

Inverse problems and machine learning in medical physics

Tomographic image reconstruction for ion imaging

Dr. Chiara Gianoli

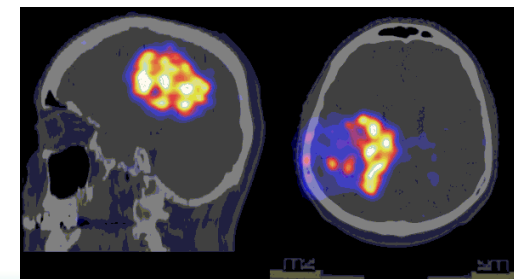
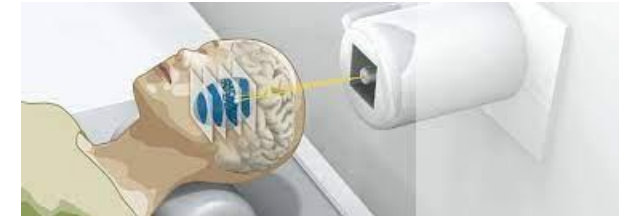
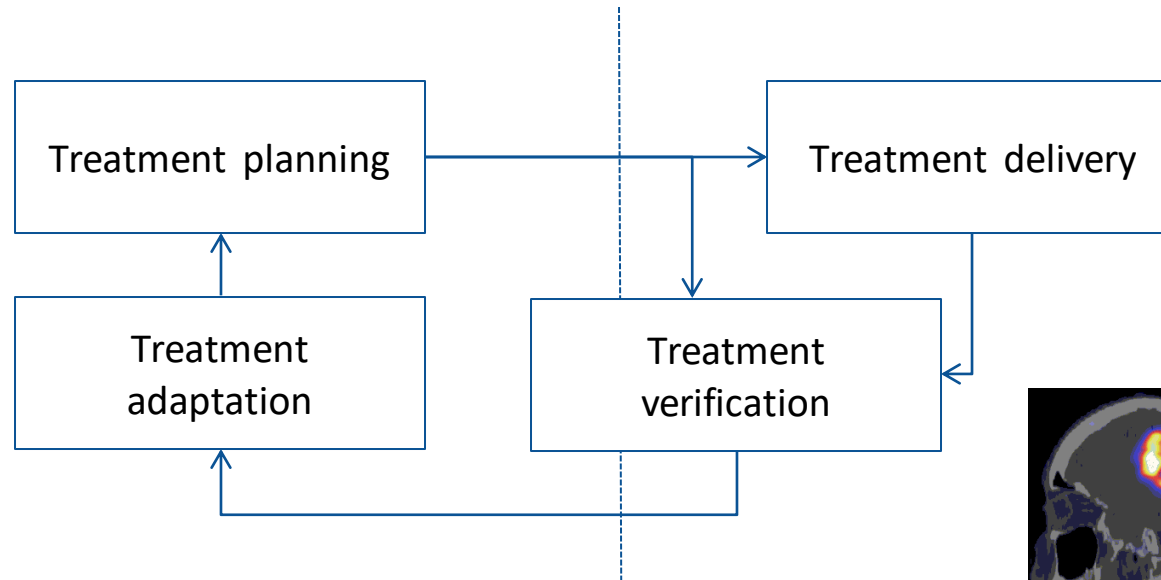
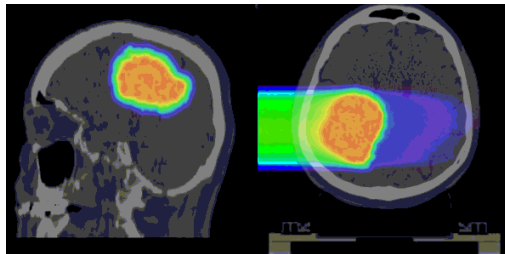
28/11/2023

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Ion imaging in ion beam therapy



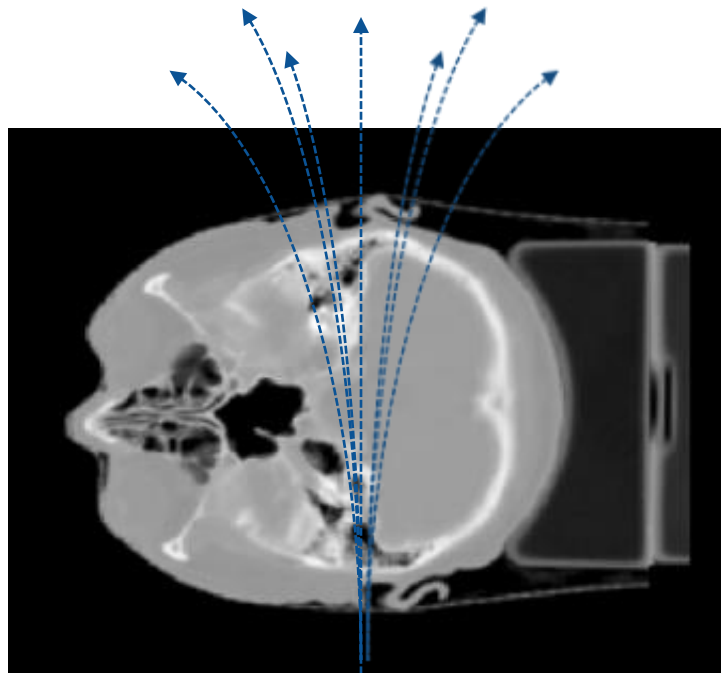
- Ion imaging can in principle enable direct assessment of the tissue stopping power and its variations due to inter-fractional anatomical changes
- Ion imaging can thus play a role not only as imaging technique for **treatment planning** but also for **treatment verification** and **adaptation** in ion beam therapy



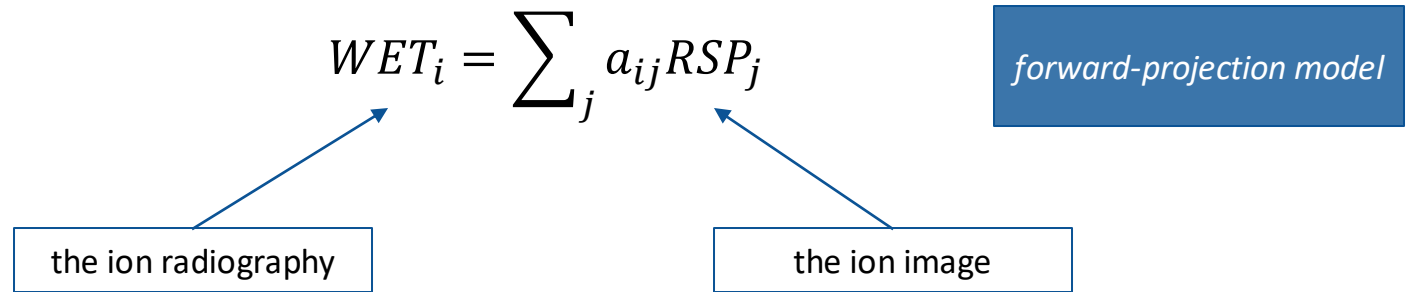
Ion imaging model



- Ion imaging offers the promise of eliminating these inaccuracies by measuring the **water equivalent thickness (WET)** and reconstructing the **relative stopping power (RSP)** of the object of interest
- In ion imaging, the **forward-projection model** describes the measured WET of the traversed object of interest as an **integral** of the RSP along a certain concept of **ion trajectory** that depends on the detector configuration



single ion or pencil beam

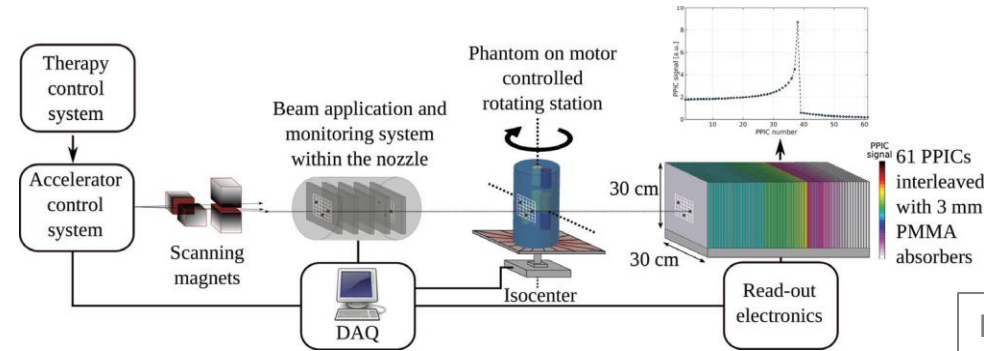
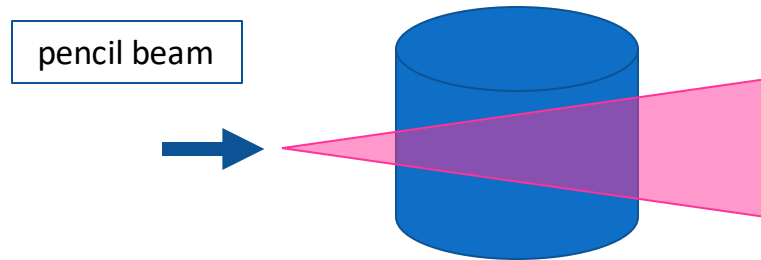


- The a_{ij} is the coefficient of the system matrix that describes the intersection length/area/volume of the trajectory i (single ion or pencil beam) with the voxel j

