Back-of-the-Envelope Physics

Winter Term 2022/23

Sheet 11

1. The relativistic Euler equations for an ideal fluid are given by

$$\partial_\nu T^{\mu\nu} = 0\,, \qquad T^{\mu\nu} = (\varrho + P)u^\mu u^\nu - P g^{\mu\nu} \tag{1}$$

Derive the nonrelativistic limit of these equations.

2. Estimate the mean free path ℓ of a photon in the sun. Using this result, estimate the time Δt it takes for a photon from the center of the sun to reach the surface.

3. Consider the matter in the sun as a nonrelativistic ideal gas.

a) Show from the equation of hydrostatic equilibrium that the average pressure $\bar{P}$, the gravitational potential energy E_g and the solar volume V are related by

$$\bar{P} = -\frac{E_g}{3V} \tag{2}$$

b) Determine the local thermal energy density dE_t/dV from the ideal gas law and derive the virial theorem for the total thermal energy E_t of the sun:

$$E_t = -\frac{E_g}{2} \quad \Leftrightarrow \quad E_{tot} \equiv E_t + E_g = -E_t \tag{3}$$

c) Using the virial theorem, find a typical temperature T for the interior of the sun (virial temperature). Note that the average particle mass in the hydrogen plasma is $\bar{m} = (m_e + m_p)/2$.