Sheet 00: Differentiation and Integration

Solution Example Problem 1: Differentiation of polynomials [1]

(a) $f'(x) = 9x^2 + 2$, $\qquad\qquad\qquad\qquad\qquad\qquad$ $f''(x) = 18x$.

(b) $f'(x) = 4x^3 - 4x$, $\qquad\qquad\qquad\qquad\qquad\qquad$ $f''(x) = 12x^2 - 4$.

Solution Example Problem 2: Derivatives involving powers, sine and cosine: product rule and chain rule [1]

Using $\sin' x = \cos x$, $\cos' x = -\sin x$, and the product and chain rules, we obtain:

(a) $f'(x) = \sin x + x \cos x$ $\qquad\qquad$ (b) $f'(x) = -\sin\big[\pi(x^2 + x)\big]\pi(2x + 1)$

(c) $f'(x) = \dfrac{2x}{(7 - x^2)^2}$ $\qquad\qquad$ (d) $f'(x) = \dfrac{1}{x + 1} - \dfrac{x - 1}{(x + 1)^2} = \dfrac{2}{(x + 1)^2}$

Solution Example Problem 3: Differentiation of powers, exponentials, logarithms [2]

(a) $f'(x) = \dfrac{1}{2\sqrt{2x^3}}$ $\qquad\qquad$ (b) $f'(x) = \dfrac{1}{2}\dfrac{1}{x^{1/2}(x+1)^{1/2}} - \dfrac{1}{2}\dfrac{x^{1/2}}{(x+1)^{3/2}} = \dfrac{1}{2}\dfrac{1}{x^{1/2}(x+1)^{3/2}}$

(c) $f'(x) = e^x(2x - 1)$ $\qquad\qquad$ (d) $f'(x) = \dfrac{d}{dx}e^{\ln 3^x} = \dfrac{d}{dx}e^{x\ln 3} = e^{x\ln 3}\ln 3 = 3^x \ln 3$

(e) $f'(x) = \ln x + \dfrac{x}{x} = \ln x + 1$ $\qquad$ (f) $f'(x) = \ln(9x^2) + x\dfrac{1}{9x^2}18x = \ln(9x^2) + 2$

Solution Example Problem 4: Elementary integrals [1]

(a) $I(x) = \displaystyle\int_1^x dy(2y^3 - 2y + 3) = \left[\tfrac{1}{2}y^4 - y^2 + 3y\right]_1^x = \boxed{\tfrac{1}{2}x^4 - x^2 + 3x - \tfrac{5}{2}}$.

(b) $I(x) = \displaystyle\int_0^x dy\, e^{3y} = \left[\tfrac{1}{3}e^{3y}\right]_0^x = \boxed{\tfrac{1}{3}(e^{3x} - 1)}$.

[Total Points for Example Problems: 5]