Ion imaging detectors

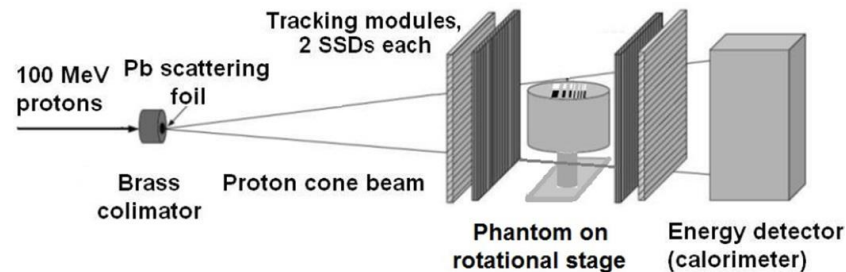
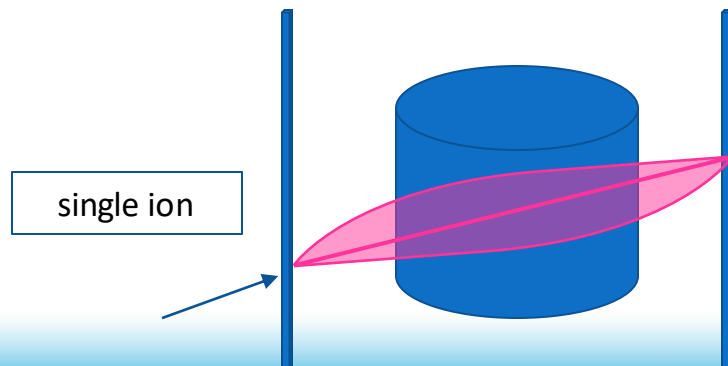


- Detectors are mainly distinguished in **list-mode** and **integration-mode** configurations
- For **integration-mode** detectors, the concept of ion trajectory can be statistically described by the multiple Coulomb scattering model



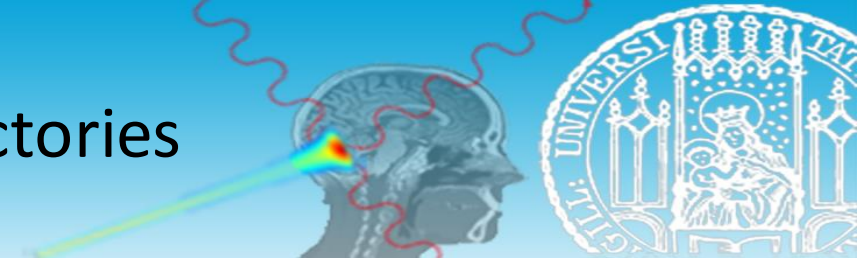
Rinaldi et al. 2013 *Phys. Med. Biol.*
Meyer, Gianoli, ... et al. 2017 *Phys. Med. Biol.*

- For **list-mode** detectors, the concept of ion trajectory can be statistically described by combining the multiple Coulomb scattering model and the Bayesian inference, as the **most likely path** algorithm



Bashkirov et al. 2016 *Nucl. Instrum. Methods Phys. Res. A*

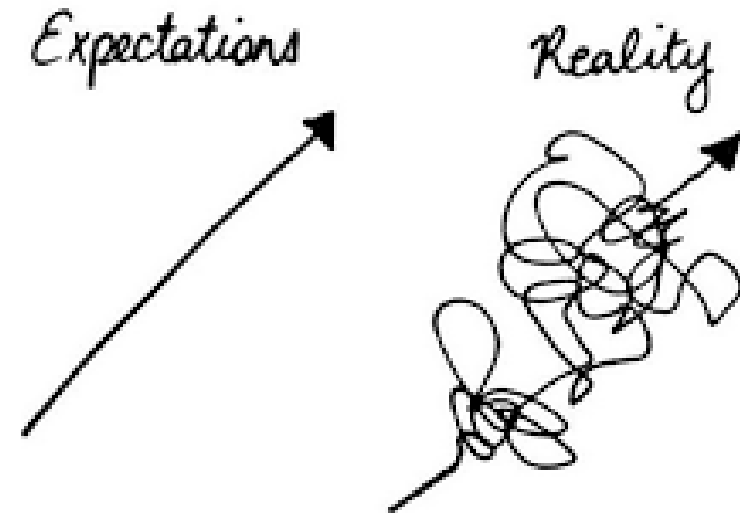
Detector configuration and ion trajectories



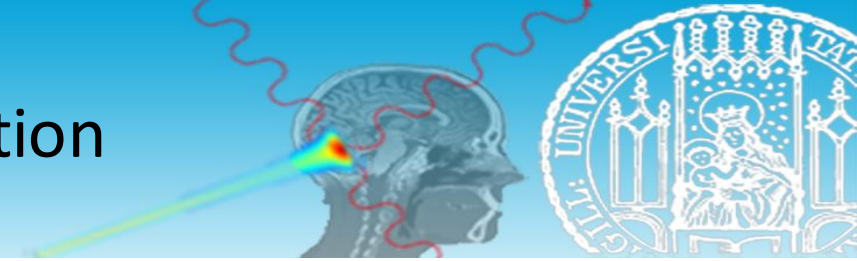
- The concept of ion trajectory for different detector configuration plays a crucial role in the forward-projection model, which is a foundation in ion imaging

$$\overrightarrow{WET} \neq A * \overrightarrow{RSP}_t \quad (t \text{ for "ground truth"})$$

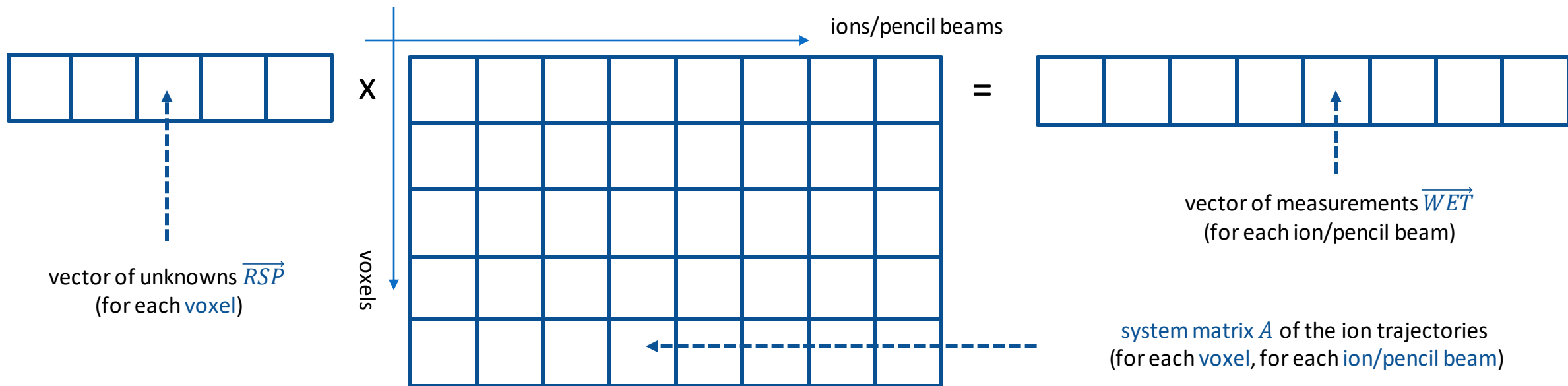
- However, in clinical scenarios the **intrinsic inconsistencies** of the forward-projection model are in the same order of magnitude of the **inaccuracies** of the semi-empirical calibration of the X-ray CT
 - Relying on Monte Carlo simulations, the normalized root mean square error between the **ion radiography** and the **forward-projection of the ground truth ion CT image** is 1-2.5% for **list-mode detector configuration** and up to 2.5-5% for **integration-mode detector configuration**



Tomographic image reconstruction

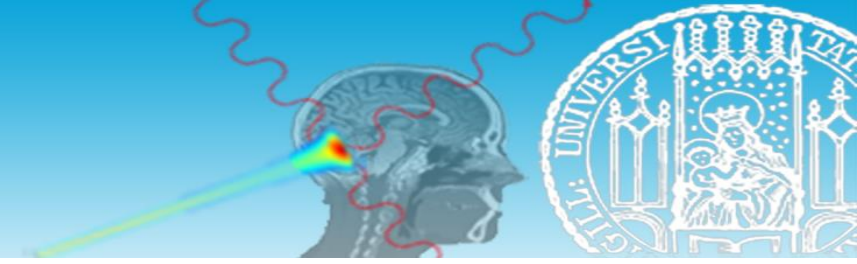


- Tomographic image reconstruction is applied to several ion radiographies, with projection angles covering 180°
- The ordered subsets simultaneous algebraic reconstruction technique (OS-SART) coupled with total variation superiorization currently represents the state-of-the-art in ion imaging^{1,2,3}
- Information redundancy mitigates the intrinsic inaccuracies of the forward-projection model⁴

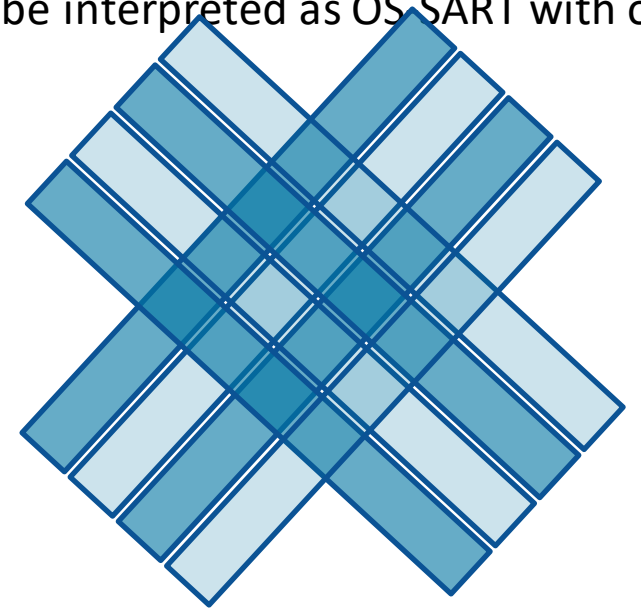
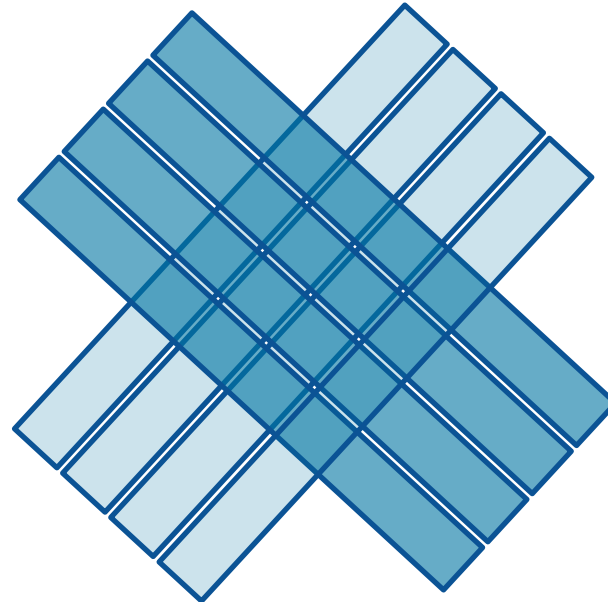


