



<https://moodle.lmu.de> → Kurse suchen: 'Rechenmethoden'

Sheet 03: Vector Product, Curves, Line Integrals

(b)[2](E/M/A) means: problem (b) counts 2 points and is easy/medium hard/advanced

Suggestions for central tutorial: example problems 3, 6, 7, 4.

Videos exist for example problems 4 (L4.3.1), 8 (V1.4.1).

Optional Problem 1: Natural parametrization of a curve [2]

Points: (a)[1](E); (b)[0.5](M); (c)[0.5](E).

Consider the curve $\mathbf{r}(t) = (t - \sin t, 1 - \cos t)^T \in \mathbb{R}^2$ for $t \in (0, 2\pi)$.

- (a) Sketch the curve qualitatively.
- (b) Determine its arc length, $s(t)$, in the time interval $(0, t)$. [Check your answer: $s(2\pi) = 8$.]
- (c) Find the natural parametrization, $\mathbf{r}_L(s)$. [Check your answer: $\mathbf{r}_L(4) = (\pi, 2)^T$.]

Optional Problem 2: Natural parametrization of a curve [4]

Points: (a)[1](M); (b)[0.5](E); (c)[0.5](E); (d)[1](E); (e)[1](E)

Consider the curve $\gamma = \{\mathbf{r}(t) \mid t \in (0, \tau)\}$, $\mathbf{r}(t) = e^{ct}(\cos \omega t, \sin \omega t)^T \in \mathbb{R}^2$, with $c \in \mathbb{R}$.

- (a) Sketch the curve for the case of $\tau = 8\pi/\omega$ and $c = 1/\tau$. [This information only applies to part (a), not for parts (b-e).]
- (b) Calculate the magnitude of the curve velocity, $\|\dot{\mathbf{r}}(t)\|$.
- (c) Calculate the arc length, $s(t)$, covered in the time interval $(0, t)$.
- (d) Determine the natural parametrization, $\mathbf{r}_L(s)$.
- (e) Check explicitly that $\left\| \frac{d\mathbf{r}_L}{ds} \right\| = 1$.

[Check your answer: for $c = \omega = \tau = 1$: (b) $\sqrt{2}e^t$, (c) $\sqrt{2}(e^t - 1)$, (d) $\mathbf{r}_L(s) = [s/\sqrt{2} + 1] \left(\cos[\ln(s/\sqrt{2} + 1)], \sin[\ln(s/\sqrt{2} + 1)] \right)^T$.]

[Total Points for Optional Problems: 6]
