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Inverse problems and machine learning in medical physics

Machine learning for tomographic image reconstruction or “deep reconstruction”

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Tomographic image reconstruction

- Tomographic image reconstruction represents the building block of medical imaging
- Tomographic image reconstruction has been classified as analytical reconstruction or numerical reconstruction
- Very recently, data-driven, deep-learning-based tomographic image reconstruction has been introduced (i.e., deep tomographic reconstruction)
  - Direct reconstruction methods
  - Unrolled iterative reconstruction methods
- The huge benefit of machine learning in reconstruction is the use of the ground truth (i.e., supervised learning), as obtained from high quality simulations or high quality measurements

Analytical reconstruction

- Tomographic image acquisition can be modelled as a Radon transform, or sinogram, of the variable describing the physical properties of the object of interest

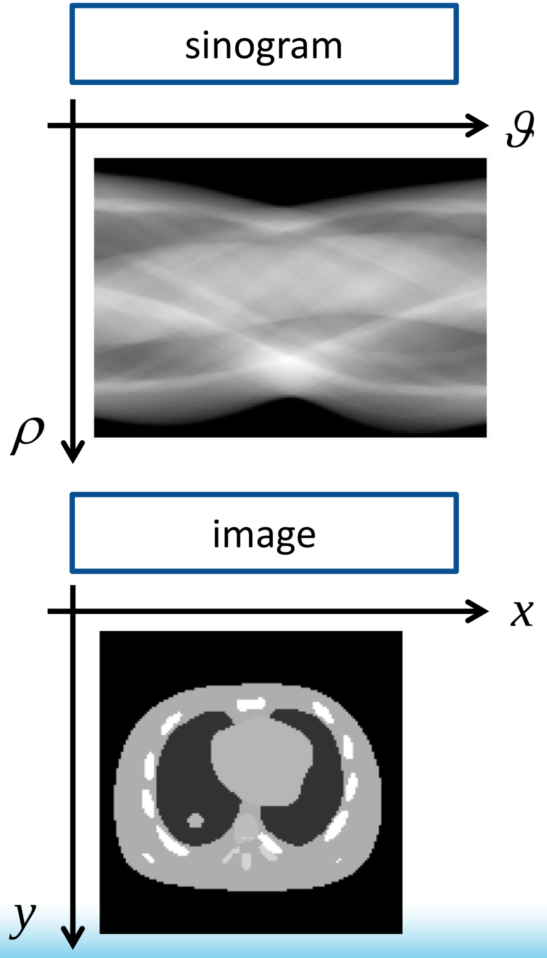
  - The Radon Transform converts an image from spatial domain to sinogram domain, by integrating the variables along the integration lines, as a function of the projection angles

  - The analytical image reconstruction is based on the Fourier slice theorem that puts in correspondence the Radon Transform with the Fourier Transform of the image (i.e., the filtered back-projection)

$$\hat{f}_\rho(w_x, w_y) = \int_{-\infty}^{+\infty} R(f) e^{-2\pi i(\rho w_\rho)} d\rho = \hat{R}(w_\rho)$$

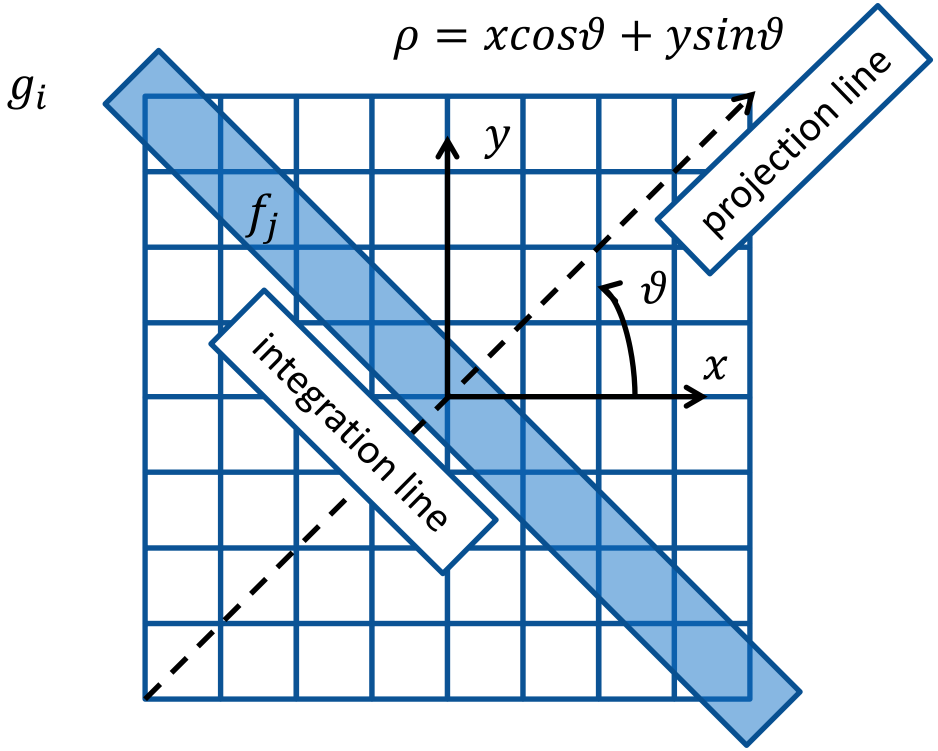
Radon transform

Fourier transform



Numerical image reconstruction

- The inverse problem of tomographic image reconstruction can be solved by means of numerical (iterative) algorithms
- Numerical algorithms can be considered as an iterative solver of a system of linear equations
  - I equations, one for each projection
  - J unknowns, one for each pixel

