LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Fakultät für Physik im WiSe 2023/24
TA1: Condensed matter physics
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https://www2.physik.uni-muenchen.de/lehre/vorlesungen/wise_24_25/TA1_theoretical_condensed_matter/index.html

Problem set 10

Problem 1 Less-classical equations of motion for Bloch electrons packets

In the lecture we already saw a semi-classical derivation of the equations of motion for the effective wave vector $\vec{K}$ of wave packets constructed from Bloch electrons in the presence of electromagnetic fields

$$\hbar\frac{d\vec{K}}{dt} = -e\left[\vec{E}(\vec{R},t) + \frac{\vec{v}(\vec{K})}{c}\times\vec{B}(\vec{R},t)\right] , \tag{1}$$

based on expanding the vector potential $\vec{A}(\hat{\vec{r}})$. In this problem, we will use a less "classical" yet more restricted approach to illustrate the crucial requirements leading to eq. (1). Consider a single-particle Hamiltonian

$$\hat{h} = \varepsilon\left((\hat{\vec{p}} + (e/c)\vec{A}(\hat{\vec{R}}))/\hbar\right) + U(\vec{R}) , \tag{2}$$

where the function $\varepsilon(\vec{k})$ denotes the dispersion relation, $U(\hat{R})$ a potential and $\hat{\vec{p}}, \hat{\vec{R}}$ are operators with canonical commutation relations $[\hat{R}_\mu, \hat{p}_\nu] = \mathrm{i}\hbar\delta_{\mu\nu}$.

Use the Heisenberg equations of motion and the identification $\hbar\vec{K} = \hat{\vec{p}} + (e/c)\vec{A}(\hat{\vec{R}})$ to show eq. (1) for this Hamiltonian.

Problem 2 Holes change signs

We consider a single-particle dispersion relation approximated near the top of the band $\vec{k}_0$

$$\varepsilon(\vec{k}) \approx \varepsilon(\vec{k}_0) - \frac{\hbar^2}{2m^*}(\vec{k}-\vec{k}_0)^2 , \tag{3}$$

with the effective mass m^*. Show that in this case wave packets constructed from Bloch states where $\vec{K} = \vec{k}_0$ obey the equations of motion

$$\frac{d^2\vec{R}}{dt^2} = +\frac{e}{m^*}\left[\vec{E}(\vec{R},t) + \frac{\vec{v}(\vec{K})}{c}\times\vec{B}(\vec{R},t)\right] . \tag{4}$$

Use eq. (3) to explain why it is consistent to interpret these equations of motion as those of a hole-excitation.