Inverse problems and machine learning in medical physics

Robotics in radiation therapy

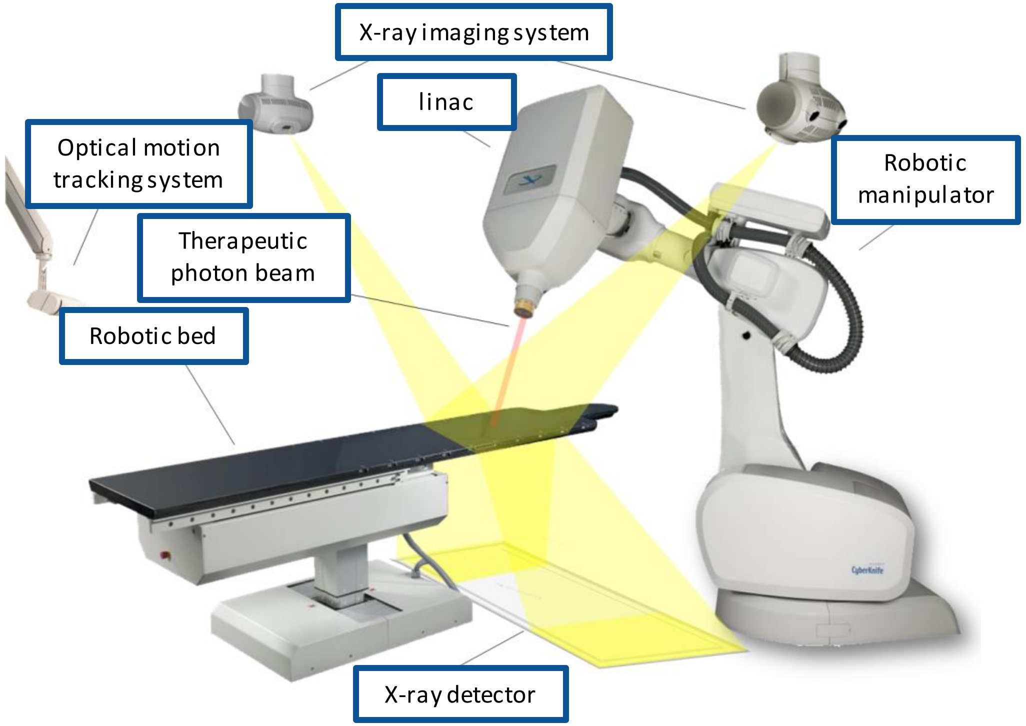
Dr. Chiara Gianoli
6/2/2024
chiara.gianoli@physik.uni-muenchen.de

The CyberKnife

- The CyberKnife is a radiation delivery system that features a linear accelerator (*linac*) directly mounted on a robot to deliver photons for radiation therapy, a fluoroscopic kV X-ray imaging for pre-treatment and intra-treatment image-guidance and an optical system for respiratory motion tracking

  - 3D conformal radiotherapy (3D-CRT), including intensity modulated radiation therapy (IMRT)

  - Stereotactic radiosurgery (SRS) and stereotactic body radiation therapy (SBRT) treatments

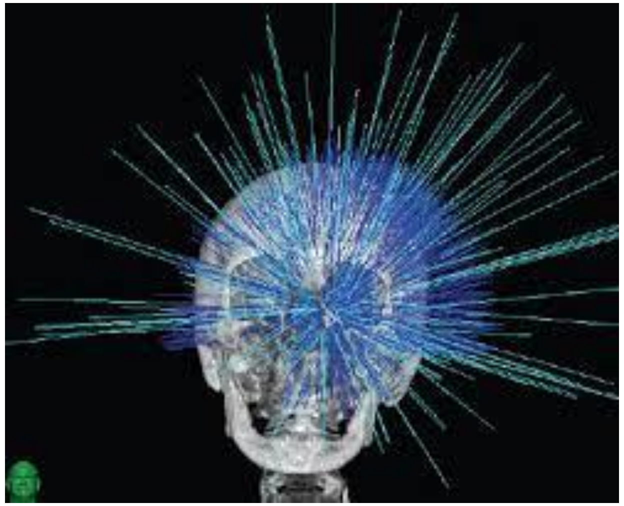


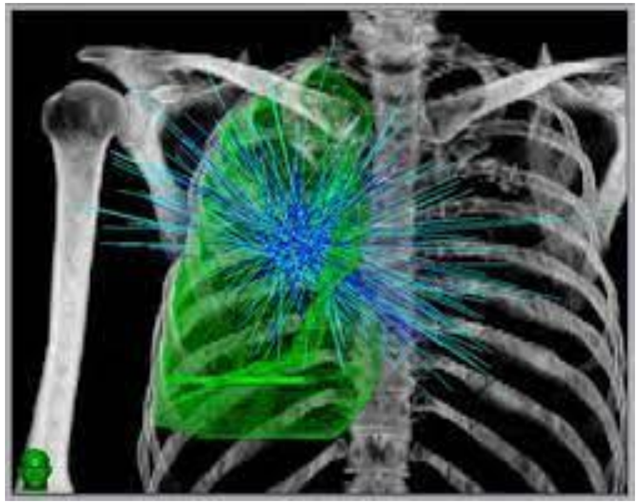
https://link.springer.com/article/10.1007/s43154-021-00072-3