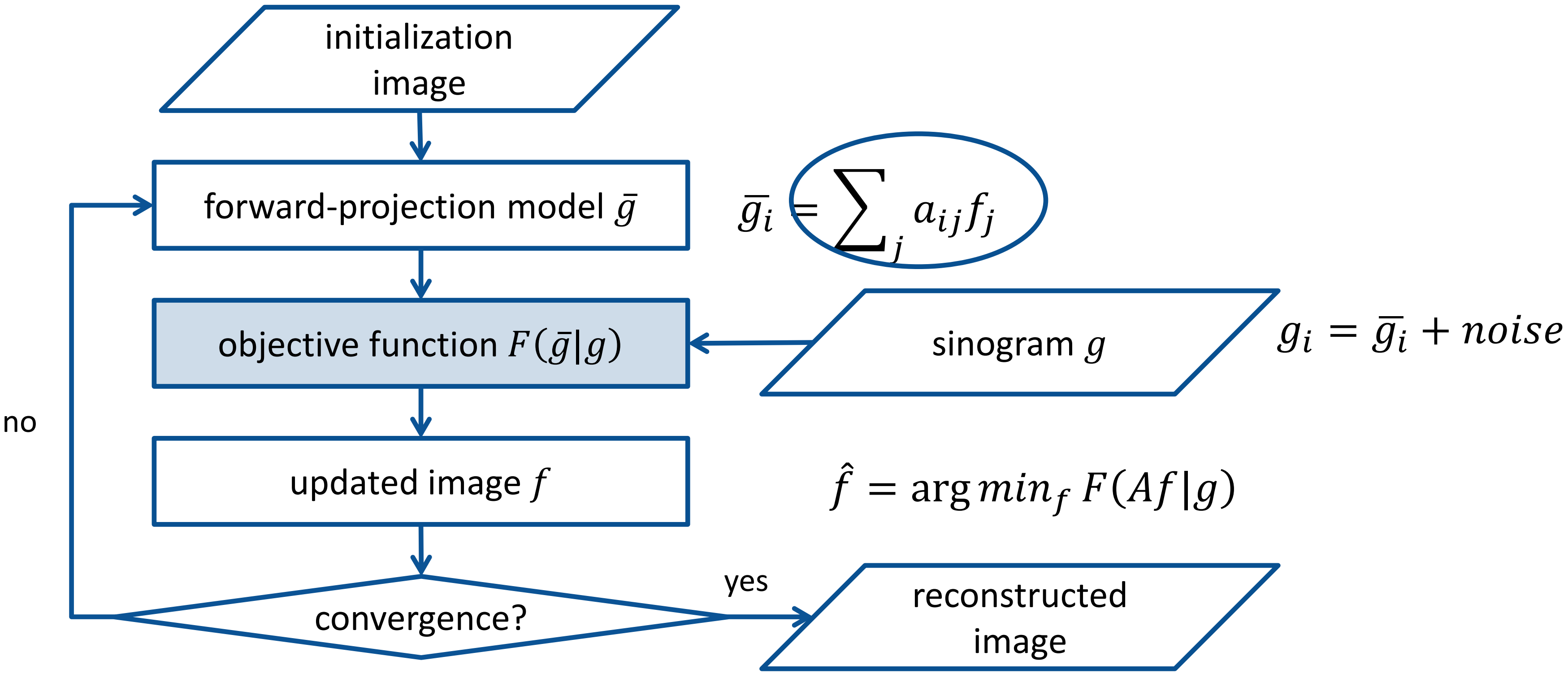


$$\begin{cases} a_{11}f_1 + a_{12}f_2 + \cdots a_{1J}f_J = g_1 \\ \cdots \\ a_{I1}f_1 + a_{I2}f_2 + \cdots a_{IJ}f_J = g_I \end{cases} \Rightarrow g_i = \sum_j a_{ij} f_j$$

- The coefficients a_{ij} (i.e., the elements of the system matrix) express the intersection area/volume of the pixel/voxel j with the integration line of the projection i

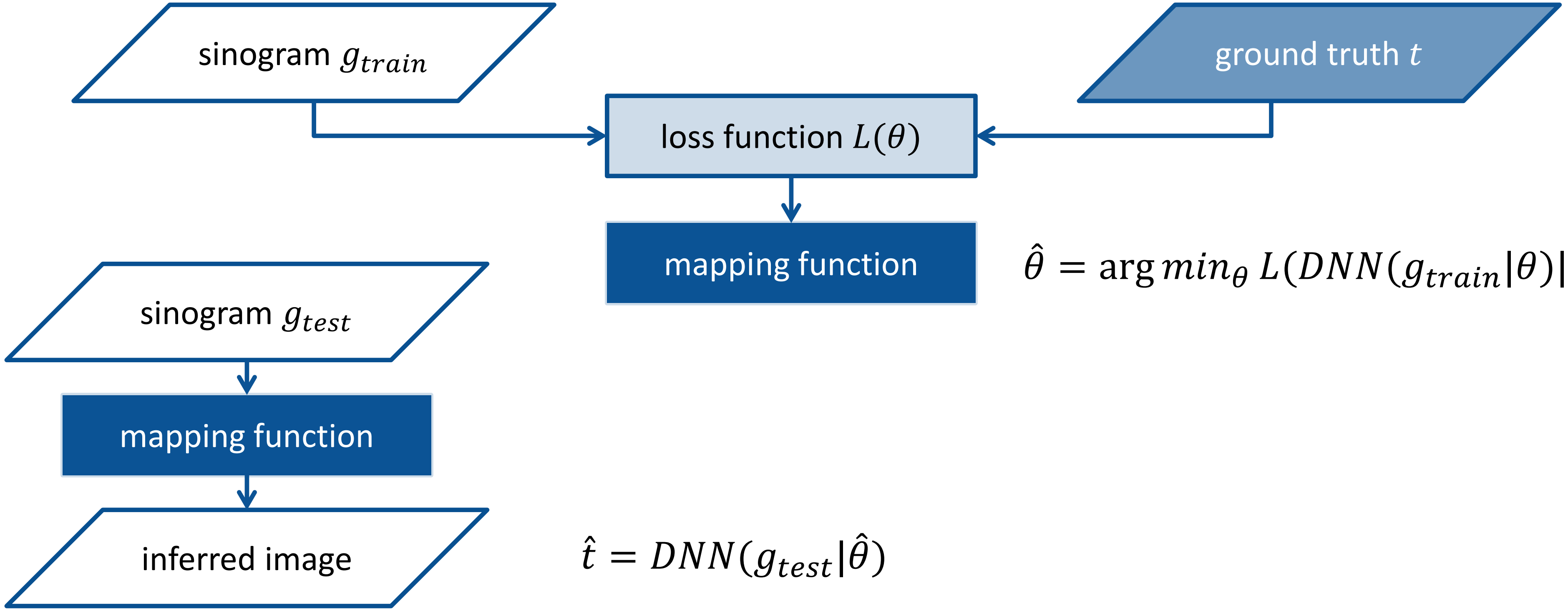
Numerical image reconstruction

- The iterative reconstruction paradigm is to find the image that minimizes the “discrepancy” between the forward-projection of the image (i.e., the model of the sinogram) and the acquired sinogram



Deep tomographic reconstruction

- The machine learning paradigm in tomographic image reconstruction is to find the parameters of the mapping function that infers the ground truth based on supervised prediction



$$\hat{\theta} = \arg min_{\theta} L(DNN(g_{train}|\theta)|t)$$

$$\hat{t} = DNN(g_{test}|\hat{\theta})$$

- Different from analytical and numerical approaches, deep learning deploys a method for reconstruction (i.e., an estimator), not a reconstruction (i.e., an estimate)

Deep tomographic reconstruction

- The back-projection is a linear mapping (i.e., matrix-vector multiplication) that can be described by a fully connected layer (i.e., linear layer) of an artificial neural network (ANN)

ground truth t

sinogram g

$$\begin{bmatrix} t_1 \\ \vdots \\ t_J \end{bmatrix} \approx \begin{bmatrix} a_{11} & & a_{1I} \\ & & \\ a_{J1} & & a_{JI} \end{bmatrix} \begin{bmatrix} g_1 \\ \vdots \\ g_I \end{bmatrix}$$

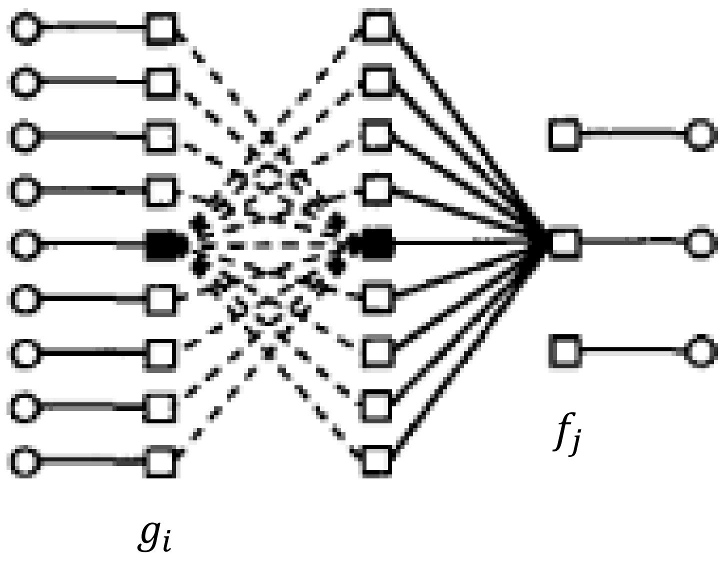
Deep tomographic reconstruction

- One of the first ML attempt to deep tomographic reconstruction was based on the "pre-calculation" of the filters for the filtered back-projection, instead of being analytically calculated each time…

  - The learnable weights (learnt based on a point source) are applied along the projection lines of the sinogram

  - The back-projection is implemented for each projection lines of the sinogram as fully connected layer with non-learnable weights (rotational and shift-invariant approximation)

Floyd, C. E. (1991). An artificial neural network for SPECT image reconstruction. *IEEE transactions on medical imaging*, 10(3), 485-487.