¹Penfold et al. 2010 *Med. Phys.* ²Meyer et al. 2019 *Phys. Med. Biol.* ³Meyer et al. 2021 *Phys. Med. Biol.*
⁴Gianoli et al. 2019 *Phys. Med.*

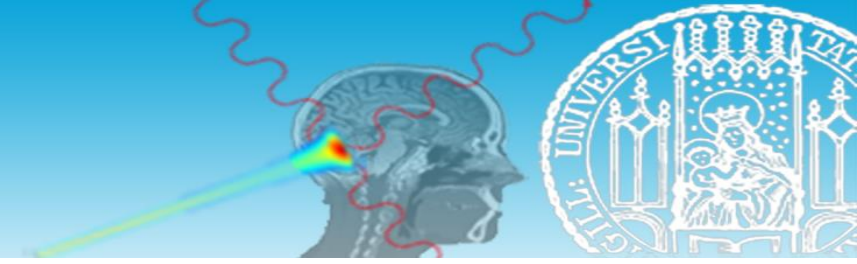
Ordered Subsets



- The ordered subsets (OS) approach is introduced to **accelerate** numerical image reconstruction and **reduce** the memory requirement for reconstruction
- In OS approach, instead of accessing all projections simultaneously for updating the image, the image is updated relying on a **subset of projections**
- ART can be interpreted as OS-SART with only one projection per subset and SART can be interpreted as OS-SART with only one subset
 - An update performed using a **single subset** is called a **sub-iteration**
 - An **iteration** is completed when **all subsets** have been processed once
 - The **convergence acceleration** is expressed in terms of number of iterations (not sub-iterations)



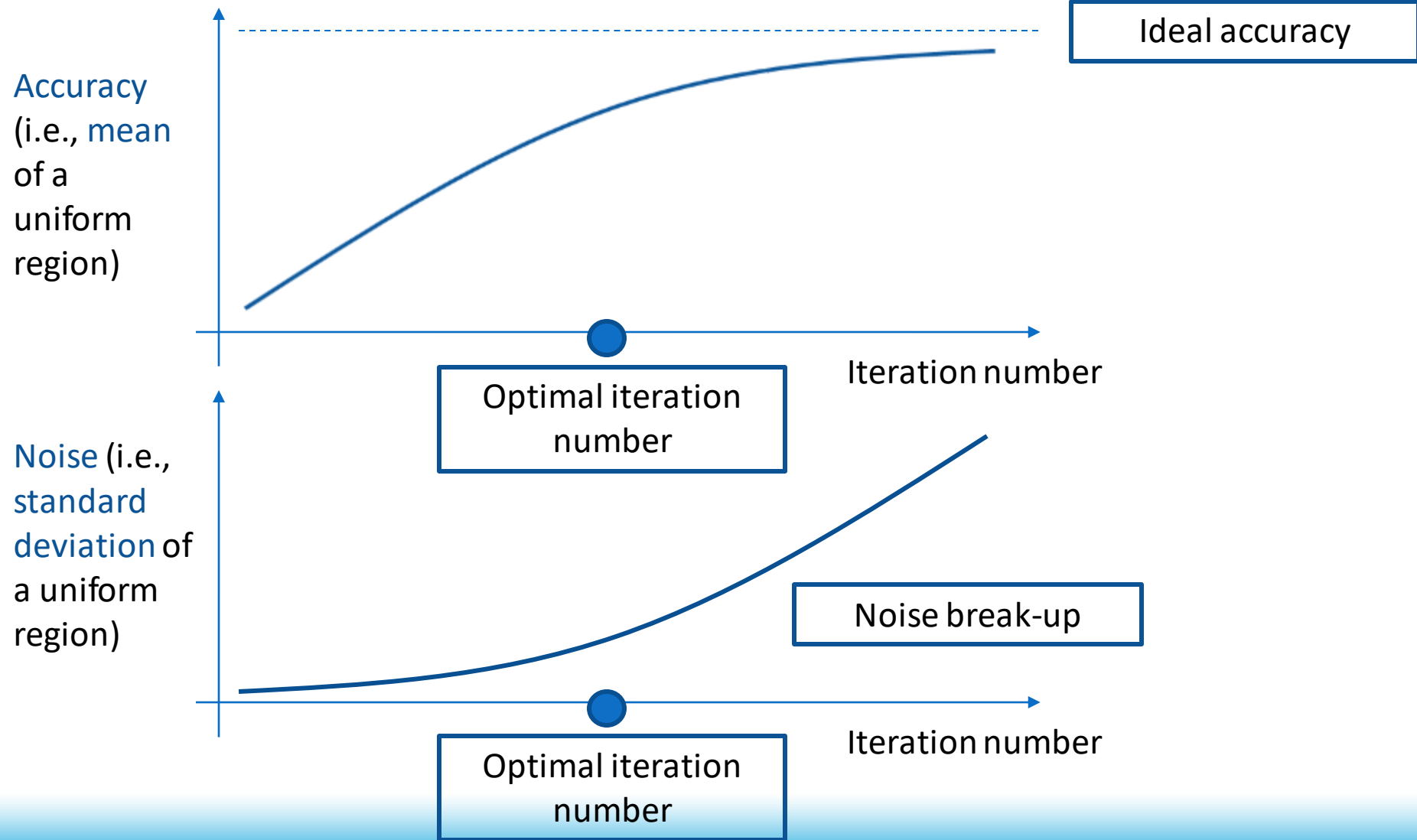
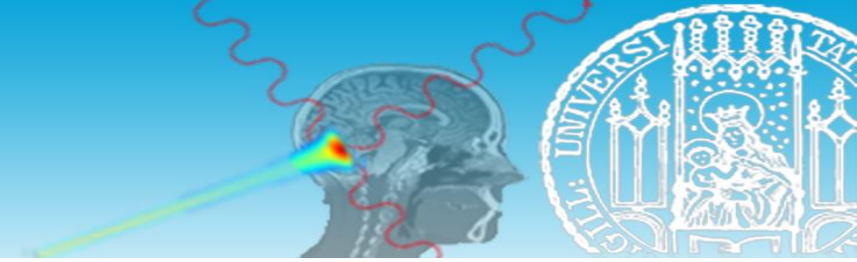
Ordered Subsets



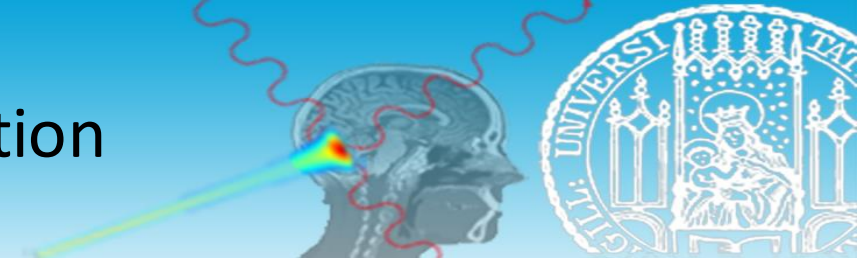
- The idea of was originally proposed for **emission tomography** and then transferred to **transmission tomography** (the SART and the ML-EM produce the maximum likelihood estimate in the Gaussian and Poisson data, respectively)
- The subset of projections $S(s)$ is employed for updating of the image, and this update, together with a different subset of projections $S(s + 1)$, is then used for calculating the next update

$$\vec{f}^{(s+1)} = \vec{f}^{(s)} - \frac{\sum_{i \in S(s)} \left(a_{ij} \cdot \left(\frac{\vec{f}^{(s)} \cdot \vec{a}_{ij}^T - \vec{g}}{\sum_j a_{ij}} \right) \right)}{\sum_i a_{ij}}$$

- The best ordering of the subsets is defined according to the **maximum angular distance** (“as orthogonal as possible”) from the previously used projections
 - This ordering further accelerates convergence as compared to **sequential** or **random** orderings
 - The **increased convergence speed** (in function of the number of iterations) and the **reduced memory requirement** (due to a reduced dimension of the system matrix) comes at the cost of an **increased noise** of the reconstructed image



Tomographic image reconstruction



- Because of the **intrinsic inconsistencies** of the forward-projection model, the optimal solution minimizing the **objective function** of the tomographic image reconstruction algorithm is not necessarily the solution that best reconstructs the ground truth image
- The fundamental approach for the mitigation of the **intrinsic inconsistencies** of the forward-projection model is to stop the tomographic image reconstruction algorithm before the solution diverges (albeit with a lower objective function), thus being referred to as **semi-convergence**
- Another approach is to use **superiorization** techniques to shift the solution at each iteration to one that is superior to the current solution
 - A superior solution is defined in terms of a certain **merit function** φ

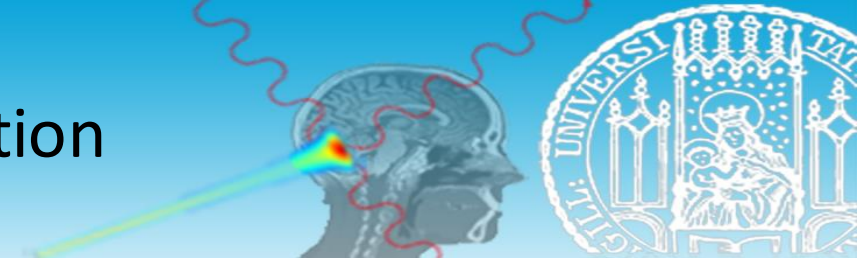
algorithm

$$x^{k+1} = f(x^k)$$

superiorized algorithm

$$x^{k+1} = f(x^k + \beta_k v^k) \quad \text{so that} \quad \varphi(x^k + \beta_k v^k) \leq \varphi(x^k)$$

