Inverse problems and machine learning in medical physics

Machine learning for tomographic image reconstruction or "deep reconstruction"

Dr. Chiara Gianoli
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chiara.gianoli@physik.uni-muenchen.de

Tomographic image reconstruction

- Tomographic image reconstruction represents the building block of medical imaging
- Tomographic image reconstruction has been classified as analytical reconstruction or as iterative reconstruction
- Very recently, data-driven, deep-learning-based tomographic image reconstruction has been introduced (i.e., deep tomographic reconstruction)
    - Direct reconstruction methods
    - Unrolled iterative reconstruction methods
- The huge benefit of machine learning in reconstruction is the use of the ground truth (i.e., supervised learning), as obtained from high quality simulations or high quality measurements

Analytical reconstruction

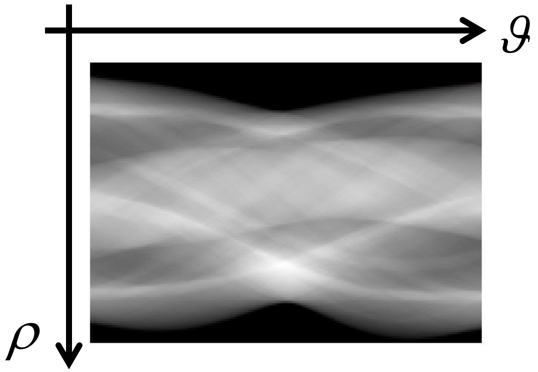
- Tomographic image acquisition can be modelled as a Radon transform, or sinogram, of the variable describing the physical properties of the object of interest
  - The Radon Transform converts an image from spatial domain to sinogram domain, by integrating the variables along the integration lines, as a function of the projection angles
  - The analytical image reconstruction is based on the Fourier slice theorem that puts in correspondence the Radon Transform with the Fourier Transform of the image (i.e., the filtered back-projection)

$$\hat{f}_{\rho}(w_x, w_y) = \int_{-\infty}^{+\infty} R(f) e^{-2\pi i(\rho w_{\rho})} d\rho = \hat{R}(w_{\rho})$$

Radon transform

Fourier transform

sinogram



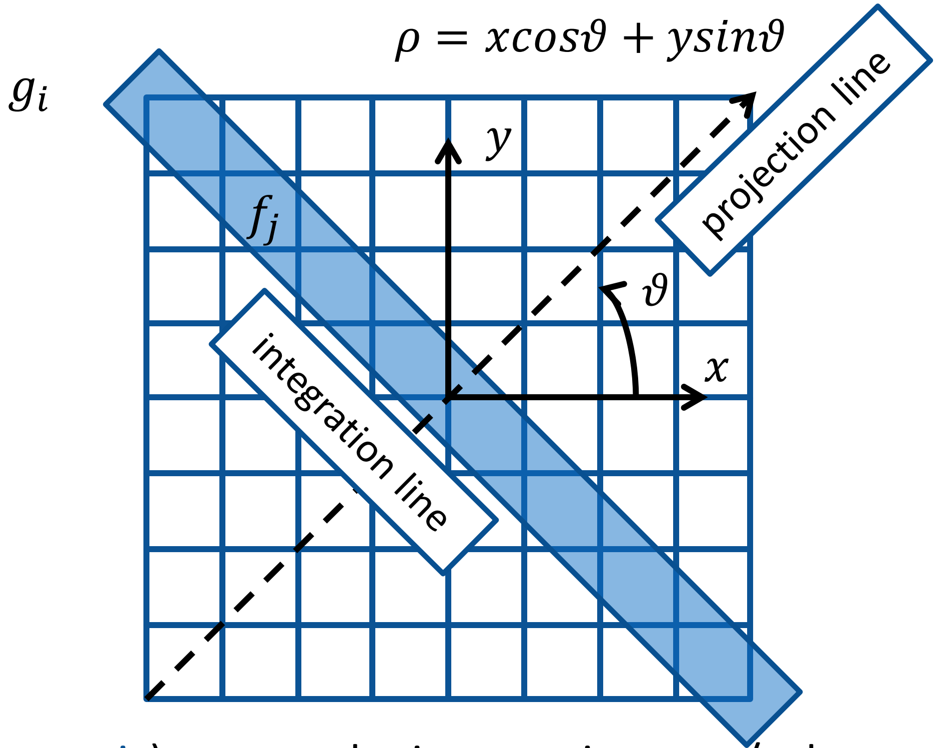
image

Iterative reconstruction

- The inverse problem of tomographic image reconstruction can be solved by means of numerical (iterative) algorithms
- Numerical algorithms can be considered as an iterative solver of a system of linear equations
  - I equations, one for each projection i
  - J unknowns, one for each pixel/voxel j