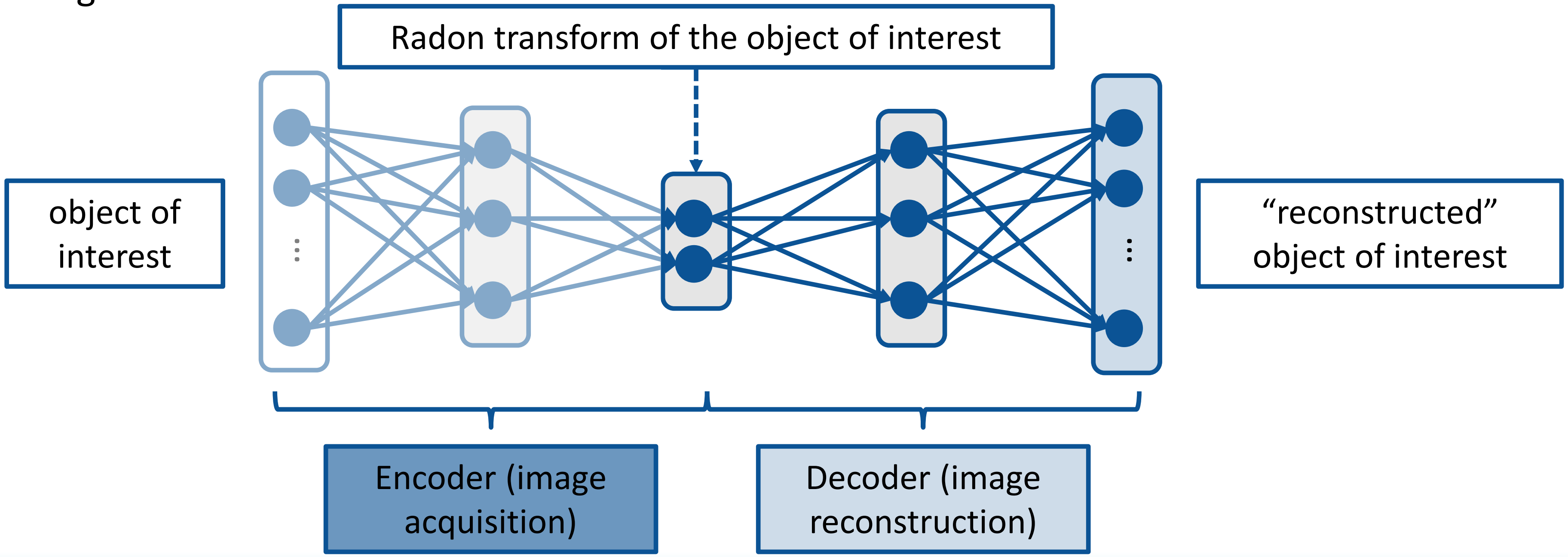
- In practice, this is suitable only for two-dimensional images

Exercise #2

- Define the image of an ideal point source (i.e., target data of the network)
- Simulate the sinogram of the ideal point source, then add noise and blur along each projection line (i.e., input data of the network)
- Implement a first fully connected network, connecting the nodes of the input layer (i.e., the projection line) to all the nodes of the hidden layer (i.e., the filtered projection line)
  - The forward-pass function is a weighed average of the inputs with unknown weights (i.e., no bias, no activation function)
- Implement a second fully connected network, connecting the nodes of the hidden layer (i.e., the filtered projection line) to all the nodes of the output layer (i.e., the image)
  - The forward-pass function is a weighed average of the inputs with known weights from the system matrix
- Train the network based on input and target data (implement the backward-pass based on the gradient descend algorithms)

Direct reconstruction methods

- The purpose of domain transform is to map the sinogram (i.e., the projections) to the image
  - The measured sinogram encodes an intermediate representation of the object of interest in the projection domain (i.e., the Radon transform), similar to an encoding function
  - The measured sinogram is subsequently reconstructed into an image by an inversion of the encoding function, similar to a decoding function



Direct reconstruction methods

- In direct reconstruction methods, the domain transformation can be learnt from the network or explicitly given as input to the network
  - Direct reconstruction methods can entail the encoding of the data (i.e., the Radon transform or the image) into a lower dimensional space (i.e., compressed sensing) and the decoding of the encoded data, typically by means of convolutional layers
    - The compressed sensing hypothesis is that a signal can be represented by and perfectly recovered from few non-zero coefficients in a suitable basis (i.e., dictionary)
      - *Wavelet* and *Shearlet* are common basis functions for image encoding and decoding (*https://www.math.uh.edu/~dlabate/SHBookIntro.pdf*)
  - Transformation into a lower dimensional space can be based on manifolds (i.e., manifold-based compressed sensing)
    - The manifold hypothesis is that high dimensional data (i.e., a continuous images) lie on low-dimensional manifolds (i.e., a point) in a high-dimensional space (i.e., an infinite dimensional vectoral space)
      - Zero-dimensional manifolds are points, one-dimensional manifolds are lines, two-dimensional manifolds are surfaces…

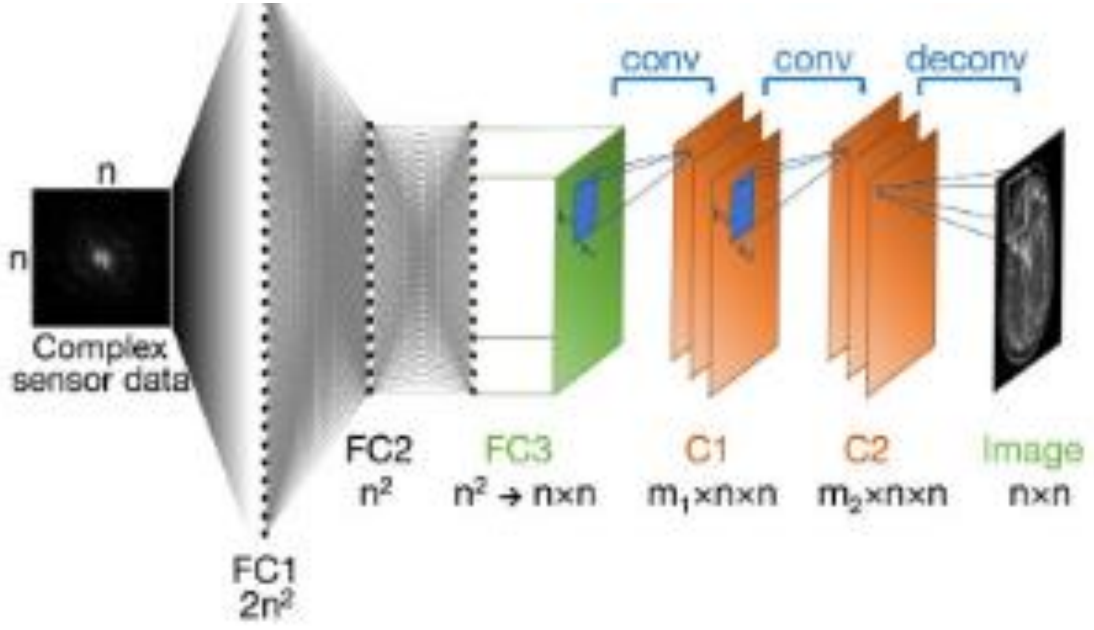
Direct reconstruction methods (AUTOMAP)

- The AUtomated TransfOrm by Manifold Approximation (AUTOMAP) is a deep neural network with feed-forward architecture, composed of multiple fully-connected layers followed by a sparse convolutional auto-encoder (i.e., encoder-decoder where the input and the output domains are the same)

- The network simultaneously learns an optimal convolutional domain (i.e., manifold space) and a sparse representations (i.e., compressed sensing) through a joint optimization (i.e., manifold encoding–decoding process)

  - Different from compressed sensing, the convolutional layers do not make hypothesis on the sparsifying transform (e.g., wavelet, shearlet…)