The CyberKnife

- The beams are delivered from fixed points in space called nodes, arranged in spherical (intracranial applications) or ellipsoidal (extracranial applications) configurations

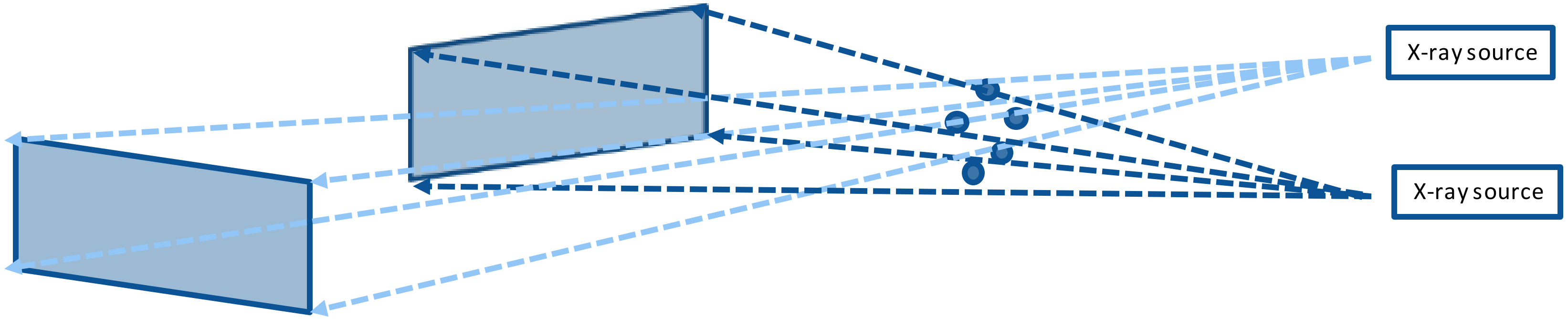
- The combination of nodes and pointing vectors (twelve for each node) provides a set of "elementary beams" to plan the treatment





The CyberKnife

- Prior to the treatment, eight radiographic X-ray images in different breathing phases are compared to the DRRs of the treatment planning X-ray CT image to determine by triangulation the transformation to be applied to the 6-DoF robotic bed for patient positioning



- During the treatment, this transformation is adjusted in real-time by moving the end-effector of the 6-DoF robotic manipulator (*linac*) according to the moving target

- The motion tracking considers:
    - Fiducial-free tumor tracking based on the optical tracking system (for external localization at 20–40 Hz) and implanted radio-opaque markers near or inside the tumor based on the X-ray imaging system (for internal localization every 30s) thus making use of external-internal correlation models
        - The model is constructed in ~30s at the beginning of the treatment and enables motion prediction for delay compensation (~ms)

https://link.springer.com/content/pdf/10.1007%2F978-3-540-69886-9.pdf

The CyberKnife

The “forward” kinematics

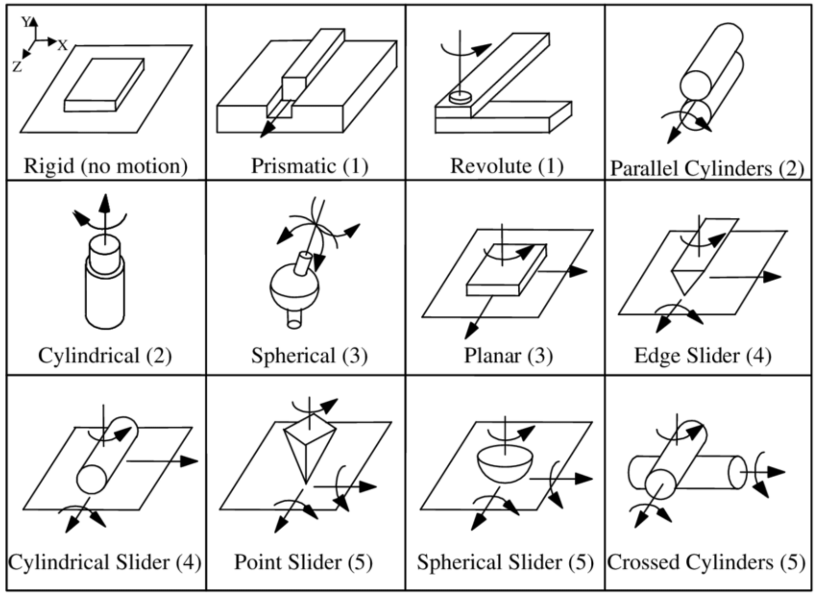
- *First of all, we have to know where the robot is…*