$$\bar{g}_i = \sum_j a_{ij} f_j$$

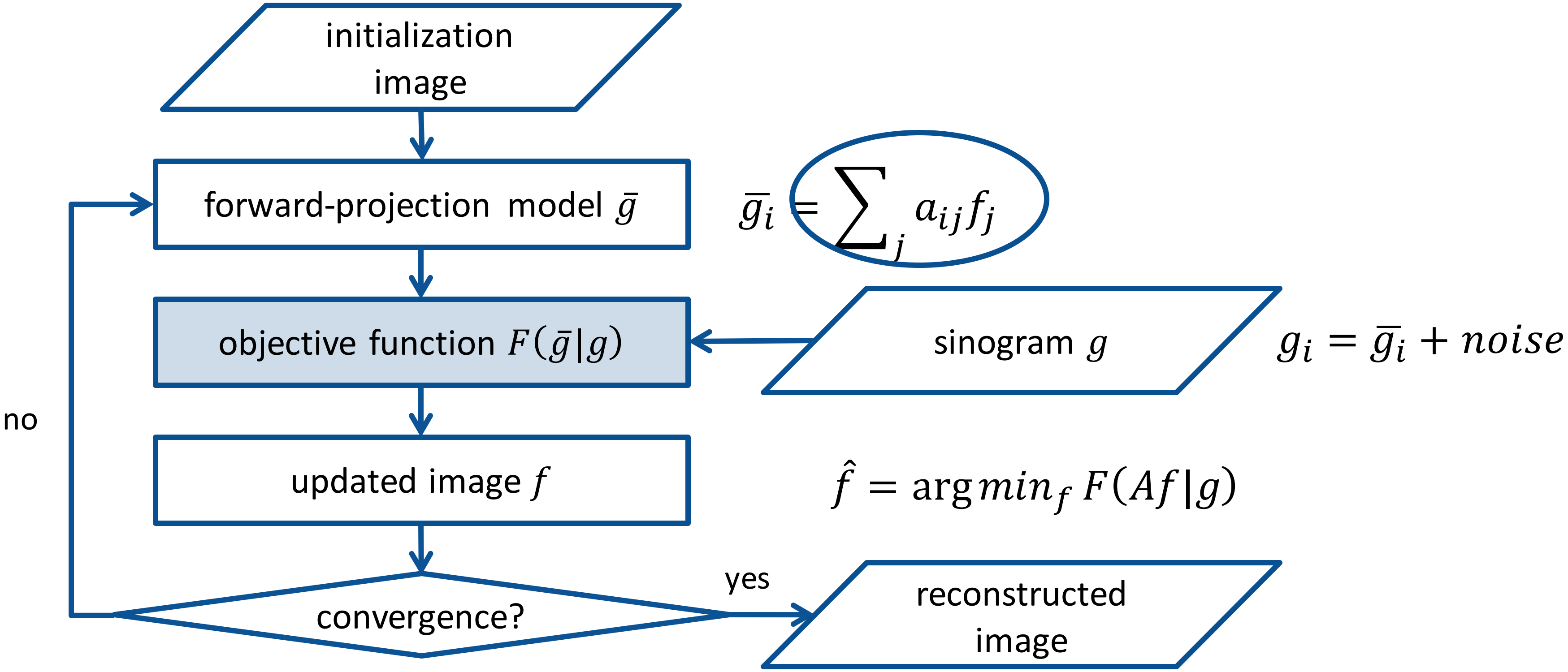
$$\begin{cases} a_{11} f_1 + a_{12} f_2 + \cdots\ a_{1J} f_J = \bar{g}_1 \\ \cdots \\ a_{I1} f_1 + a_{I2} f_2 + \cdots\ a_{IJ} f_J = \bar{g}_I \end{cases}$$