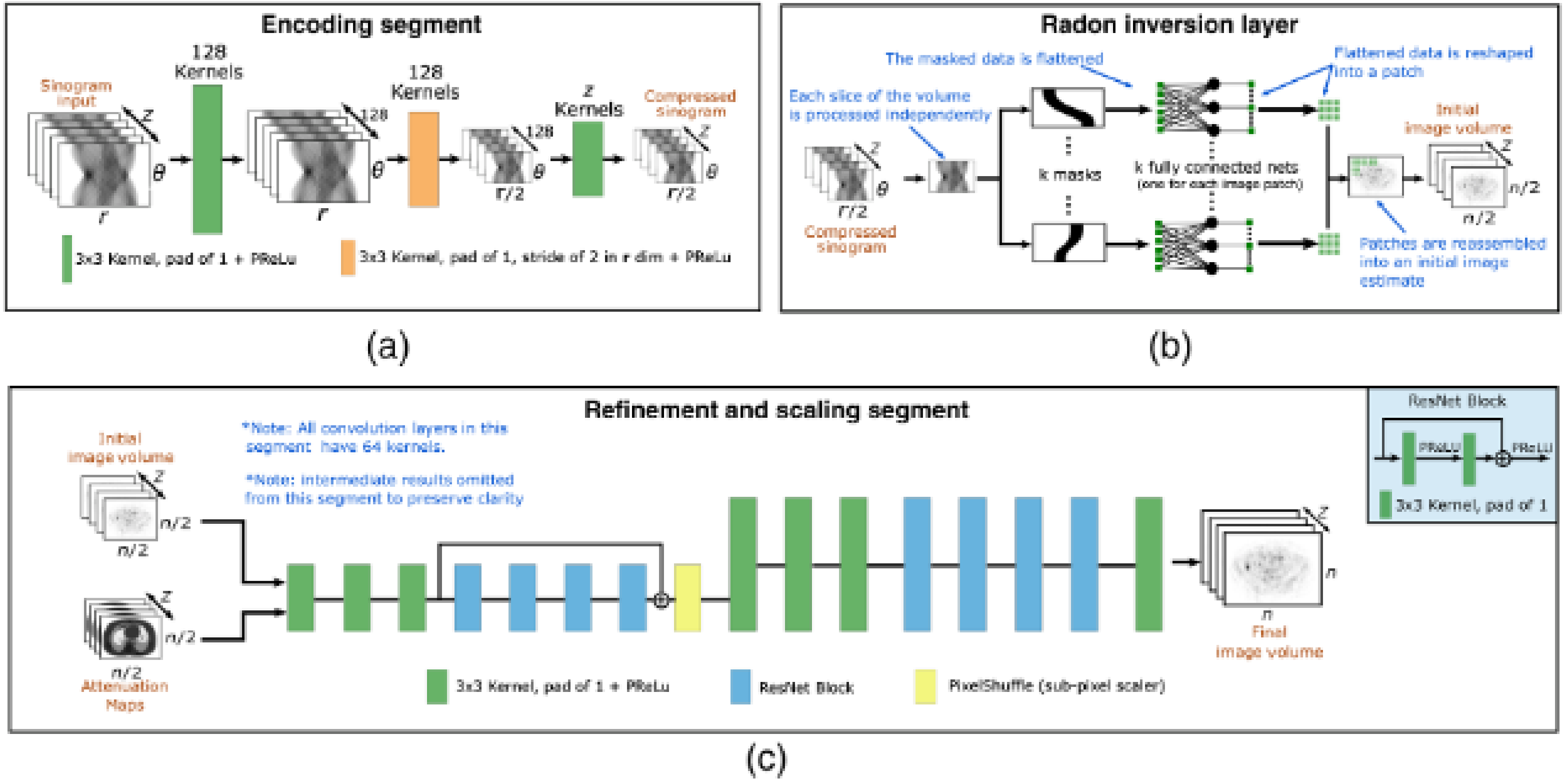
  - AUTOMAP is originally demonstrated for MRI but it is generally applicable to different image reconstruction problems



Zhu, B., Liu, J. Z., Cauley, S. F., Rosen, B. R., & Rosen, M. S. (2018). Image reconstruction by domain-transform manifold learning. *Nature*, 555(7697), 487-492.

Direct reconstruction methods (DirectPET)

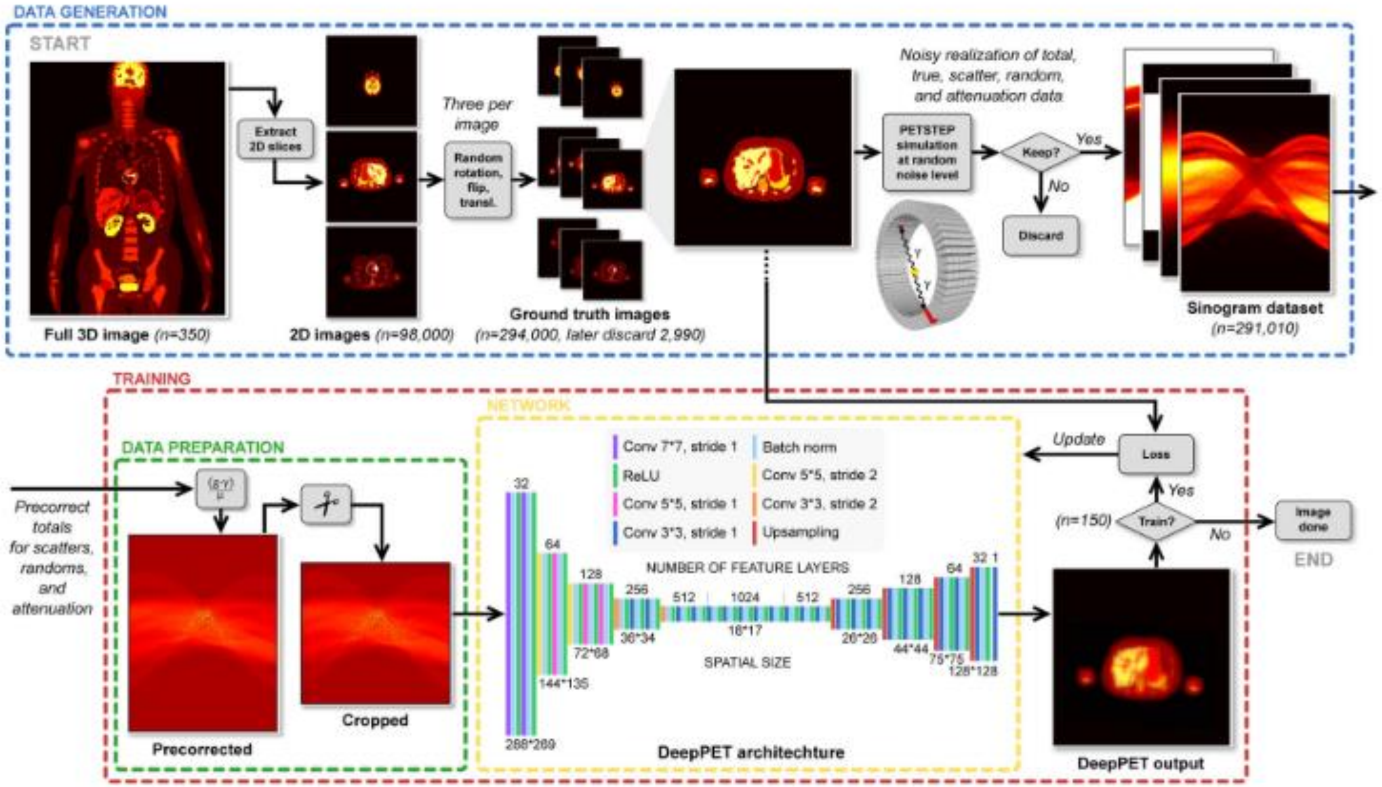
- The DirectPET is a large-scale direct neural network that performs image reconstruction by introducing a Radon inversion layer
    - An encoding segment compressing the sinogram data into a lower dimensional space
    - A domain transformation segment (i.e., Radon inversion) using sinogram data masking along with fully connected layers
    - A refinement and scaling segment enhancing and up-sampling the reconstructed image



Whiteley, W., Luk, W. K., & Gregor, J. (2020). DirectPET: full-size neural network PET reconstruction from sinogram data. *Journal of Medical Imaging*, 7(3), 032503.