- Kinematics is the science of motion that treats motion without regard to the forces which cause it (i.e., position, velocity, acceleration…)

- The forward kinematics problem is about the knowledge of the position of the end-effector, given the kinematic chain of the mechanical system

  - The kinematic chain is an assembly of links (rigid bodies) connected by joints providing degrees of freedom (DoF) to the end-effector

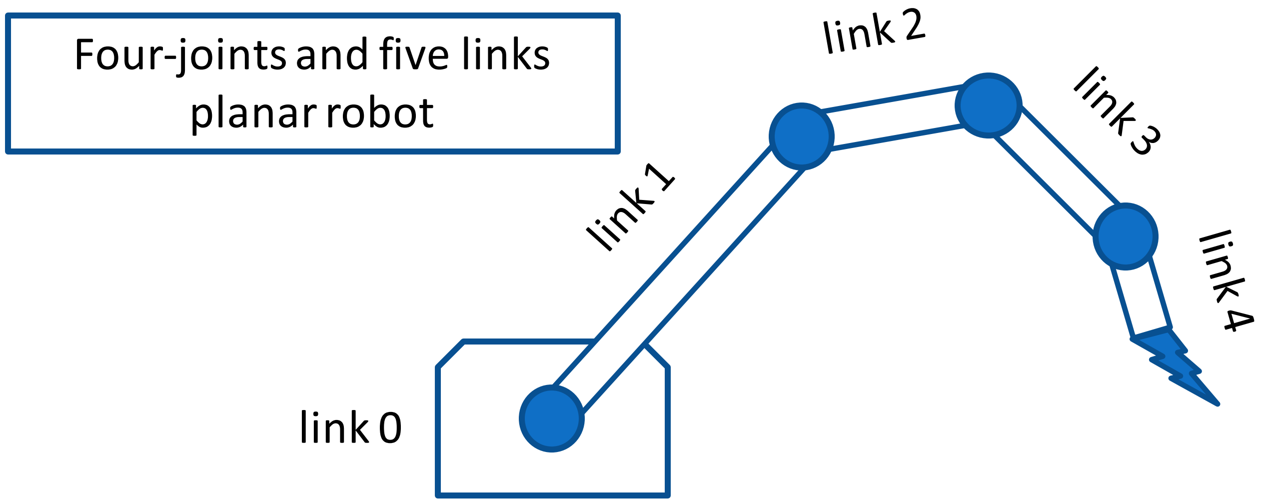
  - the number of DoF of the end-effector is determined by the DoF of all the joints (i.e., 1-DoF for each revolute joint)



- The kinematic chain is described by the joint variables (i.e., the angles for the revolute joints)

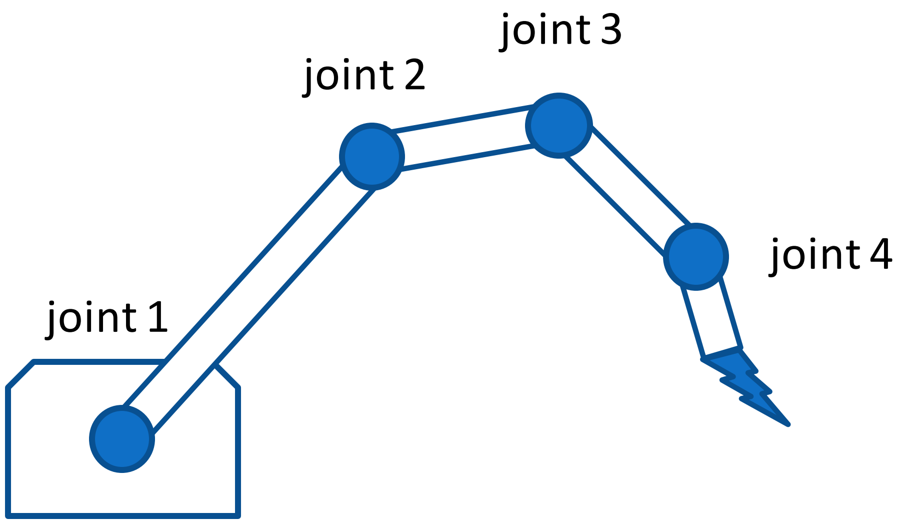
The “forward” kinematics

- In order to unequivocally localize the position of the end-effector in function of the angles of the kinematic chain (joints and links), the Denavit–Hartenberg convention (D-H convention) is commonly adopted
  - The *n* links are numbered from 0 (the base of the kinematic chain) to *n-1* (the end-effector)