Tomographic image reconstruction



- In transmission imaging (i.e., ion imaging), the merit function φ is typically the total variation
- For a two-dimensional (2D) image representation in i and j of the image vector x^k is defined as:

$$\varphi(x^k) = \sum_i \sum_j \sqrt{(x_{i+1,j}^k - x_{i,j}^k)^2 + (x_{i,j+1}^k - x_{i,j}^k)^2}$$

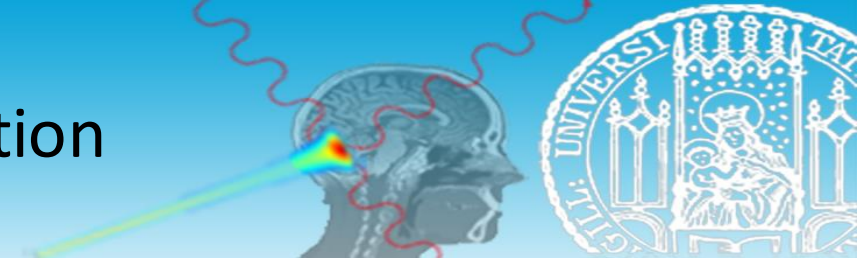
$x_{i-1,j-1}$	$x_{i,j-1}$	$x_{i+1,j-1}$
$x_{i-1,j}$	$x_{i,j}$	$x_{i+1,j}$
$x_{i-1,j+1}$	$x_{i,j+1}$	$x_{i+1,j+1}$

- $\beta_k v^k$ is the perturbation term, with β_k real non-negative numbers (typically $0 < \beta_k < 1$) and v^k the perturbation vector, typically defined as non-ascending direction of the merit function φ at x^k

$$v^k = -\frac{\nabla\varphi(x^k)}{\|\nabla\varphi(x^k)\|_2} = -\frac{s^k}{\|s^k\|}$$

- The perturbation vector is calculated as the negative of the normalized sub-gradient s^k (generalized concept of derivative for convex functions which are not necessarily differentiable) of the total variation $\varphi(x^k)$ for the image vector x^k

Tomographic image reconstruction



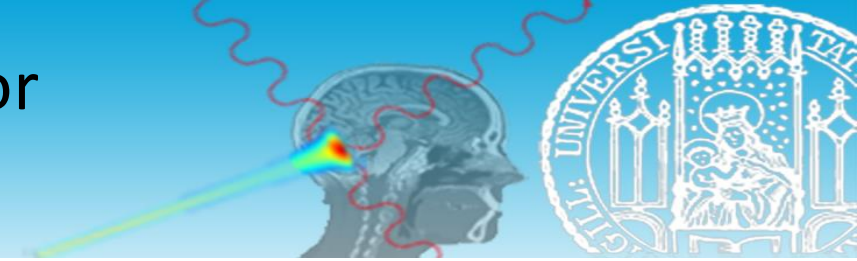
- Another approach is to use **regularization** technique to add a **penalty function** to the objective function
 - The penalty function enforces a desired feature on the reconstructed image

$$x_{\min} = \operatorname{argmin}_x F(x) + \lambda \phi(x)$$

- $\phi(x)$ is the **penalty function** (i.e., **total variation** or smoothness-related functions) and λ is a parameter controlling its weighting
- The regularization changes the problem!
- The optimal value of λ depends on the level of noise in the data
 - λ too small under-regularizes (i.e., not substantially improve image quality)
 - λ too large over-regularizes, resulting typically in an oversmoothed image

Defrise, M., Vanhove, C., & Liu, X. (2011). An algorithm for total variation regularization in high-dimensional linear problems. *Inverse Problems*, 27(6), 065002.

Tomographic image reconstruction for list-mode detector configuration

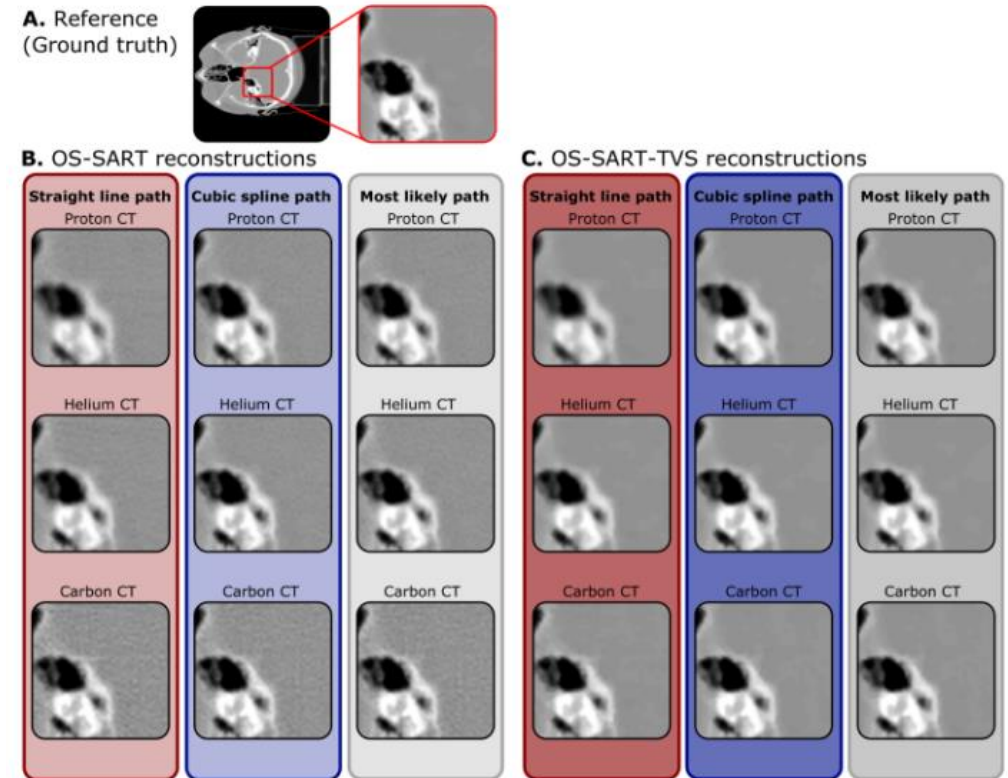


- The superiorization of the algorithm for tomographic image reconstruction introduces a “perturbation” of the solution in tomographic domain in order to reduce, and not necessarily minimize, a merit function φ (i.e., the total variation)
- The superiorization changes the algorithm by adding a shifting step but it does not change the problem!

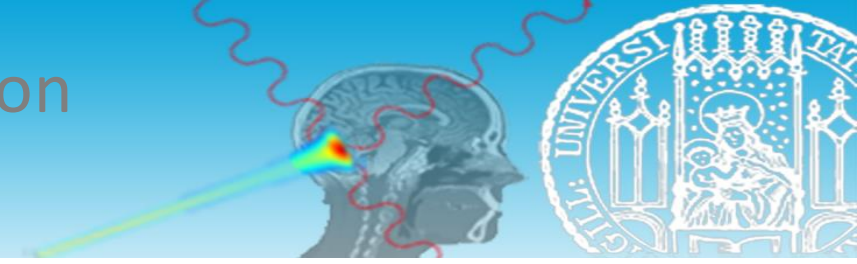
```

set  $k = 0, l = 0, \beta_k = 1$ ;
while repeat for each subset over the requested number of cycles do
  if  $\|s^k\| \geq 0$  then
     $v^k = -\frac{s^k}{\|s^k\|}$ ;
  else
     $v^k = s^k$ ;
  end
  while loop do
    set  $y^k = x^k + \beta_k v^k$ ;
    if  $\Phi(y^k) \leq \Phi(x^k)$  then
       $x^k = y^k$ ;
      set loop = false;
    end
     $\beta_k = \beta_k / 2$ ;
  end
   $x^{k+1} = P_Q x^k$ ;
   $k = k + 1$ ;
end
  
```

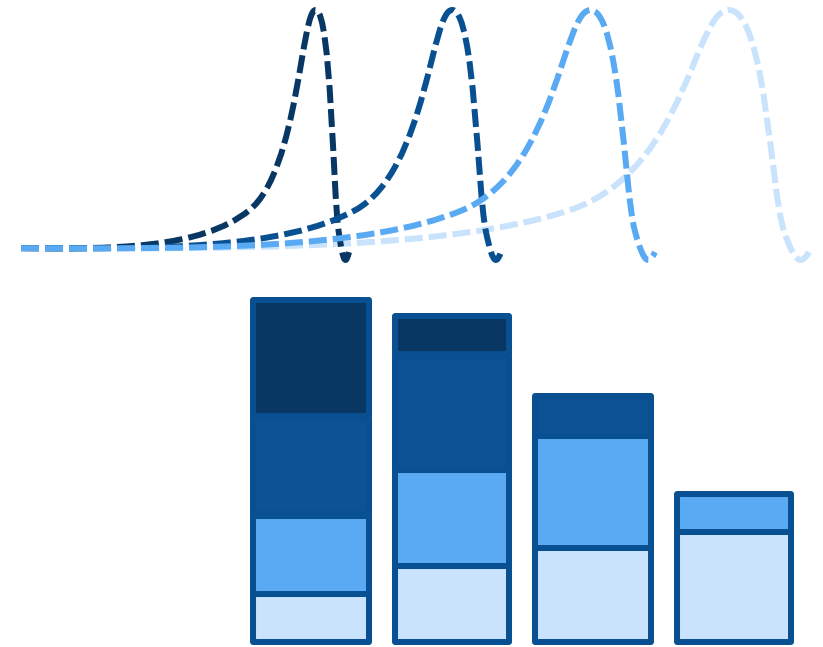
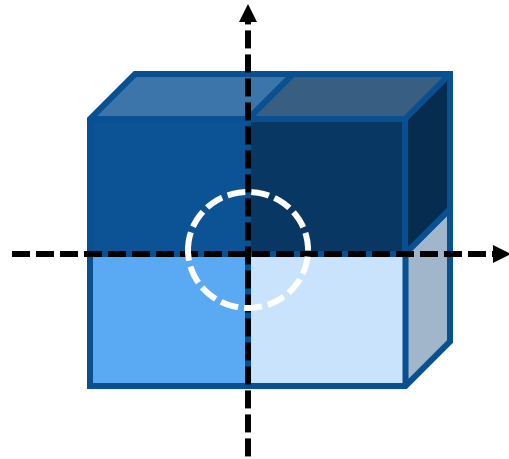
P_Q is the operator that updates the OS-SART algorithm



