\sigma'}}\sum_{\vec{q}} V(\vec{q})\hat{c}_{\vec{k}-\vec{q},\sigma}^{\dagger}\hat{c}_{\vec{k}'+\vec{q},\sigma'}^{\dagger}\hat{c}_{\vec{k}',\sigma'}\hat{c}_{\vec{k},\sigma}\,. \tag{1}$$

Here, $V(\vec{q})$ denotes the Fourier transform of the interaction potential and $\lambda \ll 1$ parametrizes the perturbation strengths.