- The coefficients a_{ij} (i.e., the elements of the system matrix) express the intersection area/volume of the pixel/voxel j with the integration line of the projection i

Iterative reconstruction

- The iterative reconstruction paradigm is to find the image that minimizes the “discrepancy” between the forward-projection of the image (i.e., the model of the sinogram) and the acquired sinogram



Deep tomographic reconstruction

- The machine learning paradigm in tomographic image reconstruction is to find the parameters of the mapping function that infers the ground truth based on supervised prediction

sinogram g_{train}

ground truth t

loss function $L(\theta)$

mapping function

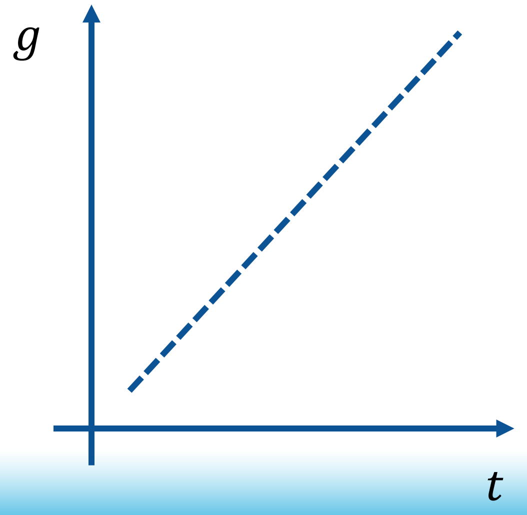
$$\hat{\theta} = \arg min_{\theta} L(DNN(g_{train}|\theta)|t)$$

sinogram g_{test}

mapping function

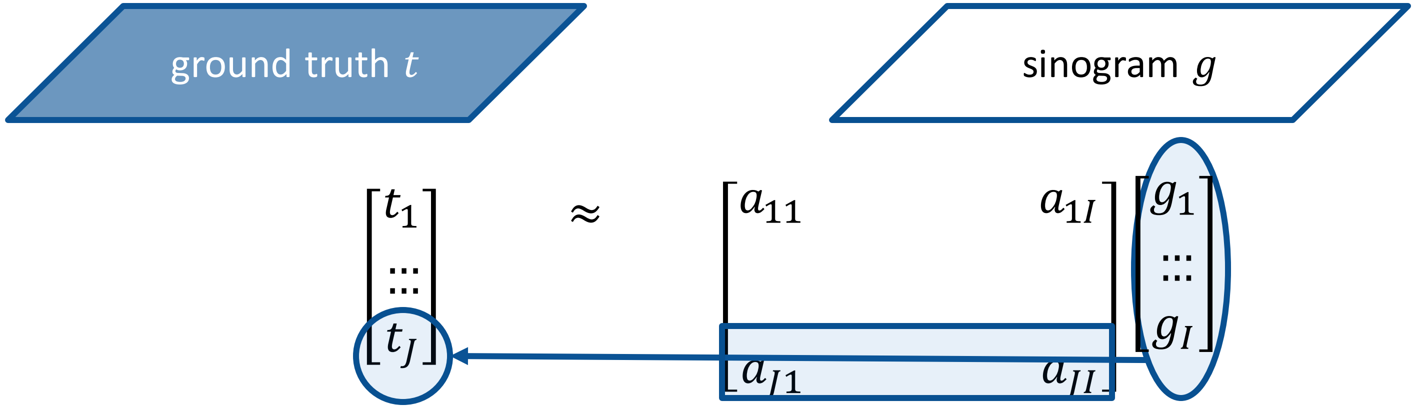
inferred image

$$\hat{t} = DNN(g_{test}|\hat{\theta})$$



Deep tomographic reconstruction

- The back-projection is a linear mapping (i.e., matrix-vector multiplication) that can be described by a fully connected layer (i.e., linear layer) of an artificial neural network (ANN)