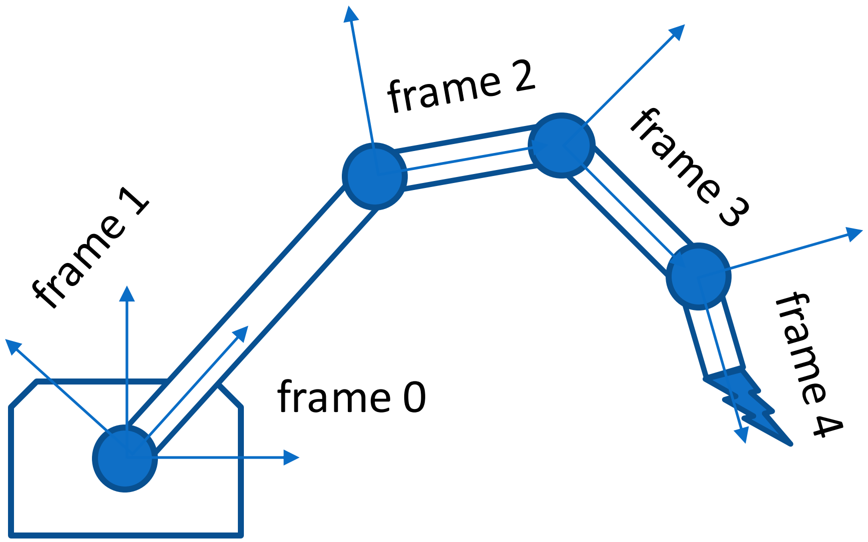
The “forward” kinematics

- In order to unequivocally localize the position of the end-effector in function of the angles of the kinematic chain (joints and links), the Denavit–Hartenberg convention (D-H convention) is commonly adopted
  - The *n-1* joints are numbered from 1 to *n-1* so that the joint *n* connects the link *n-1* (the base of the kinematic chain) to the link *n* (the end-effector)



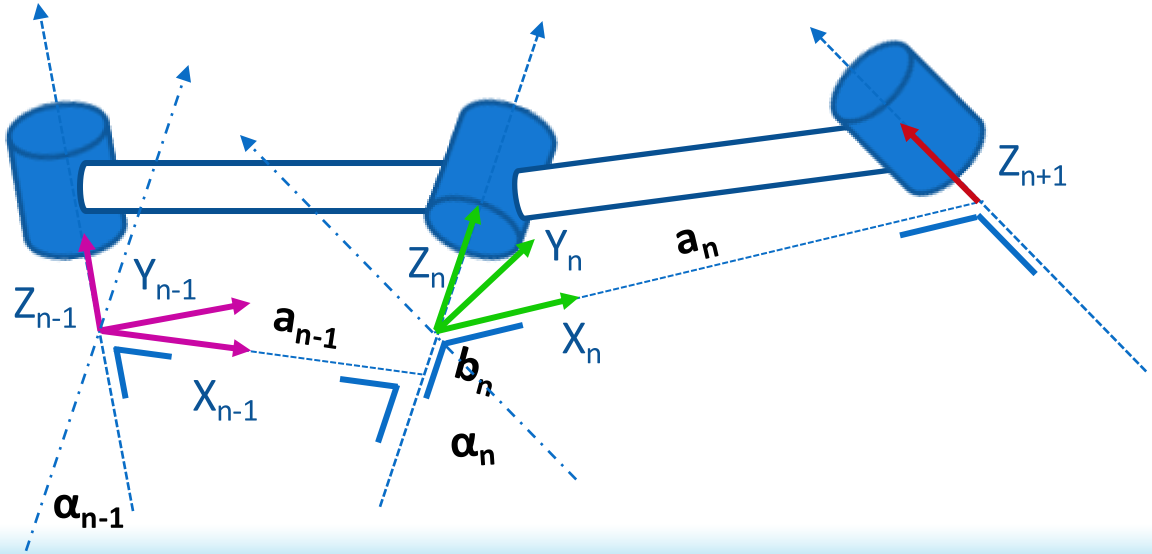
The "forward" kinematics

- In order to unequivocally localize the position of the end-effector in function of the angles of the kinematic chain (joints and links), the Denavit–Hartenberg convention (D-H convention) is commonly adopted
  - A frame *n* is defined at the joint n