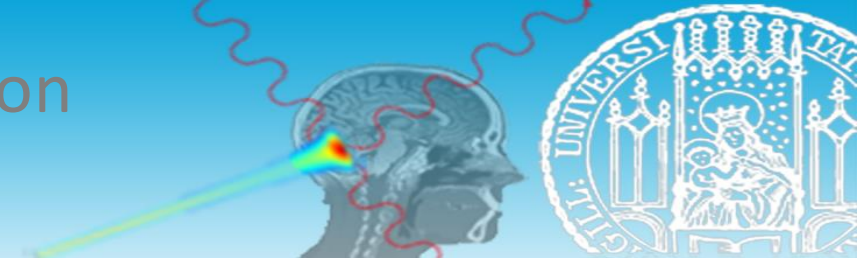
Integration-mode detector configuration for pencil beams



- In integration-mode detector configuration, the Bragg peak signal for each pencil is discretized according to the multiple layers (i.e., channels) or according to the multiple initial energies in a single layer
- Due to lateral inhomogeneity traversed by the pencil beam, the Bragg peak signal results in a **linear combination** of elementary Bragg peak signals



- The Bragg peak of the component with the larger WET (i.e., the shorter range) takes advantages from the Bragg peaks of the components with smaller WET

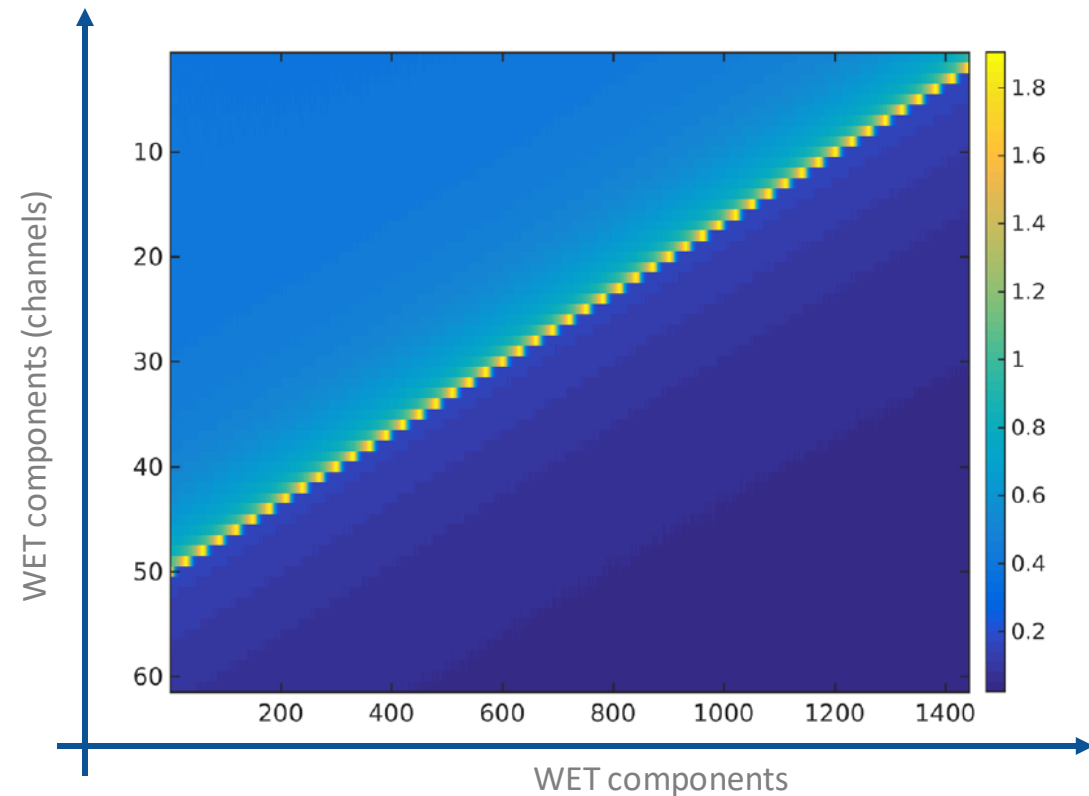


- Linear decomposition^{1,2} (inverse problem) is applied to retrieve the WET histogram as WET occurrence for each WET component by solving the system of linear equations $\overrightarrow{BP} = LUT * \overrightarrow{WET}$

- \overrightarrow{BP} is the discretized Bragg peak signal
 - \overrightarrow{WET} is the unknown vector of WET occurrences
 - LUT is the look-up-table of individual Bragg peak signals for each WET component
- The least square optimization is based on Euclidean distance minimization

$$\operatorname{argmin}_{\overrightarrow{WET}} \frac{1}{2} \left\| LUT * \overrightarrow{WET} - \overrightarrow{BP} \right\|_2^2$$

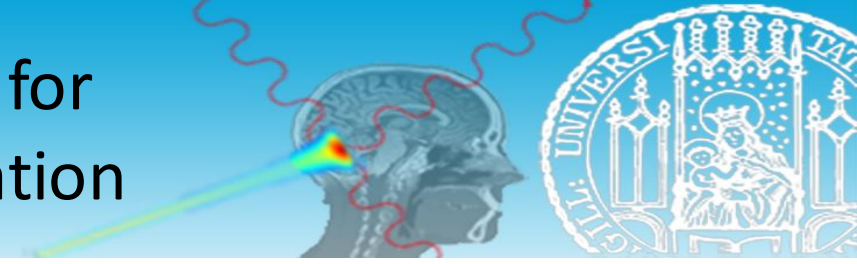
- An histogram of WET occurrences for each WET component is obtained



¹Krah et al. 2015 *Phys. Med. Biol.*

²Meyer et al. 2017 *Phys. Med. Biol.*

Tomographic image reconstruction for integration-mode detector configuration

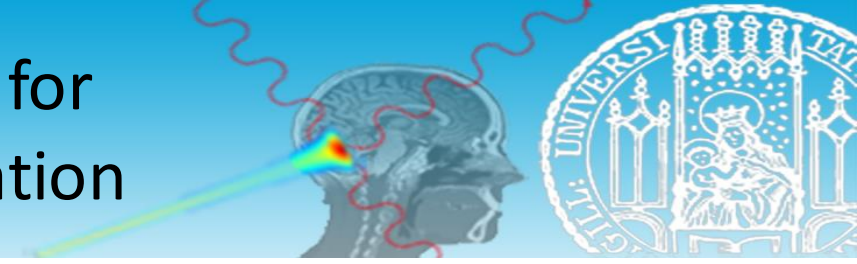


- Defining p_{ik} the WET components and w_{ik} the WET occurrences, the tomographic image reconstruction for integration-mode detector configuration deals with the additional **channel dimension** of the WET histogram (WET_{hist})
- The typical approach, the WET histogram is reduced to a single **WET component**
 - $WET_i = \max\{p_{ik}\}$ the WET component with maximum WET occurrence (WET_{mode} or WET_{max})
 - $WET_i = \text{mean}(p_{ik}w_{ik})$ the weighted averaged WET components (WET_{mean})

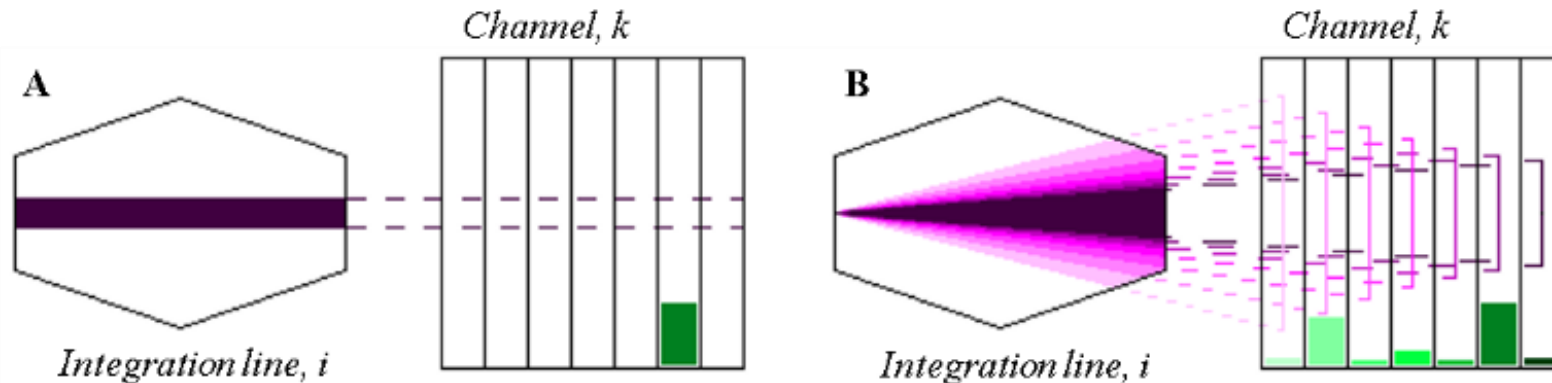
$$WET_i = \sum_j a_{ij} RSP_j$$

- Therefore, the integration line is assumed as straight or coinciding to the **mean ion trajectory** of the pencil beam
 - Analytical or numerical algorithms for tomographic image reconstruction are applied

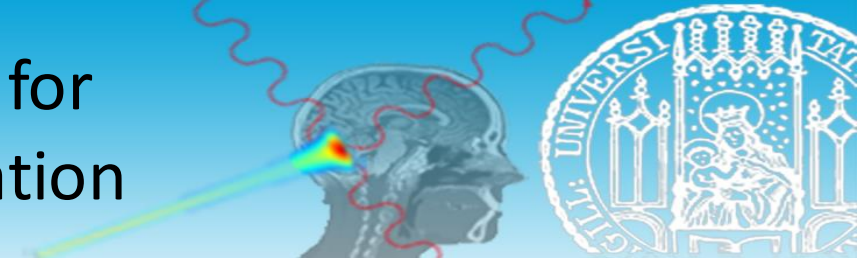
Tomographic image reconstruction for integration-mode detector configuration