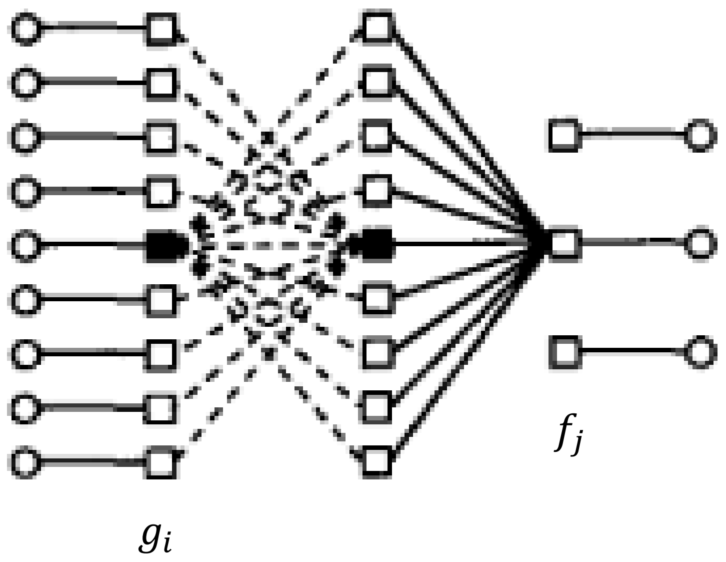
$$\begin{bmatrix} t_1 \\ \vdots \\ t_J \end{bmatrix} \approx \begin{bmatrix} a_{11} & & a_{1I} \\ & & \\ a_{J1} & & a_{JI} \end{bmatrix} \begin{bmatrix} g_1 \\ \vdots \\ g_I \end{bmatrix}$$

Deep tomographic reconstruction

- One of the first ML attempt to deep tomographic reconstruction was based on the “pre-calculation” of the filters for the filtered back-projection, instead of being analytically calculated each time…

  - The learnable weights (learning based on a point source) are applied along the projection lines of the sinogram

  - The back-projection is implemented for each projection lines of the sinogram as fully connected layer with non-learnable weights (rotational and shift-invariant approximation)



Floyd, C. E. (1991). An artificial neural network for SPECT image reconstruction. *IEEE transactions on medical imaging*, 10(3), 485-487.

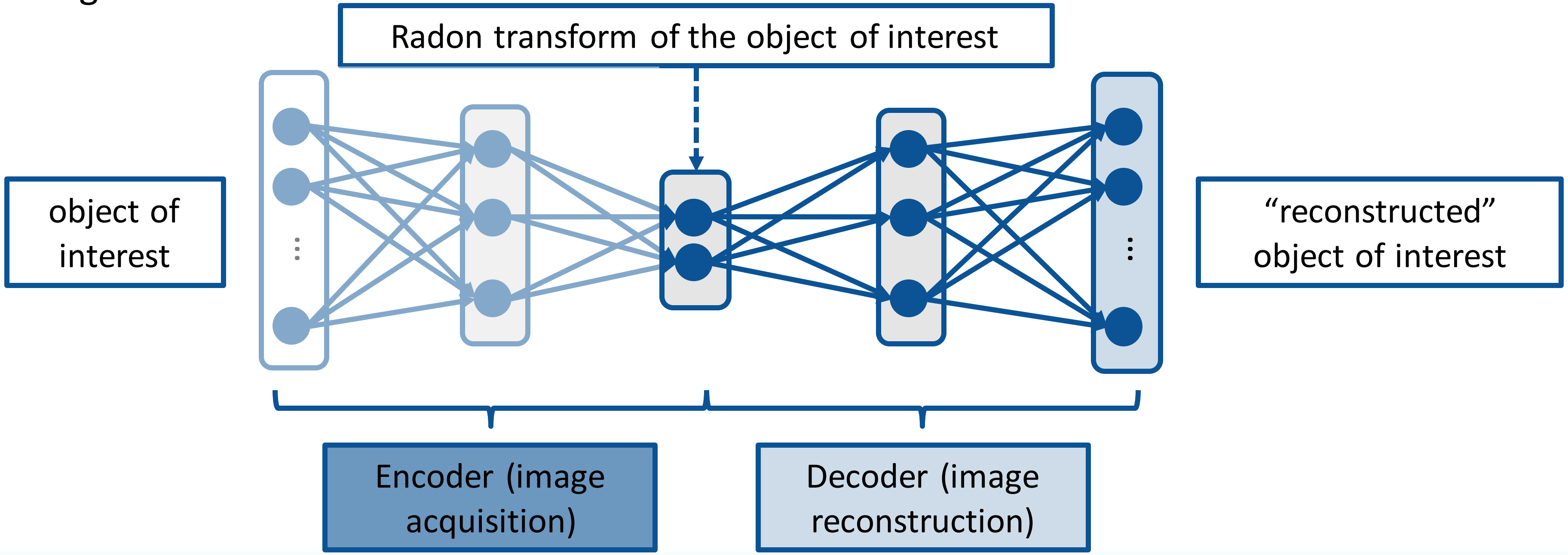
- In practice, this is suitable only for two-dimensional images