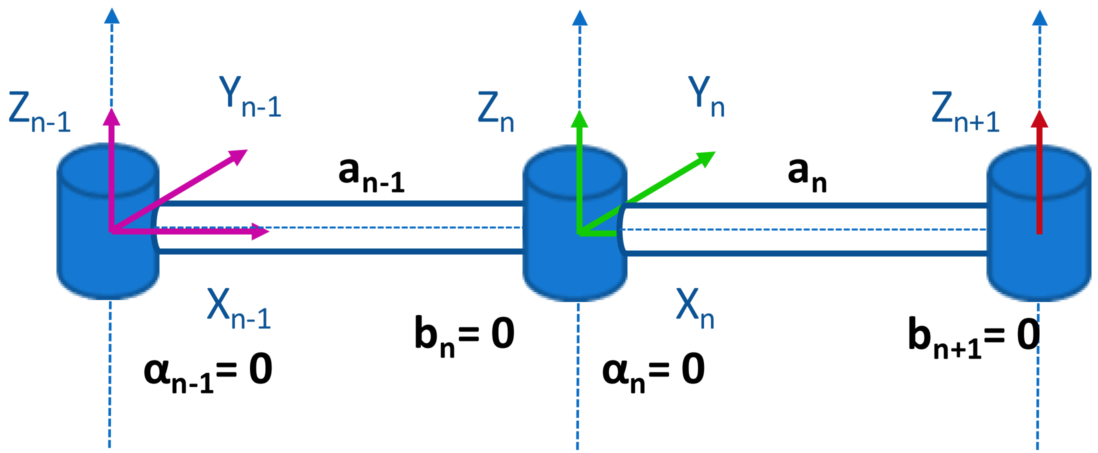
The “forward” kinematics

- Z_n is defined as the rotational axis of the revolute joint n
  - If Z_n and Z_{n+1} are skew lines, X_n is defined along the common perpendicular of Z_n and Z_{n+1} (the shortest distance between two skew lines is the distance between their intersection points with their common perpendicular), from Z_n to Z_{n+1}
  - The origin of the frame n is defined at the intersection point with Z_n
  - Y_n is defined according to the right-hand frame



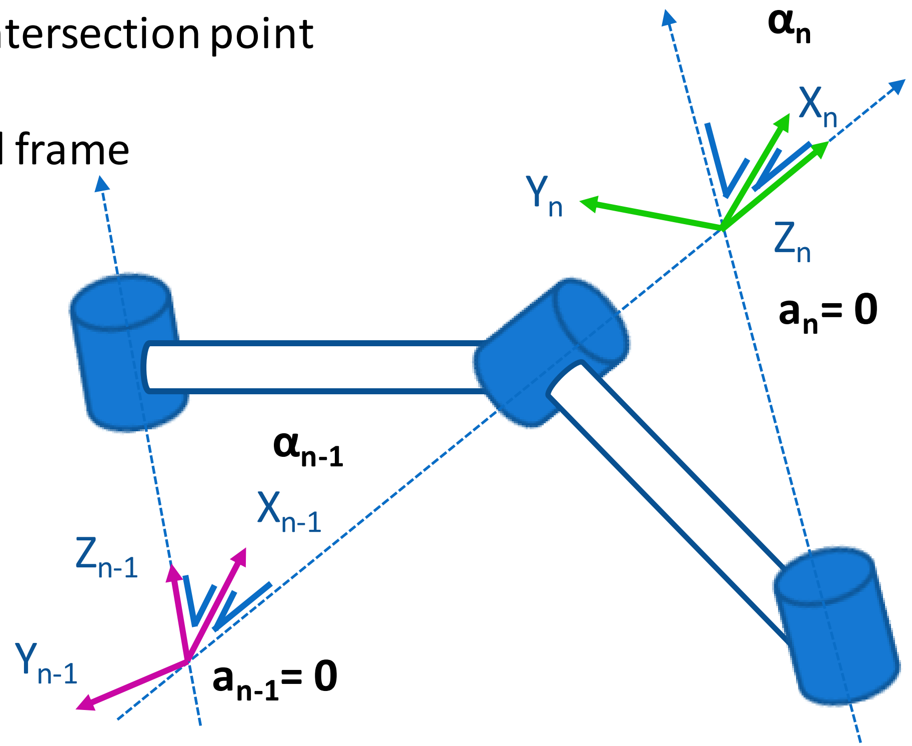
The “forward” kinematics

- Z_n is defined as the rotational axis of the revolute joint n
  - If Z_n and Z_{n+1} are parallel lines (i.e., planar robot), X_n is defined along the common normal of Z_n and Z_{n+1}, from Z_n to Z_{n+1}
  - The origin of the frame n is set on the joint n
  - Y_n is defined according to the right-hand frame



The "forward" kinematics

- Z_n is defined as the rotational axis of the revolute joint n
  - If Z_n and Z_{n+1} are intersecting lines, X_n is defined by the vector product between Z_n and Z_{n+1} (i.e., according to the right-hand frame)
  - The origin of the frame n is set on the intersection point
  - Y_n is defined according to the right-hand frame