- Alternatively, WET_{hist} is handled within **numerical** tomographic image reconstruction (ART&SART)
 - WET components and WET occurrences are entirely exploited
 - The integration line is defined according to the **scattering model** (conical Gaussian) for each **WET component**
 - The WET components are spatially assigned (the *mean ion trajectory* is valid only for WET_{max} and WET_{mean})



Tomographic image reconstruction for integration-mode detector configuration



- The SART algorithm is considered for numerical tomographic image reconstruction

$$f_j^{n+1} = f_j^n + \frac{\sum_i a_{ij} \cdot \frac{g_i - \sum_j a_{ij} \cdot f_j^n}{\sum_j a_{ij}}}{\sum_i a_{ij}}$$

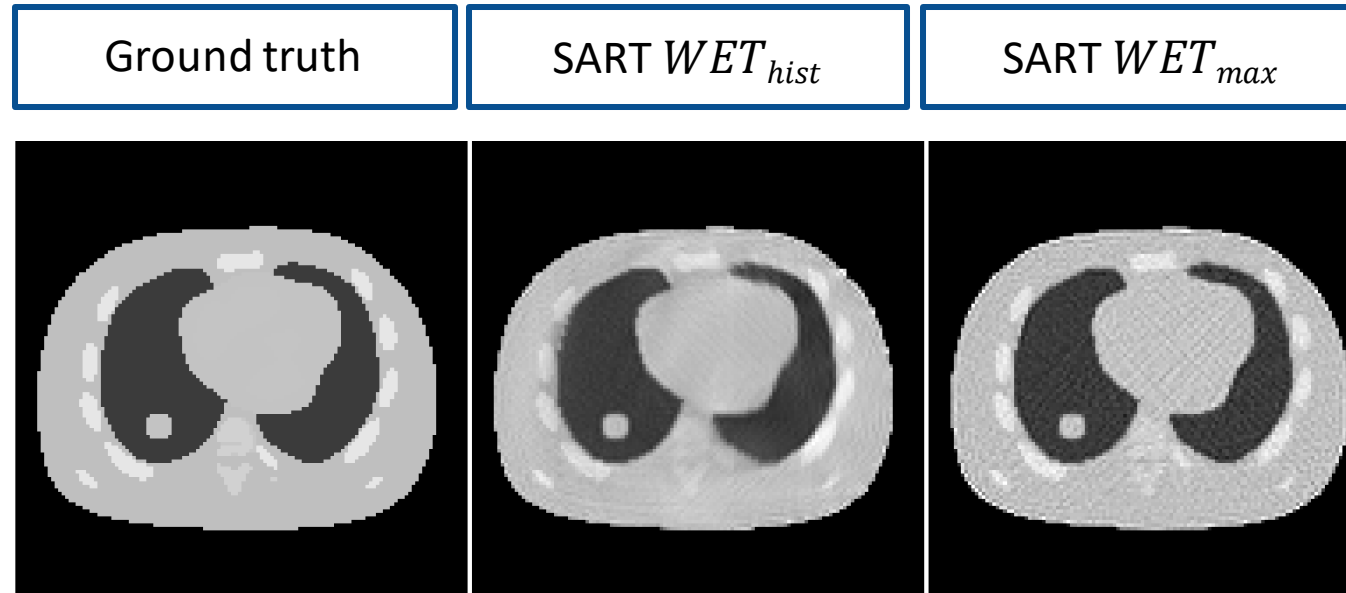
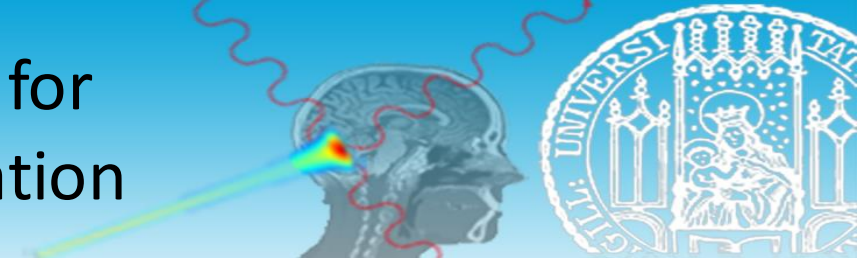
- The SART algorithm is modified to handle the additional channel dimension

$$m_{ik} = \sum_k w_{ik} a_{(ik)j} \quad g_i = \sum_k w_{ik} g_{ik}$$

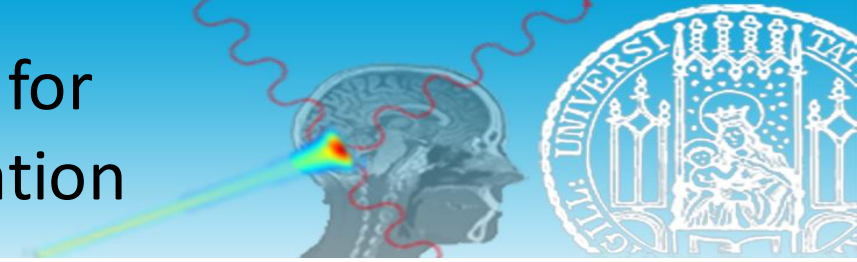
where $a_{(ik)j}$ describe the **conical Gaussian for each channel** (indexing j the pixel/voxel, k the channel and i the measurement)

$$f_j^{n+1} = f_j^n + \frac{\sum_i m_{ij} \cdot \frac{g_i - \sum_j m_{ij} \cdot f_j^n}{\sum_j m_{ij}}}{\sum_i m_{ij}}$$

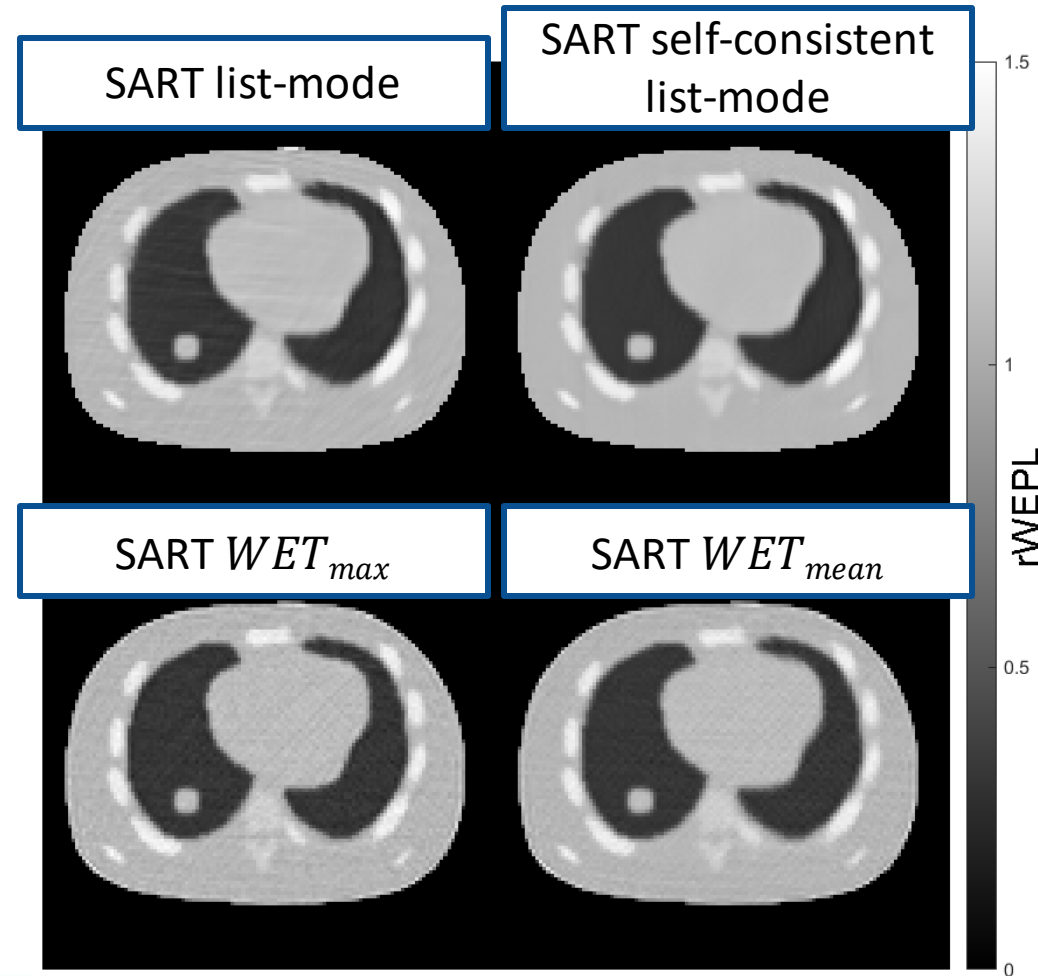
Tomographic image reconstruction for integration-mode detector configuration



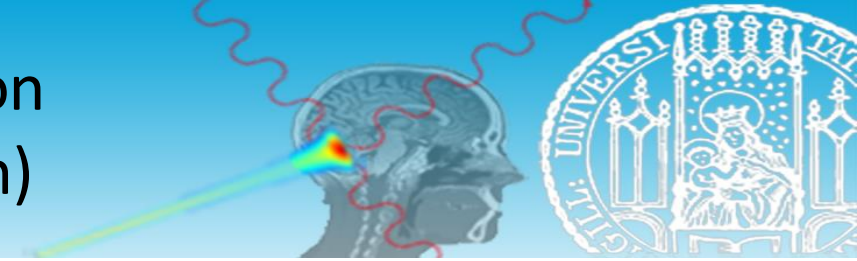
Tomographic image reconstruction for integration-mode detector configuration