Exercise #2

- Define the image of an ideal point source (i.e., target data of the network)
- Simulate the sinogram of the ideal point source, then add noise and blur along each projection line (i.e., input data of the network)
- Implement a first fully connected network, connecting the nodes of the input layer (i.e., the projection line) to all the nodes of the hidden layer (i.e., the filtered projection line)
    - The forward-pass function is a weighed average of the inputs with unknown weights (i.e., no bias, no activation function)
- Implement a second fully connected network, connecting the nodes of the hidden layer (i.e., the filtered projection line) to all the nodes of the output layer (i.e., the image)
    - The forward-pass function is a weighed average of the inputs with known weights from the system matrix
- Train the network based on input and target data (implement the backward-pass based on the gradient descend algorithms)

Direct reconstruction methods

- The purpose of domain transform is to map the sinogram (i.e., the projections) to the image
  - The measured sinogram encodes an intermediate representation of the object of interest in the projection domain (i.e., the Radon transform), similar to an encoding function
  - The measured sinogram is subsequently reconstructed into an image by an inversion of the encoding function, similar to a decoding function



Direct reconstruction methods

- In direct reconstruction methods, the domain transformation can be explicitly learnt from the network or explicitly given as input to the network
  - Direct reconstruction methods can entail the encoding of the Radon transform into a lower dimensional space (i.e., compressed sensing) and the decoding of the encoded Radon transform, typically by means of convolutional layers
    - The compressed sensing hypothesis is that a signal can be represented by and perfectly recovered from few non-zero coefficients in a suitable basis (i.e., dictionary)
      - *Wavelet* and *Shearlet* are common basis functions (*https://www.math.uh.edu/~dlabate/SHBookIntro.pdf*)
  - Transformation into a lower dimensional space can be based on manifolds (i.e., manifold-based compressed sensing)
    - The manifold hypothesis is that high dimensional data (i.e., a continuous images) lie on low-dimensional manifolds (i.e., a point) in a high-dimensional space (i.e., an infinite dimensional vectoral space)
      - Zero-dimensional manifolds are points, one-dimensional manifolds are lines, two-dimensional manifolds are surfaces…

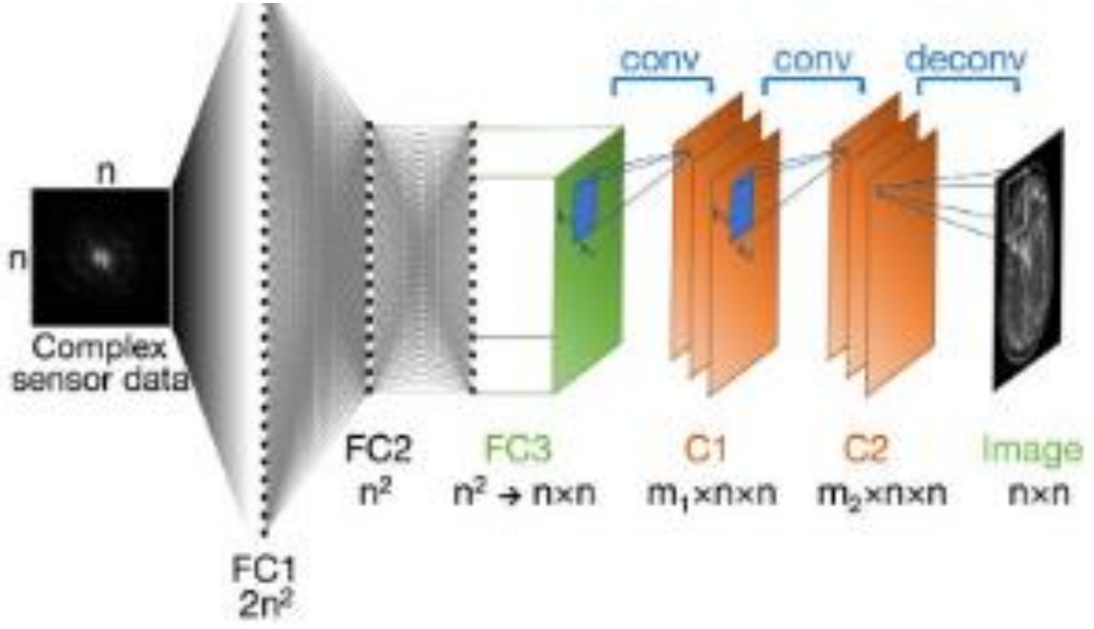
Direct reconstruction methods (AUTOMAP)