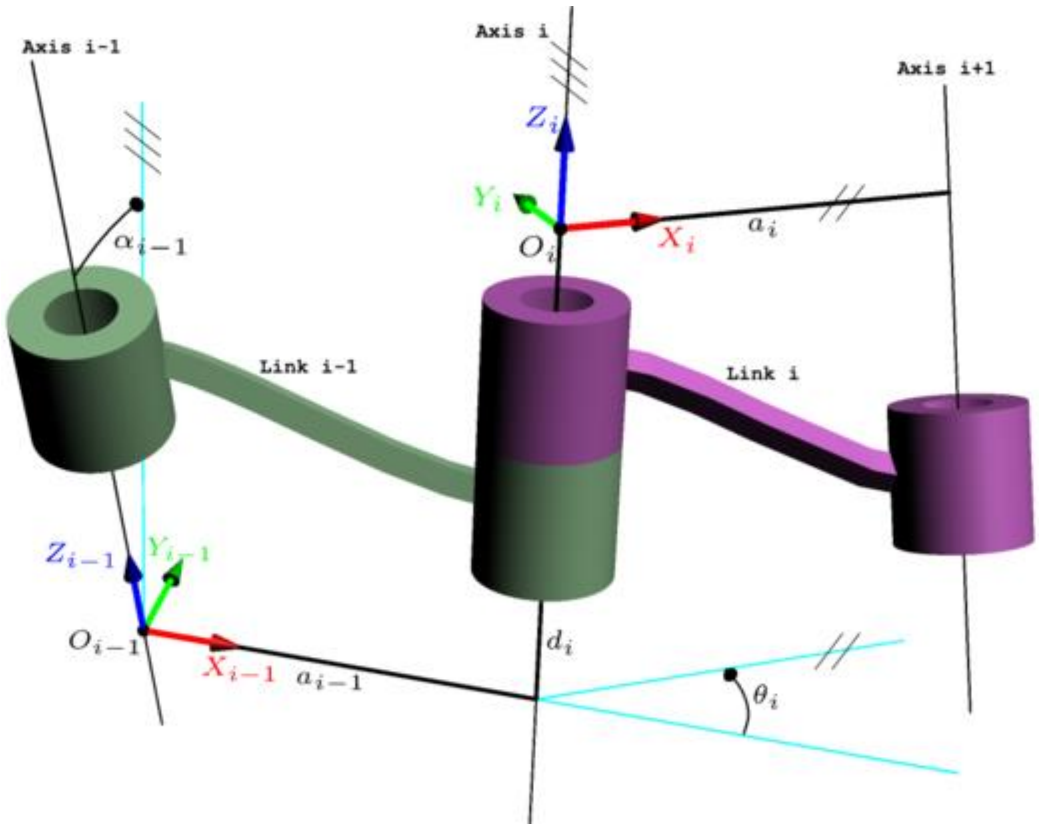


The "forward" kinematics

- Based on this convention, four Denavit–Hartenberg parameters (D-H parameters) are defined for each joint
  - **I D-H parameter a_n** (link length)
    - a_n is the absolute distance between Z_n and Z_{n+1}
  - **II D-H parameter b_n** (joint offset)
    - b_n is the distance along Z_n between X_{n-1} and X_n
  - **III D-H parameter α_n** (twist angle)
    - α_n is the angle between Z_n and Z_{n+1} across X_n (positive if counterclockwise)
  - **IV D-H parameter ϑ_n** (joint angle)
    - ϑ_n is the angle between X_{n-1} and X_n across Z_{n-1} (positive if counterclockwise)

The “forward” kinematics

- Based on this convention, four Denavit–Hartenberg parameters (D-H parameters) are defined for each joint



LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

The “forward” kinematics

- The forward kinematics describe the transformation of the frame n with respect to the frame n-1 as a composition of rotations and translations in the Denavit-Hartenberg matrix (D-H matrix)

$$^{n-1}_{n}T = R_X(\boldsymbol{\alpha}_{n-1})D_X(\boldsymbol{a}_{n-1})R_Z(\boldsymbol{\vartheta}_n)D_Z(\boldsymbol{b}_n)$$

