LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Fakultät für Physik im WiSe 2023/24
TMP - TA4: Condensed Matter Many-Body-Physics and Field Theory II
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https://www2.physik.uni-muenchen.de/lehre/vorlesungen/wise_23_24/TMP-TA4/index.html

Sheet 6:

Hand-out: Tuesday, Nov. 28, 2023; Solutions: Tuesday, Dec. 05, 2023

Problem 1 Linear spin wave theory and magnon excitations

In this problem, we study the ordered phase of a ferromagnetic quantum Heisenberg model, and explore its low-energy excitations, that are magnons. The ferromagnetic Heisenberg model is descibed by the Hamiltonian

$$\hat{H} = -J \sum_{\langle i,j \rangle} \left[\hat{S}_i^z \hat{S}_j^z + \frac{1}{2} \left(\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+ \right) \right], \tag{1}$$

where $J > 0$, and $\langle i, j \rangle$ denotes nearest neighbor sites. We first look into the ground state properties of $\hat{H}$.

(1.a) Construct all the possible ground states of the Heisenberg model, and find their degeneracy.

(1.b) Calculate the ground state energy E_G.

We now investigate the elementary low-energy excitations of $\hat{H}$. Consider the ground state $|\emptyset\rangle$ where all spins are in the $-z$ direction. An excitation can be created by applying $\hat{S}^+$ on a single spin.

(1.c) Construct the eigenstate $|\psi_1\rangle$ of the first excited state. This a state with a spin wave. *Hint: Use the translational invariance of the system.*

(1.d) Calculate the excitation energy and discuss its long wavelength behavior.

We now study the spectrum of magnons, that are quantized spin wave excitations of a magnetically ordered system. Assume again the ground state $|\emptyset\rangle$ in the last section, where S (the total spin) is sufficiently large.

(1.e) Find an expression for the Hamiltonian $\hat{H}$ in terms of Holstein-Primakoff transformation, and discuss the bosonic nature of the excitations of the Hamiltonian.

(1.f) Compute the spectrum of the magnons. *Hint: for spins along $-z$, the Holstein-Primakoff transformation is defined by* $\hat{S}^z = -S + \hat{a}^\dagger \hat{a}$, $\hat{S}^+ = \sqrt{2S}\hat{a}^\dagger \sqrt{1 - \frac{\hat{a}^\dagger \hat{a}}{2S}}$, $\hat{S}^- = \sqrt{2S}\sqrt{1 - \frac{\hat{a}^\dagger \hat{a}}{2S}}\hat{a}$.

Problem 2 Journal Club

In the journal club, we will discuss the following paper.
Mourigal, M., Enderle, M., Klöpperpieper, A. et al. Fractional spinon excitations in the quantum Heisenberg antiferromagnetic chain. Nature Phys 9, 435–441 (2013). https://doi.org/10.1038/nphys2652.
Read the paper and try to answer the following question.

(2.a) How spin excitations fractionalize in spin-$1/2$ Heisenberg model?

(2.b) What is the primary method used in the experiments and what are the underlying physical principles?

(2.b) What is the goal of the experiment? How do they achieve this goal?

(2.c) How the experiments confirm the theoretical prediction of spinons?