Direct reconstruction methods (Deep PET)

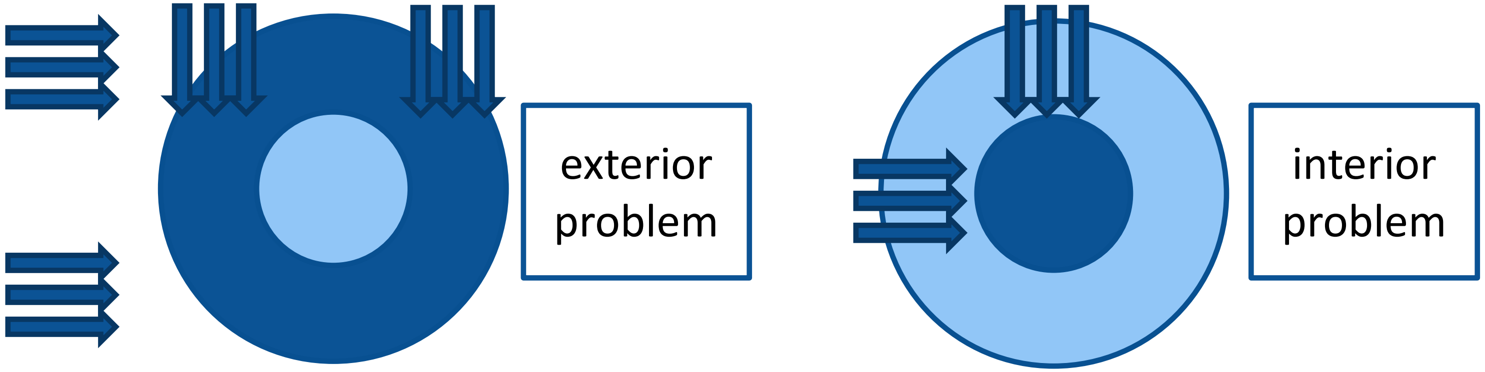
- The Deep PET is a convolutional encoder–decoder network without fully connected layers
  - Shift-invariant mapping of the convolution to encode sinogram data into feature maps (convolutional encoding)
    - Spatial down-sampling to introduce space variance (needed for domain transform)
    - Convolutional decoding with spatial up-sampling



Häggström, I., Schmidtlein, C. R., Campanella, G., & Fuchs, T. J. (2019). DeepPET: A deep encoder–decoder network for directly solving the PET image reconstruction inverse problem. *Medical image analysis*, 54, 253-262.

Direct reconstruction methods (iCT-Net)

- The intelligent CT network (iCT-Net) is a deep neural network based on multi-channel convolutional layers intended for image reconstruction of truncated data (i.e., the limited and sparse angle problem, the exterior problem and the interior problem)

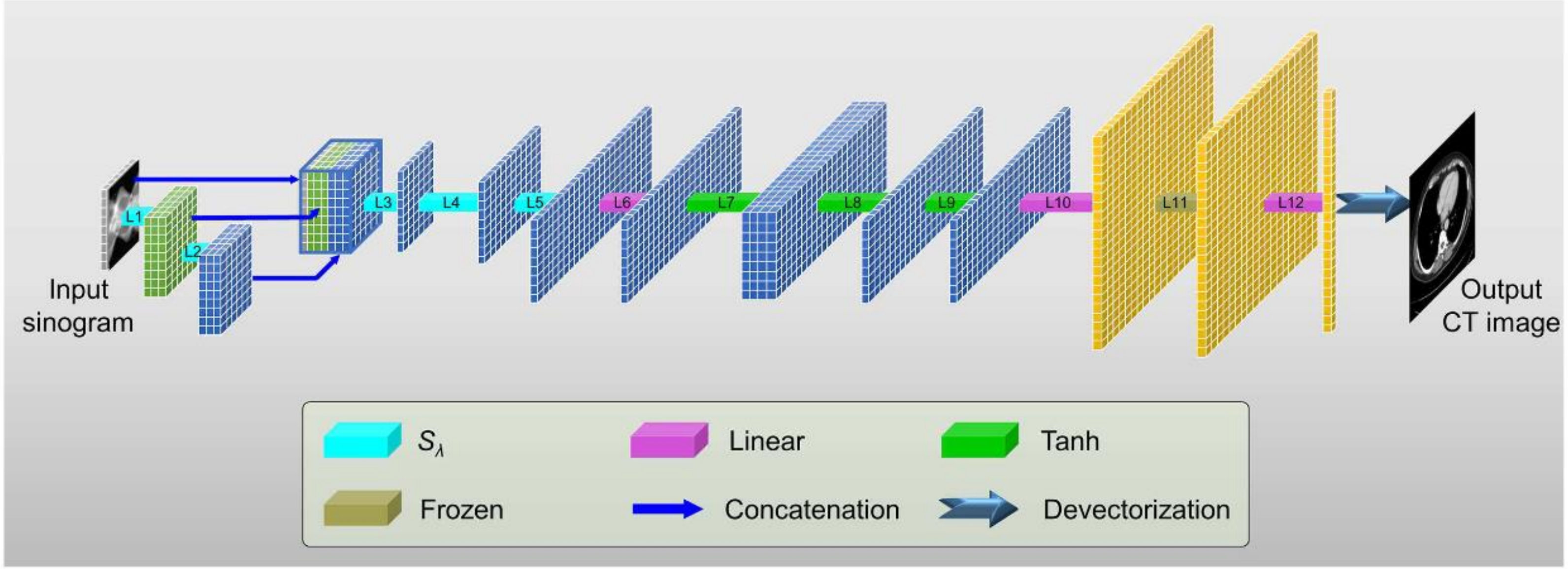


  - The design of the iCT-Net is inspired by the filtered back-projection pipeline which consists of three major cascaded steps

Li, Y., Li, K., Zhang, C., Montoya, J., & Chen, G. H. (2019). Learning to reconstruct computed tomography images directly from sinogram data under a variety of data acquisition conditions. *IEEE transactions on medical imaging*, 38(10), 2469-2481.

Direct reconstruction methods (iCT-Net)

- Five convolutional layers (L1-L5 in figure) to suppress noise and convert a sparse-view sinogram into a dense-view sinogram (i.e., manifold learning or pre-processing step)
- Four convolutional layers (L6-L9 in figure) to extract features (i.e., filtering)
- A fully connected layer (L10 in figure) to perform the domain transform from the extracted feature space to image space (i.e., back-projection step)
- Two convolutional layers (L11-L12 in figure) to learn a combination of the image from each view (i.e., summation step but with learnable summation weights)
  - The rotational symmetry of the back-projection is explicitly implemented to reduce the dimensionality of the network



Li, Y., Li, K., Zhang, C., Montoya, J., & Chen, G. H. (2019). Learning to reconstruct computed tomography images directly from sinogram data under a variety of data acquisition conditions. *IEEE transactions on medical imaging*, 38(10), 2469-2481.

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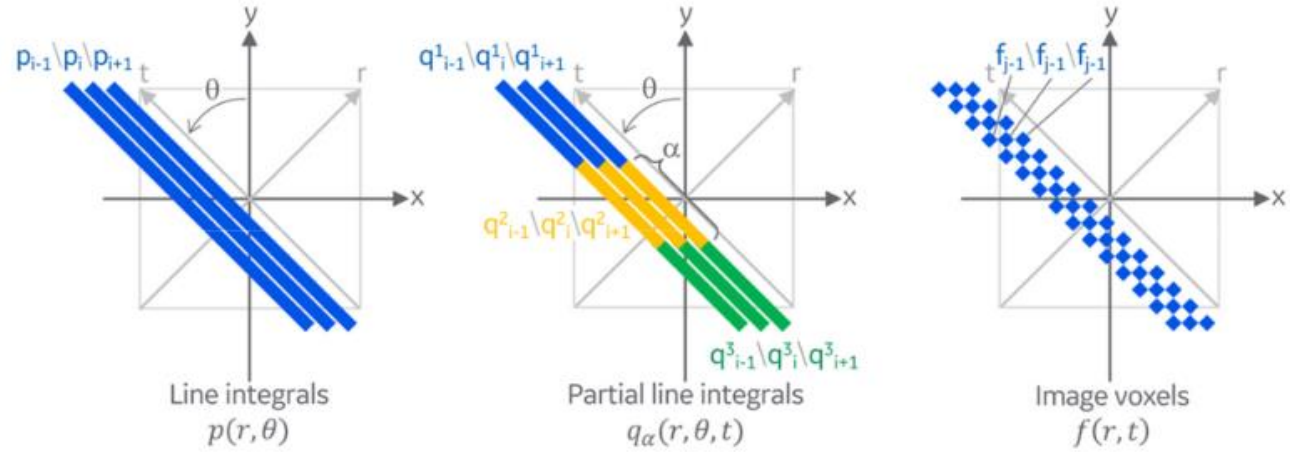
Direct reconstruction methods (hierarchical deep learning)