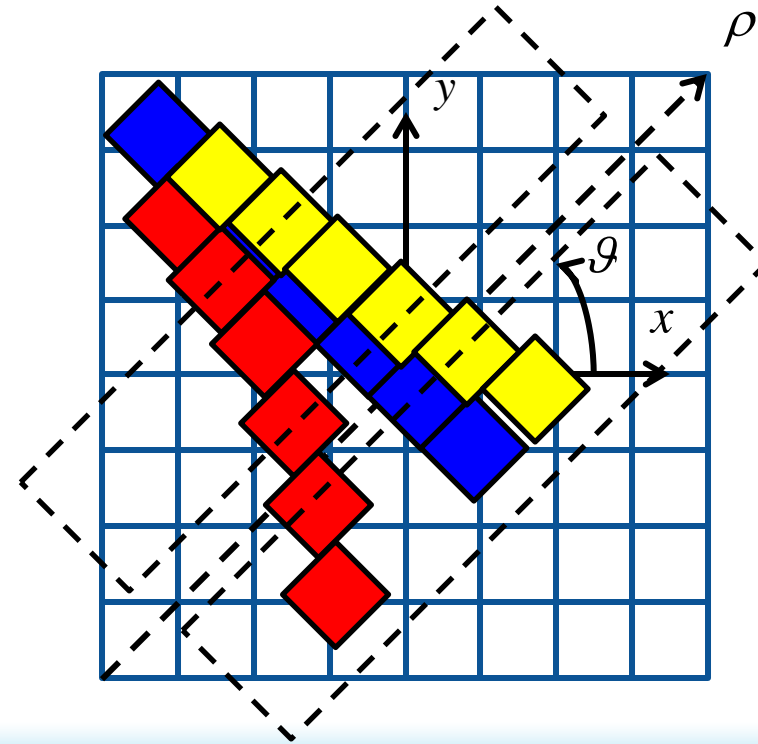
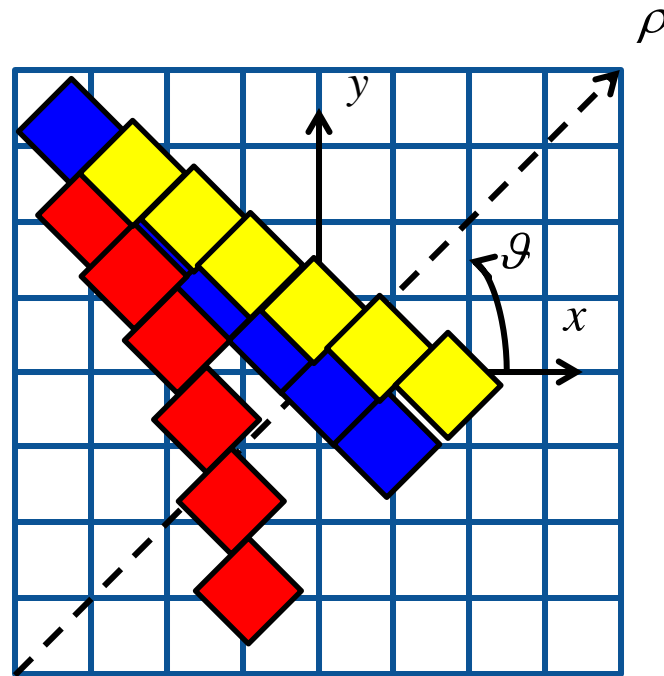
- Comparison between list-mode and integration-mode detector configurations
- The **self-consistent** tomographic image reconstruction pretends to know the exact trajectories of the protons (to overcome the ill-posed nature of the inverse problem in ion imaging)



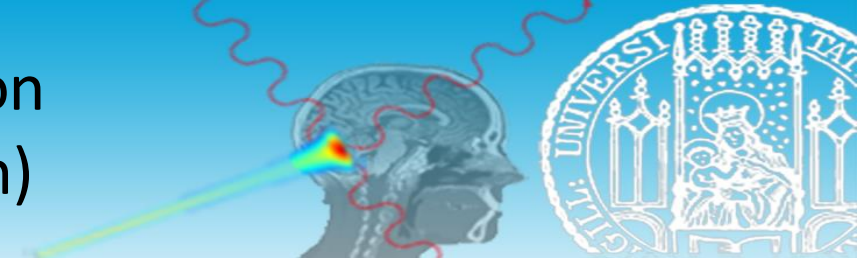
Tomographic image reconstruction (list-mode detector configuration)



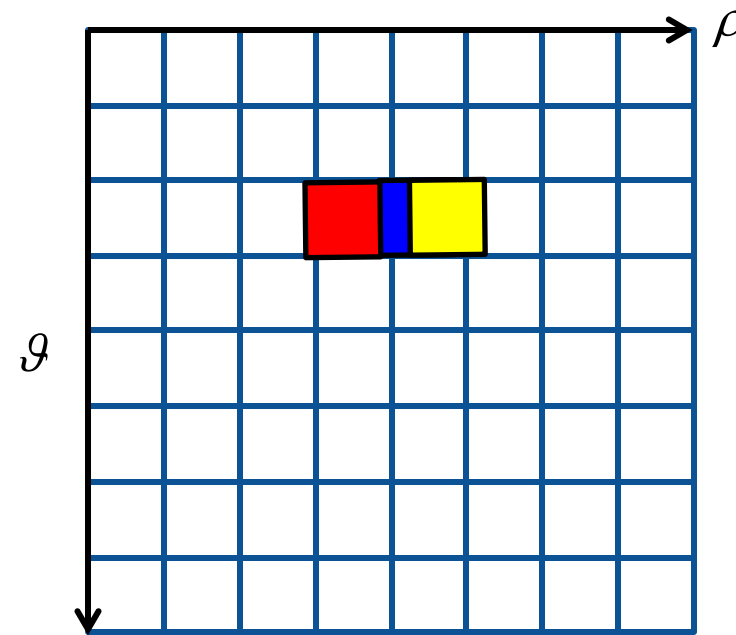
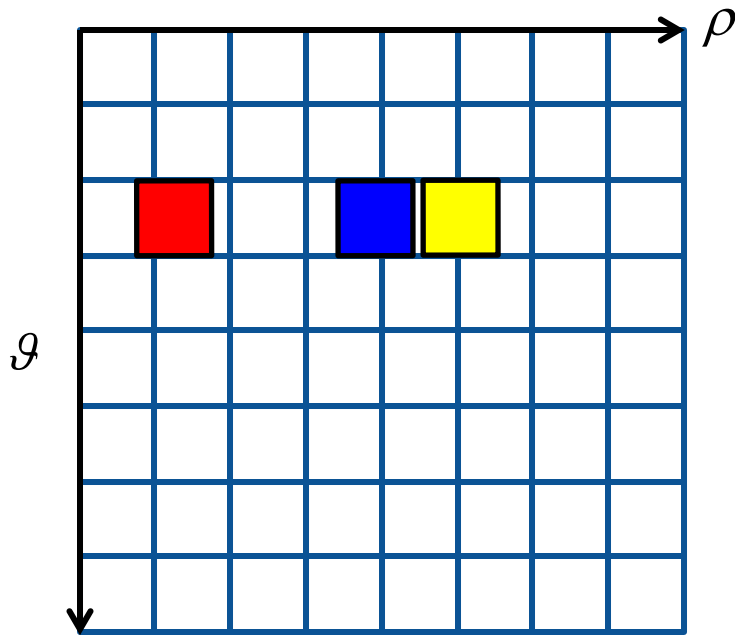
- The MLP is the estimation of the proton trajectory that can be easily adopted in the system matrix of numerical reconstruction
- Alternatively, the estimation of the proton trajectory can be used to implement a “modified” FBP, based on a distance-driven binning of projections for individual source positions



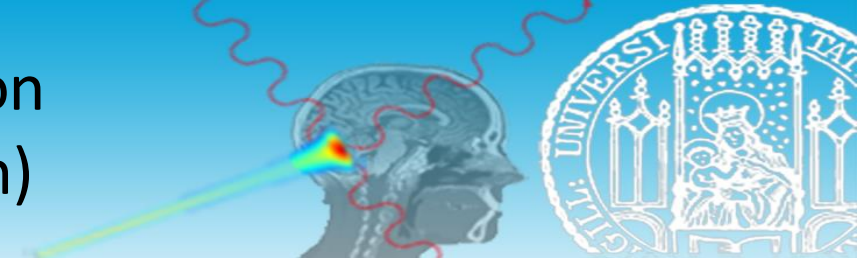
Tomographic image reconstruction (list-mode detector configuration)



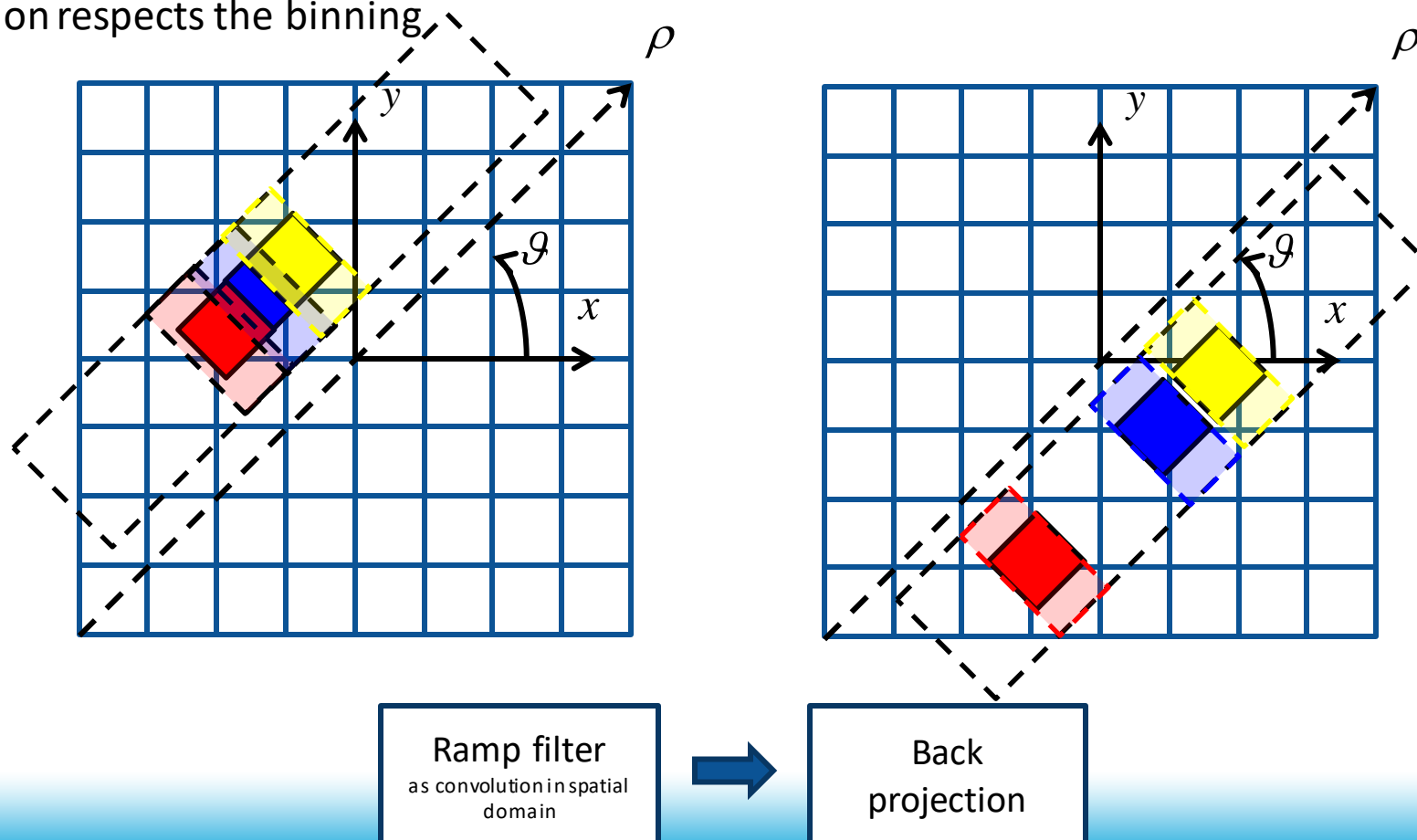
- The projection is binned (subdivided) according to the **source to detector distance**
- The binning provides the sinogram an additional dimension, whose size is defined by the number of bins