  - Translation along Z_n equal to $\boldsymbol{b}_n$
  - Rotation across Z_n equal to $\boldsymbol{\vartheta}_n$
  - Translation along X_{n-1} equal to $\boldsymbol{a}_{n-1}$
  - Rotation across X_{n-1} equal to $\boldsymbol{\alpha}_{n-1}$

$$^{n-1}_{n}T = \begin{bmatrix} cos(\boldsymbol{\vartheta}_n) & -sin(\boldsymbol{\vartheta}_n) & 0 & \boldsymbol{\alpha}_{n-1} \\ sin(\boldsymbol{\vartheta}_n)cos(\boldsymbol{\alpha}_{n-1}) & cos(\boldsymbol{\vartheta}_n)cos(\boldsymbol{\alpha}_{n-1}) & -sin(\boldsymbol{\alpha}_{n-1}) & -sin(\boldsymbol{\alpha}_{n-1})\,\boldsymbol{b}_n \\ sin(\boldsymbol{\vartheta}_n)sin(\boldsymbol{\alpha}_{n-1}) & cos(\boldsymbol{\vartheta}_n)sin(\boldsymbol{\alpha}_{n-1}) & cos(\boldsymbol{\alpha}_{n-1}) & cos(\boldsymbol{\alpha}_{n-1})\,\boldsymbol{b}_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation matrix $^{n-1}_{n}R$

Translation vector $^{n-1}_{n}t$

The “forward” kinematics

- The forward kinematics describe the transformation of the end-effector frame with respect to the base frame (i.e., frame 0) as a composition of D-H matrixes

$$ {}^{0}_{e-e}T = {}^{0}_{1}T \dots {}^{n-1}_{e-e}T = \prod {}^{n-1}_{n}T = \begin{bmatrix} {}^{0}_{e-e}R & & & {}^{0}_{e-e}t \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

- The position of the end-effector in the base frame (i.e., frame 0) ${}^{0}P$ is determined by the matrix-vector product of the composed D-H matrix with the position of the end-effector in the end-effector frame ${}^{e-e}P$
  - The frame at the end-effector can be arbitrarily defined

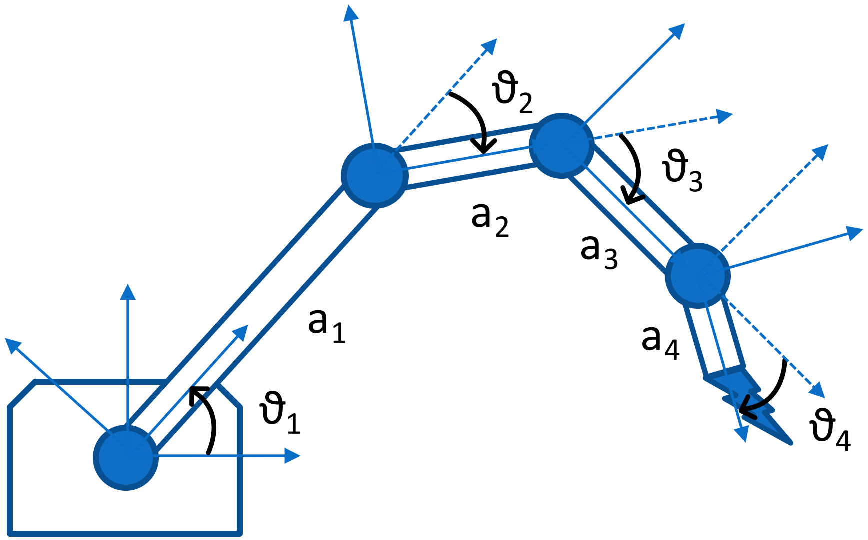
$$ {}^{0}P = {}^{0}_{e-e}T\,{}^{e-e}P $$

Descriptor of the forward kinematics

The "forward" kinematics

Four-joints and five links planar robot

Link	b_n	ϑ_n	a_n	α_n
1	0	ϑ_1	a_1	0
2	0	ϑ_2	a_2	0
3	0	ϑ_3	a_3	0
4	0	ϑ_4	a_4	0



$${}^0_4T = {}^0_1T\ {}^1_2T\,{}^2_3T\ {}^3_4T$$

$${}^0_4T = \begin{bmatrix} cos(\vartheta_1) & -sin(\vartheta_1) & 0 & 0 \\ sin(\vartheta_1) & cos(\vartheta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\vartheta_2) & -sin(\vartheta_2) & 0 & a_1 \\ sin(\vartheta_2) & cos(\vartheta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\vartheta_3) & -sin(\vartheta_3) & 0 & a_2 \\ sin(\vartheta_3) & cos(\vartheta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\vartheta_4) & -sin(\vartheta_4) & 0 & a_3 \\ sin(\vartheta_4) & cos(\vartheta_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The “inverse” kinematics

- *Finally, we have to know where the robot goes…*
- The inverse kinematics problem is about the knowledge of the kinematic chain of the mechanical system, given the desired position of the end-effector (i.e., the task)
- The solution of the inverse kinematics problem is defined within the workspace of the mechanical system
  - If the task is outside the workspace, the solution does not exist
  - If the solution exists, this can be single or multiple (infinite) depending on the DoF of the mechanical system

The “inverse” kinematics

- The number of unknowns is defined by the DoF of the joints of the mechanical system

- The forward kinematics given by ${}^{0}_{e-e}T$ provide 16 equations but 4 of them are trivial. Among the remaining 12 equations, 3 equations are relevant to the position-vector ${}^{0}_{e-e}t$ and 9 equations are relevant the rotation-matrix ${}^{0}_{e-e}R$. In the rotation-matrix only 3 equations are independent. The number of equations is therefore 6.