- In the hierarchical deep learning, the image reconstruction is fully learnt by interpreting the domain transform as a continuum of intermediate representations between the input and output data
    - A partial line integral is proposed as an intermediate representation between line integral and voxel according to a hierarchical framework
    - The reconstructed image is made by voxels, which are essentially line integrals over the "length" of the voxel



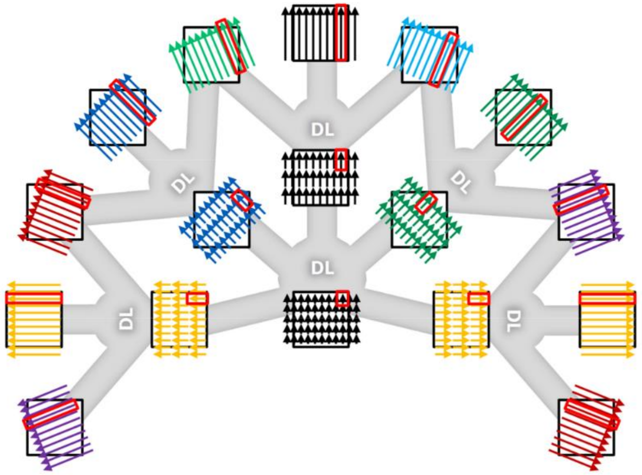
Fu, L., & De Man, B. (2019, May). A hierarchical approach to deep learning and its application to tomographic reconstruction. In *15th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine* (Vol. 11072, p. 1107202). SPIE.

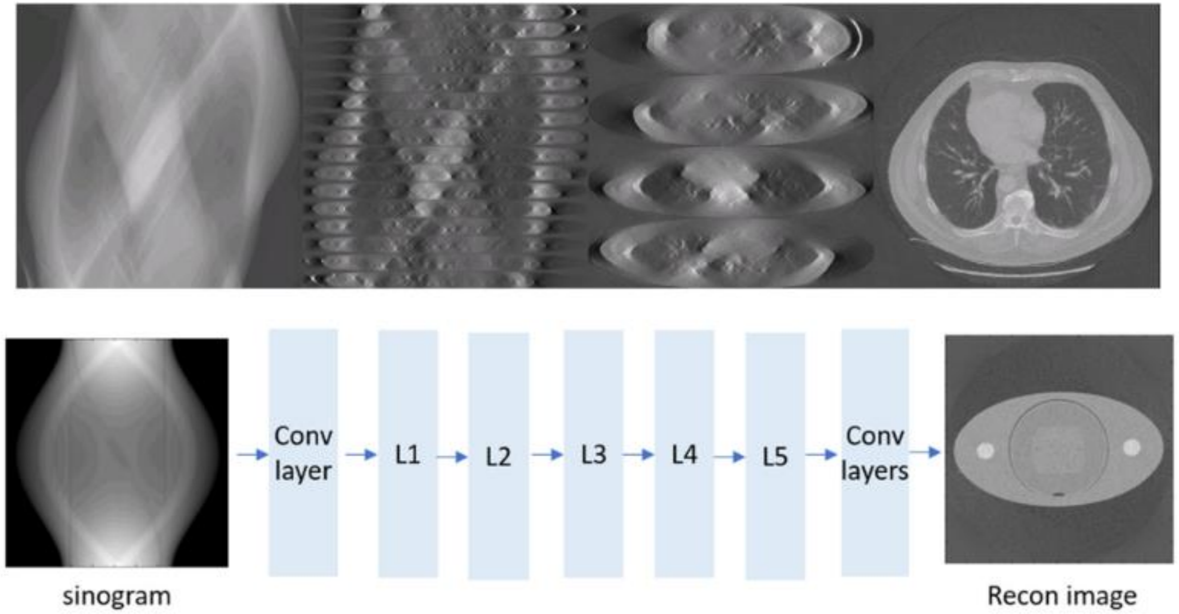
Direct reconstruction methods (hierarchical deep learning)

- The estimation of the partial line integrals only requires the line integrals that are at nearby angular positions and at nearby radial positions

- Similarly, the estimation of the voxel values requires as inputs only the partial line integrals that are at nearby radial and depth positions

    - Sparse connections layers

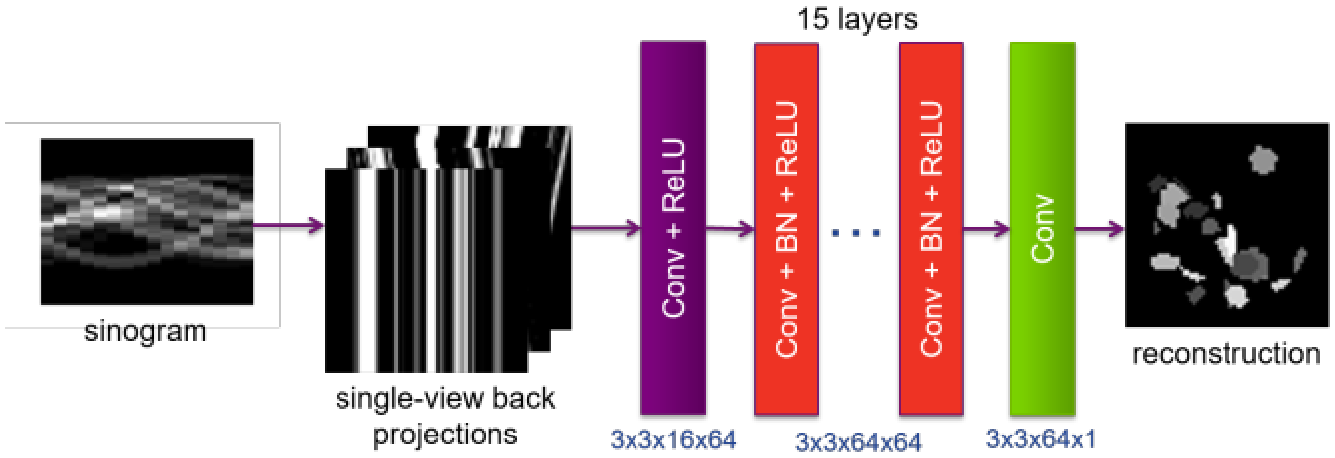




Fu, L., & De Man, B. (2019, May). A hierarchical approach to deep learning and its application to tomographic reconstruction. In *15th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine* (Vol. 11072, p. 1107202). SPIE.

Direct reconstruction methods (DBP)

- With the Deep Back-Projection (DBP) the geometrical relationship between the projection domain and the image domain is encoded in single-view back-projections that are stacked and then fed as input into the convolutional neural network
  - Typically, image reconstruction needs non-linear and shift-variant mapping, as introduced by fully connected layers, down-sampling (encoding) and then up-sampling (decoding)
  - In this case, the spatial invariance of the purely convolutional neural network is retained



Ye, D. H., Buzzard, G. T., Ruby, M., & Bouman, C. A. (2018, November). Deep back projection for sparse-view CT reconstruction. *In 2018 IEEE Global Conference on Signal and Information Processing (GlobalSIP)* (pp. 1-5). IEEE.

