Back-of-the-Envelope Physics

Winter Term 2022/23

Sheet 2

1. Estimate the ground-state energy of the harmonic oscillator in quantum mechanics by using the uncertainty relation $p \cdot x \sim \hbar$ and minimizing the energy.

2. Consider the innermost electron in an atom with nuclear charge Z. At what values of Z do we have to worry about relativistic effects?

3. Compute the area A of an arbitrary triangle with sides a, b, c directly, taking c as the base line and using the expression $A = ch/2$, with h the height of the triangle above the base c. Show the equivalence of the result with Heron's formula.

4. Show that Heron's formula for the area of a triangle implies Pythagoras' theorem.

5. The one-dimensional diffusion equation for a density $n = n(x,t)$ is given by

$$\partial_t n = D\,\partial_x^2 n \tag{1}$$

with diffusion constant D.

Show that for $t > 0$, $n = G(x,t)$ with

$$G(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left[-\frac{x^2}{4Dt}\right]$$

is a solution of the diffusion equation. Determine the normalization of G

$$\int_{-\infty}^{\infty} dx\, G(x,t)$$

and the initial distribution $G(x,0)$.

Write down the general solution $n(x,t)$ of (1) for $t > 0$ when $n(x,0) = f(x)$, with $f(x)$ a given function.

Finally, evaluate and interpret

$$\langle x^2 \rangle \equiv \int_{-\infty}^{\infty} dx\, x^2\, G(x,t)$$