- The kinematic equations are nonlinear and transcendental, their solution is not always easy (or even possible) in a closed-form

    - For a 6-DoF robot, there are 6 equations and 6 unknowns. In this case, the analytical solution of the inverse kinematic problem is feasible

The "inverse" kinematics

- Numerical methods (i.e., iterative optimization algorithm) based on approximation and derivatives of the forward-kinematics function for finding the local minimum

- If n the number of joint variables, the forward-kinematics function map a point in the joint space to an end-effector position in the workspace

$$p(x): \mathbb{R}^n \rightarrow \mathbb{R}^3$$

- Given the initial position of the system $p_0 = p(x_0)$, the task is defined as $p_1 = p(x_0 + \Delta x)$

- Given the Jacobian of the forward-kinematics function $J_p(x_0)$, whose size is 6 x n, the Taylor series expansion of the forward-kinematics function, valid for small Δx, is calculated as:

$$p(x_1) \approx p(x_0) + J_p(x_0)\Delta x$$

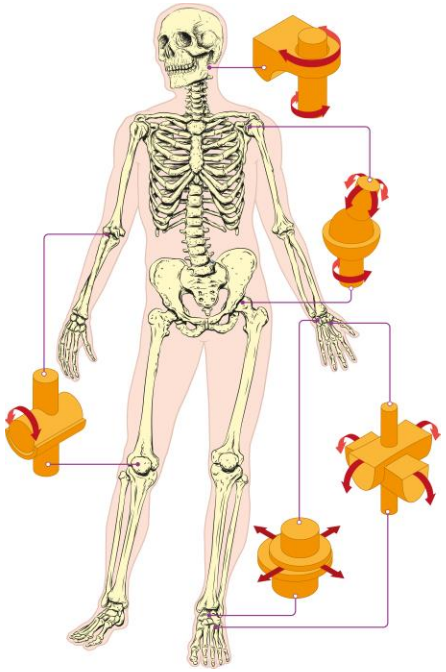
- By calculating the (pseudo) inverse ${J_p}^{inv}(x_0)$, the updating step is defined as:

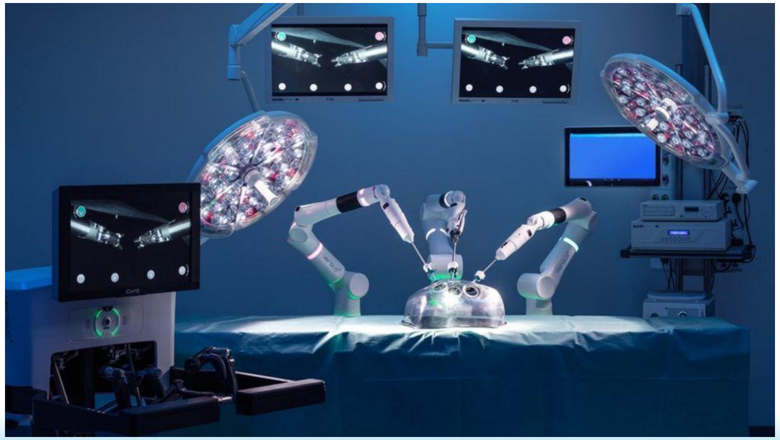
$\Delta x \approx {J_p}^{inv}(x_0)\Delta p(x_0)$ with $\Delta p = p(x_0 + \Delta x) - p(x_0)$, and thus $\Delta x_{k+1} \approx {J_p}^{inv}(x_k)\Delta p_k$

Robotics and artificial intelligence

- The human body is a mechanical system made of joints and links that can implement a task
- The human senses define the control system that can provide information about the task
- The human brain is the intelligence system that can decide the task based on the sensor information

→ artificial intelligence-driven robots





Robotic surgery

Robotics and artificial intelligence

- In radiation oncology, the task is executed based on the patient model in the treatment planning scenario
- The task is adapted based on imaging and sensor systems for monitoring the treatment delivery scenario, thus adapting the *real* patient to the patient model (i.e., patient positioning) or vice versa, to adapt the patient model to the *real* patient (i.e., treatment adaptation and tumor tracking)
  - Correction models are defined (i.e., anatomical correction models, external-internal correlation models)
  - Model-free adaptive tasks based on AI