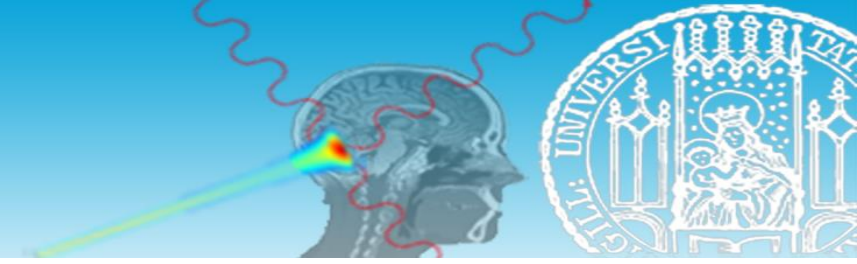
Tomographic image reconstruction (list-mode detector configuration)



- The filtering, enabled by the distance-driven binning, is applied to each binned projection of the sinogram (without filtering the method is simply a back-projection along the MLP)
- The back-projection respects the binning

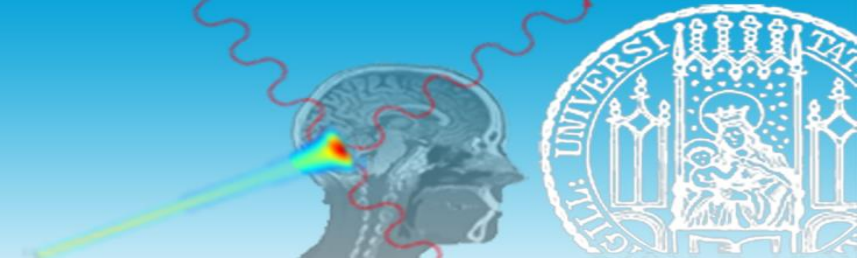


Exercise #1



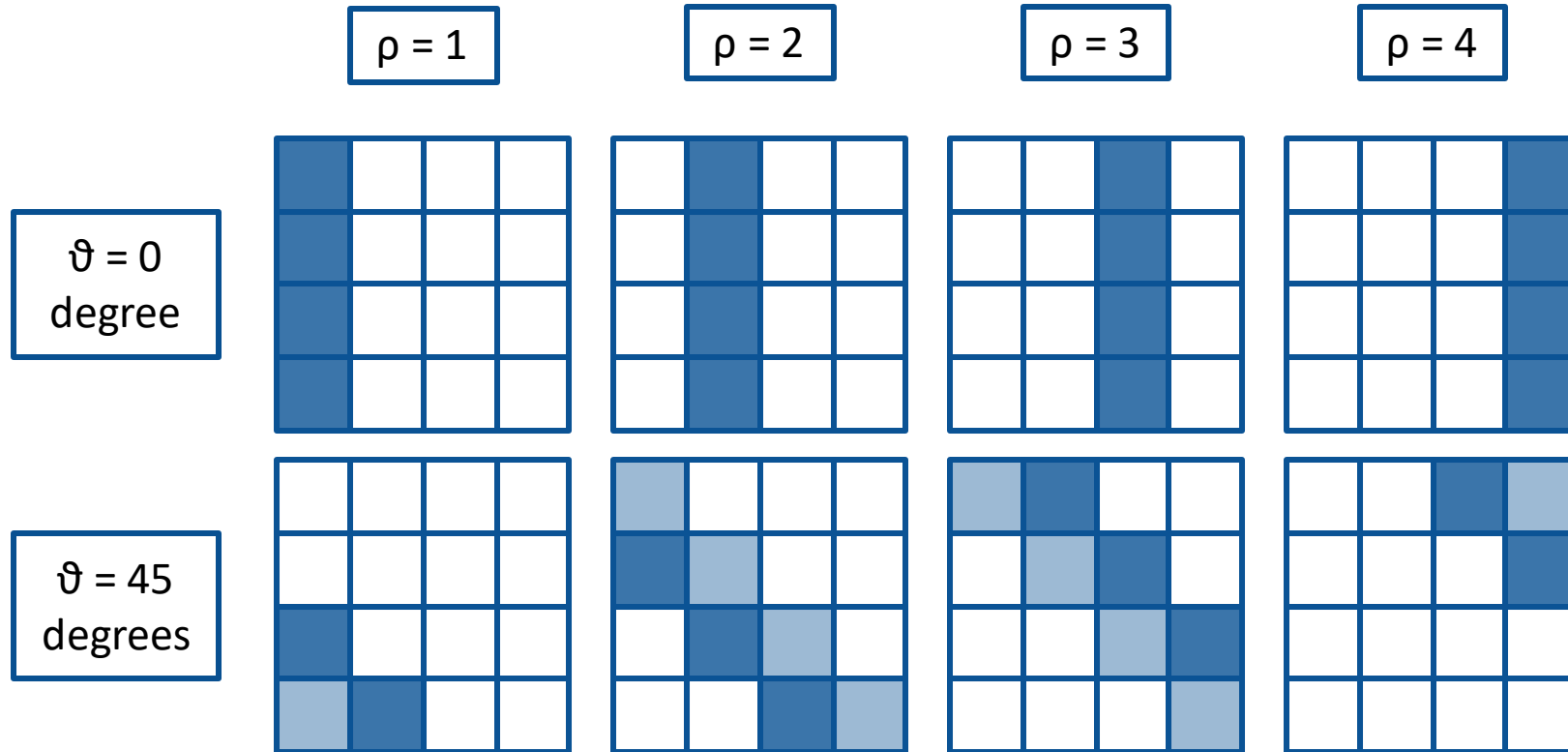
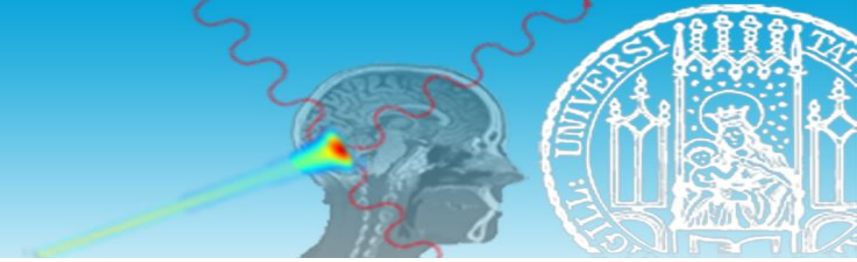
- Analytical calculation of the sinogram or Radon Transform where each projection is calculated as **line integrals** of the **image intensity**
 - Choose the image (phantom.png)
 - Choose the number of projection lines ($n_p = 128$) and the number of projection angles ($n_\theta = 180$ with spacing $\Delta\theta = 1$ degree)
 - For loop over projection angles
 - Rotate the image matrix according to the projection angle and integrate the image matrix along the straight integration lines (instead of rotating the integration lines)
 - Store the resulting vector as a column in the sinogram matrix

Exercise #1

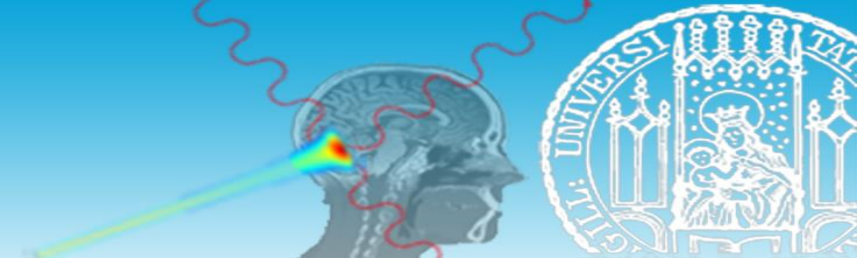


- Analytical calculation of the system matrix
 - Choose the number of projection lines ($n_p = 128$) and the number of projection angles ($n_\vartheta = 180$ with spacing $\Delta\vartheta = 1$ degree)
 - For loop over projection angles
 - For loop over projections
 - Create image matrix made of a column in correspondence of the projection
 - Rotate the image matrix according to the projection angle
 - Store the resulting vector as a column in the system matrix
- Analytical calculation of the sinogram or Radon Transform as **forward-projection** of the **image** to be compared to the previous sinogram
 - Are they different? Why?

Exercise #1



Exercise #1



- Implementation of the numerical tomographic image reconstruction algorithm SART
 - Take a **sinogram** and the **system matrix**
 - Initialize the vectorized image as vector of zero
 - For loop over **iterations**
 - Updating formula of the SART