Unrolled iterative reconstruction methods

- Interpretability and generalizability in medical applications play fundamental roles but deep neural networks are usually difficult to interpret because of the huge number of parameters
  - Algorithm unrolling or unfolding is proposed to improve the interpretability and the generalizability (i.e., overfitting) of the deep neural network
    - Explicitly connected to the iterative algorithms used in imaging (and signal processing)
    - Explicitly based on domain knowledge as in imaging (and signal processing)
  - If the normal operator of the forward-projection model is a convolution (i.e., denoising and deblurring in the back-projection model), convolutional neural networks take part of unrolled iterative reconstruction methods

$$\begin{bmatrix} t_1 \\ \vdots \\ t_J \end{bmatrix} \approx \begin{bmatrix} a_{11} & & a_{1J} \\ & & \\ a_{J1} & & a_{JJ} \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_J \end{bmatrix}$$

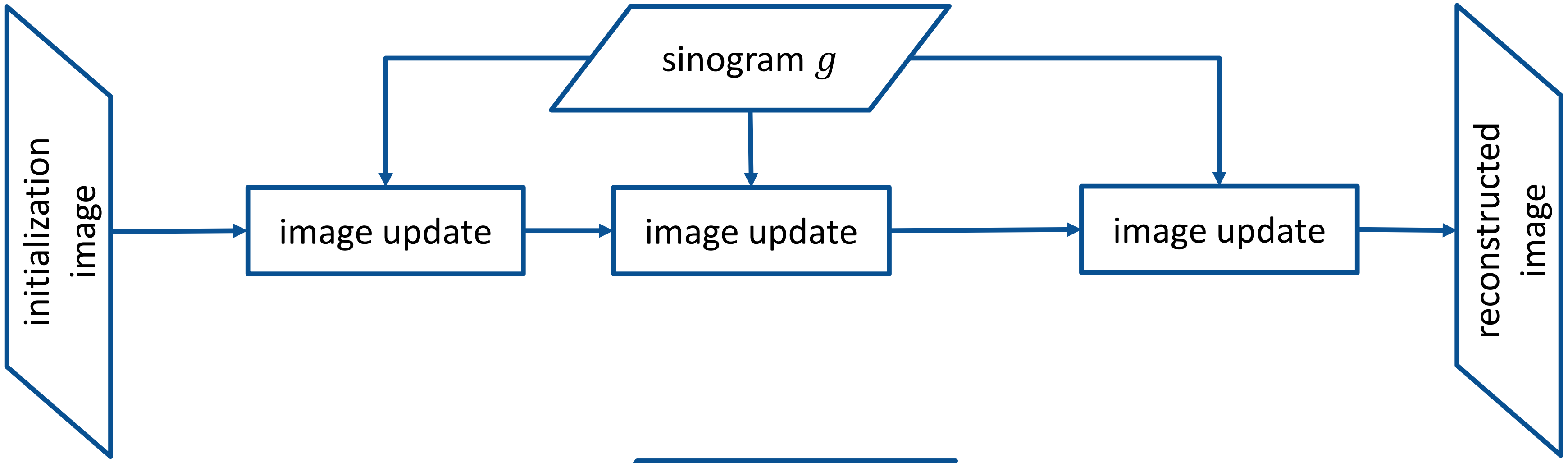
    - Denoising and deblurring can be described by convolutional layers

Unrolled iterative reconstruction methods

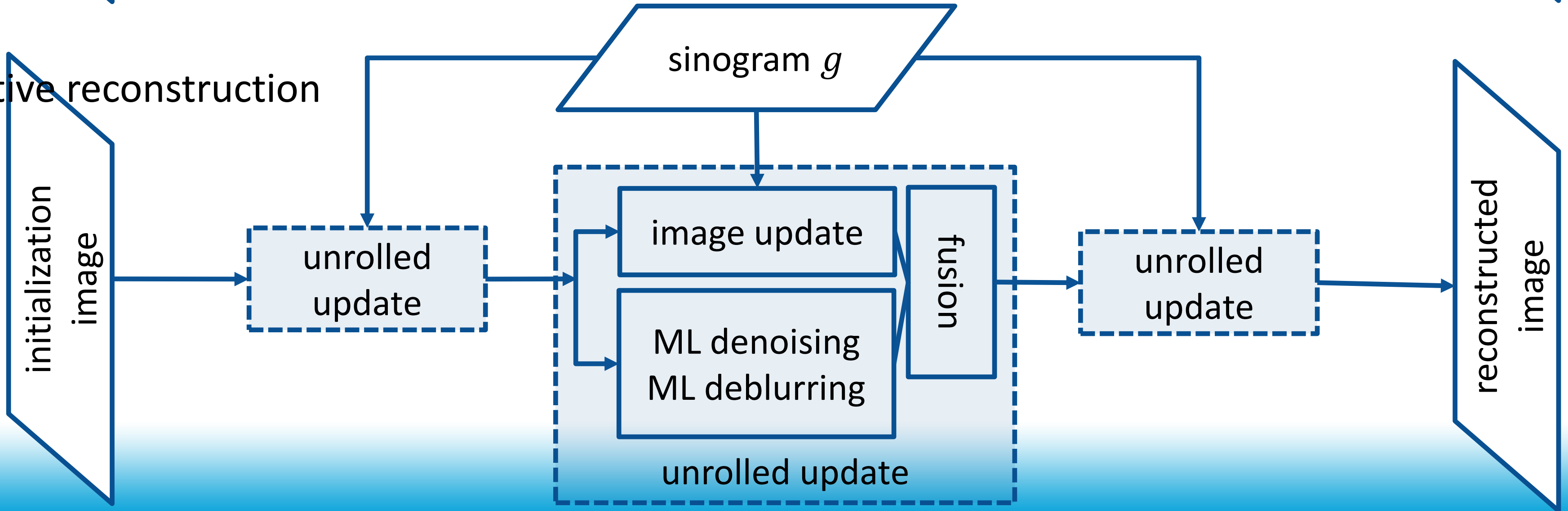
- Each iteration of the algorithm is represented as one layer of the network
  - Concatenating these layers forms a deep neural network
  - The number of layers in a deep network is typically much smaller than the number of iterations required in an iterative algorithm
- The network can be trained through all layers (i.e., end-to-end training) or layer per layer (i.e., sequential training)

Unrolled iterative reconstruction methods

- "Unfolded" iterative reconstruction



- Unrolled iterative reconstruction

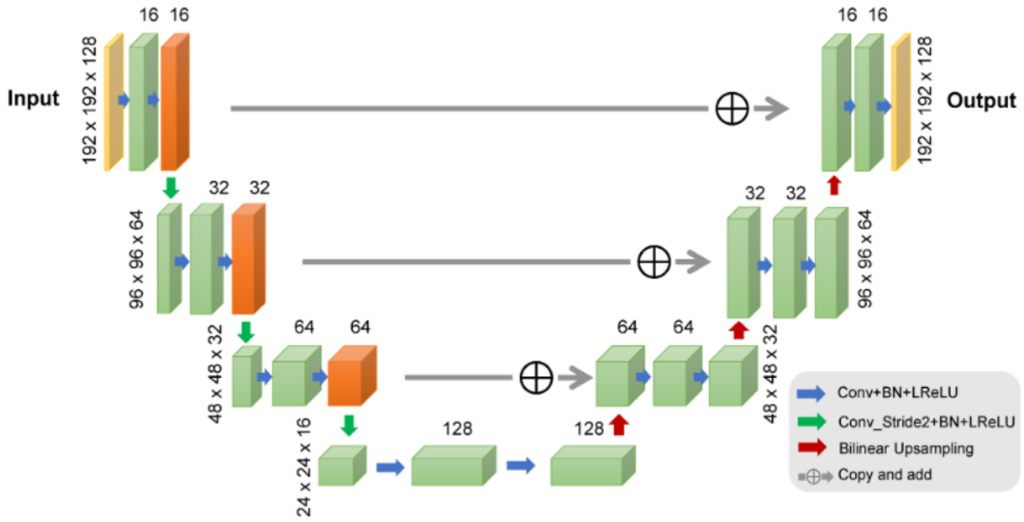


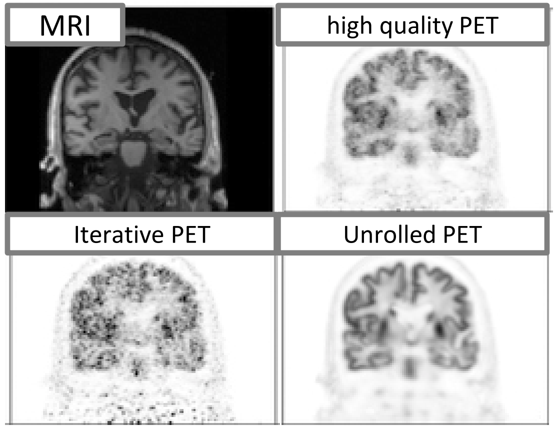
Unrolled iterative reconstruction methods

- Unrolled iterative reconstruction methods are typically image domain networks intended to reduce the image noise due to low count statistics PET sinograms based on prior information from high quality imaging

