Back-of-the-Envelope Physics

Winter Term 2022/23

Sheet 3

1. Determine the dimensions $[\vec{E}]$, $[\vec{B}]$, $[\varphi]$, $[\vec{A}]$, $[c]$, $[e]$ in the system of natural units (Gaussian system with $\hbar = c = 1$). Express the dimensions in units of energy.

2. Simplify the expression

$$\vec{A} \times (\vec{B} \times \vec{C})$$

by first arguing that it must be a linear combination of $\vec{B}$ and $\vec{C}$. Next, obtain the coefficients of $\vec{B}$ and $\vec{C}$ from dimensional analysis, up to numerical factors. Finally, fix the numerical factors from suitable (and simple) special cases.

Use the resulting formula to evaluate

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{V})$$

3. Show through explicit calculation that ($r \equiv |\vec{x}|$)

$$\left(\frac{1}{c^2}\partial_t^2 - \Delta\right) G(t,r) = 4\pi\delta(t)\delta(\vec{x}) \qquad \text{for} \qquad G(t,r) = \frac{\delta(t - r/c)}{r}$$

Use that

$$-\Delta\frac{1}{r} = 4\pi\delta(\vec{x})$$

4. Compute $\Delta(1/r)$ for $r \neq 0$. Use both cartesian and spherical coordinates for the Laplace operator Δ.

5. The charge density ϱ and the current density $\vec{j}$ of a moving point charge e can be written as

$$\varrho(t,\vec{x}) = e\,\delta(\vec{x} - \vec{r}(t))\,, \qquad \vec{j}(t,\vec{x}) = e\,\vec{v}(t)\,\delta(\vec{x} - \vec{r}(t))\,,$$

where $\vec{r}(t)$ is the trajectory of the charge and $\vec{v}(t) = d\vec{r}(t)/dt$ its velocity.

Show by explicit calculation that the continuity equation holds for this case.