  - The residual network, as a modified fully convolutional network (U-net), is trained using patient-specific prior information (i.e., MRI image) and the measured data (i.e., PET sinogram) so that no prior training pairs are needed

  - The training of the network is embedded in the tomographic image reconstruction algorithm (i.e., the training of the network is iterated based on the iteratively reconstructed image…)

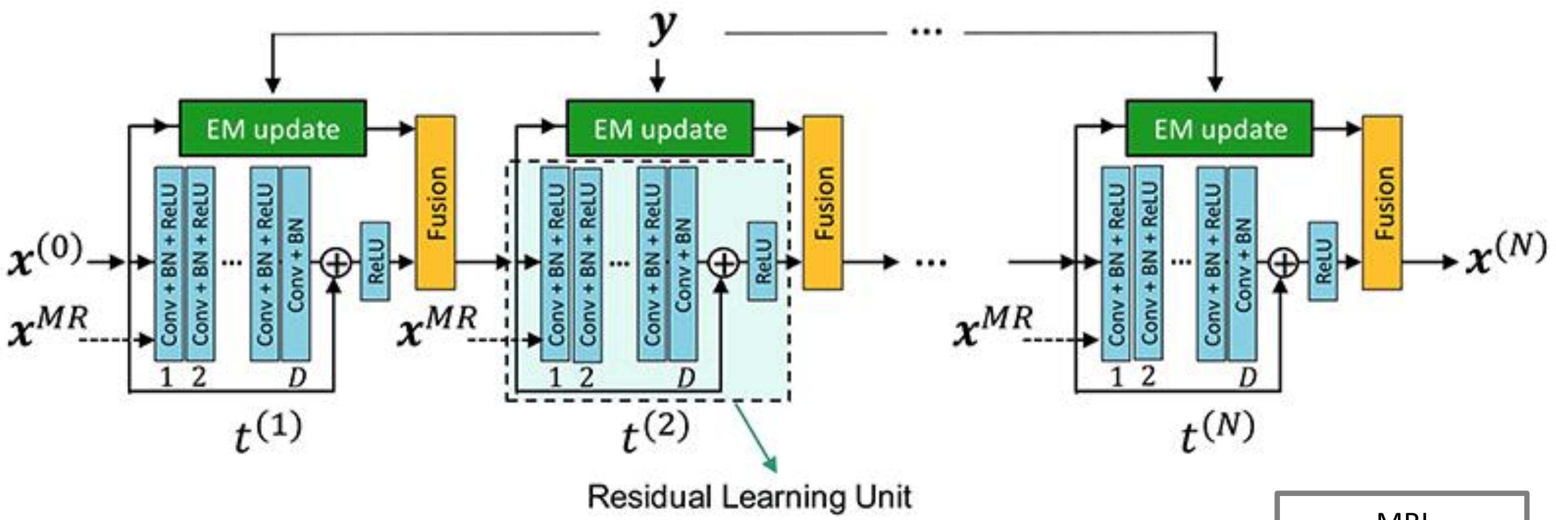


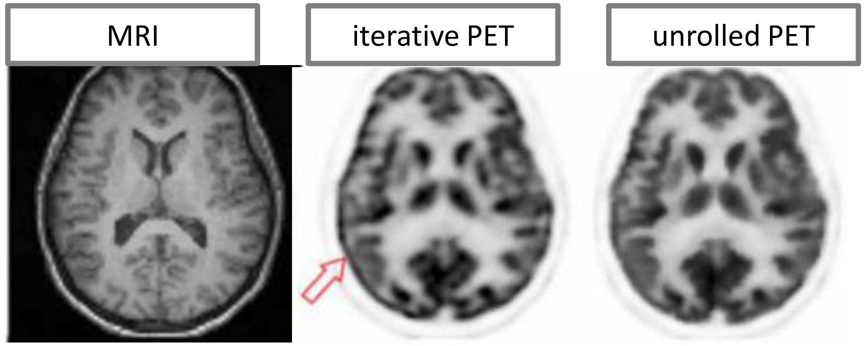


Gong, K., Catana, C., Qi, J., & Li, Q. (2018). PET image reconstruction using deep image prior. *IEEE transactions on medical imaging*, 38(7), 1655-1665.

Unrolled iterative reconstruction methods

- Residual network based on convolutional layers trained during reconstruction



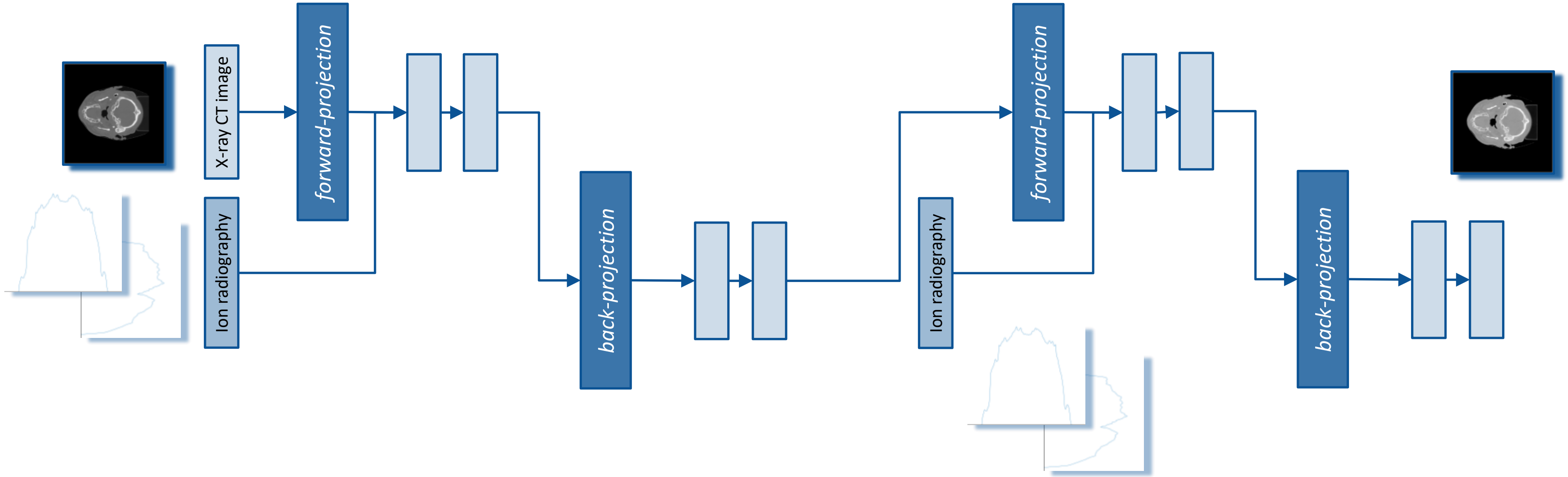


Mehranian, A., & Reader, A. J. (2020). Model-based deep learning PET image reconstruction using forward–backward splitting expectation–maximization. *IEEE transactions on radiation and plasma medical sciences*, 5(1), 54-64.

Unrolled iterative reconstruction methods

- The state-of-the-art in deep learning for the solution of inverse problems is the Learned PrimalDual architecture (i.e., physics-informed design), which delegates to the network also the image update

  - The Learned PrimalDual architecture explicitly contains the forward- and back-projections models, which are applied to the input data multiple times, as an unrolled tomographic image reconstruction algorithm



Adler, J., & Öktem, O. (2018). Learned primal-dual reconstruction. *IEEE transactions on medical imaging*, 37(6), 1322-1332.

Outlook and conclusions

- Analytical image reconstruction is based on the Fourier Slice Theorem that mathematically defines the correspondence between the sinogram and the image

- Numerical image reconstruction is based on the forward-projection model that defines the correspondence between the measurements and the unknowns, so that the iterative algorithm matches the forward-projection model with the measurements

- Deep image reconstruction is instead based on the learning of the ground truth image

  - The network can include the domain conversion in different ways

    - By means of fully connected layers, thus informing the network about the space variance of the correspondence between image and sinogram (i.e., learnt from the network)

    - By means of encoding and decoding branches, thus introducing a certain space variance to the network (i.e., learnt from the network)

    - Explicitly given as input to the network in form of back-projection tensor