- The AUtomated TransfOrm by Manifold Approximation (AUTOMAP) is a deep neural network with feed-forward architecture, composed of multiple fully-connected layers followed by a sparse convolutional auto-encoder (i.e., encoder-decoder where the input and the output domains are the same)

- The network simultaneously learns an optimal convolutional domain (i.e., manifold space) and a sparse representations (i.e., compressed sensing) through a joint optimization (i.e., manifold encoding–decoding process)

  - Different from compressed sensing, the convolutional layers do not make hypothesis on the sparsifying transform (e.g., wavelet, shearlet…)

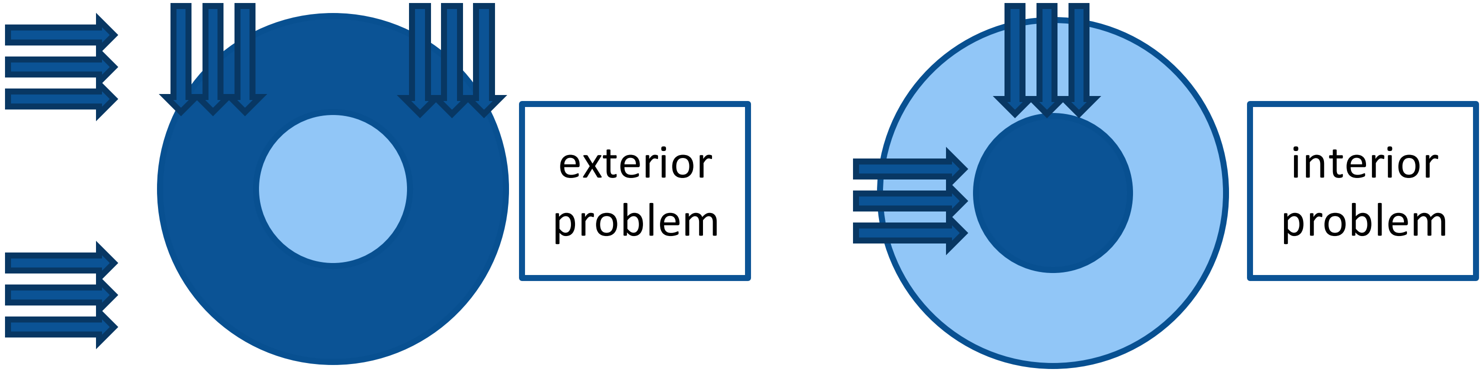
  - AUTOMAP is originally demonstrated for MRI but it is generally applicable to different image reconstruction problems



Zhu, B., Liu, J. Z., Cauley, S. F., Rosen, B. R., & Rosen, M. S. (2018). Image reconstruction by domain-transform manifold learning. *Nature*, 555(7697), 487-492.

Direct reconstruction methods (iCT-Net)

- The intelligent CT network (iCT-Net) is a deep neural network based on multi-channel convolutional layers intended for image reconstruction of truncated data (i.e., the limited and sparse angle problem, the exterior problem and the interior problem)



- The design of the iCT-Net is inspired by the filtered back-projection pipeline which consists of three major cascaded steps

Pre-processing of projection data (i.e., noise and beam hardening correction) → Projection data filtering → Back-projection for domain transform

Li, Y., Li, K., Zhang, C., Montoya, J., & Chen, G. H. (2019). Learning to reconstruct computed tomography images directly from sinogram data under a variety of data acquisition conditions. *IEEE transactions on medical imaging*, 38(10), 2469-2481.

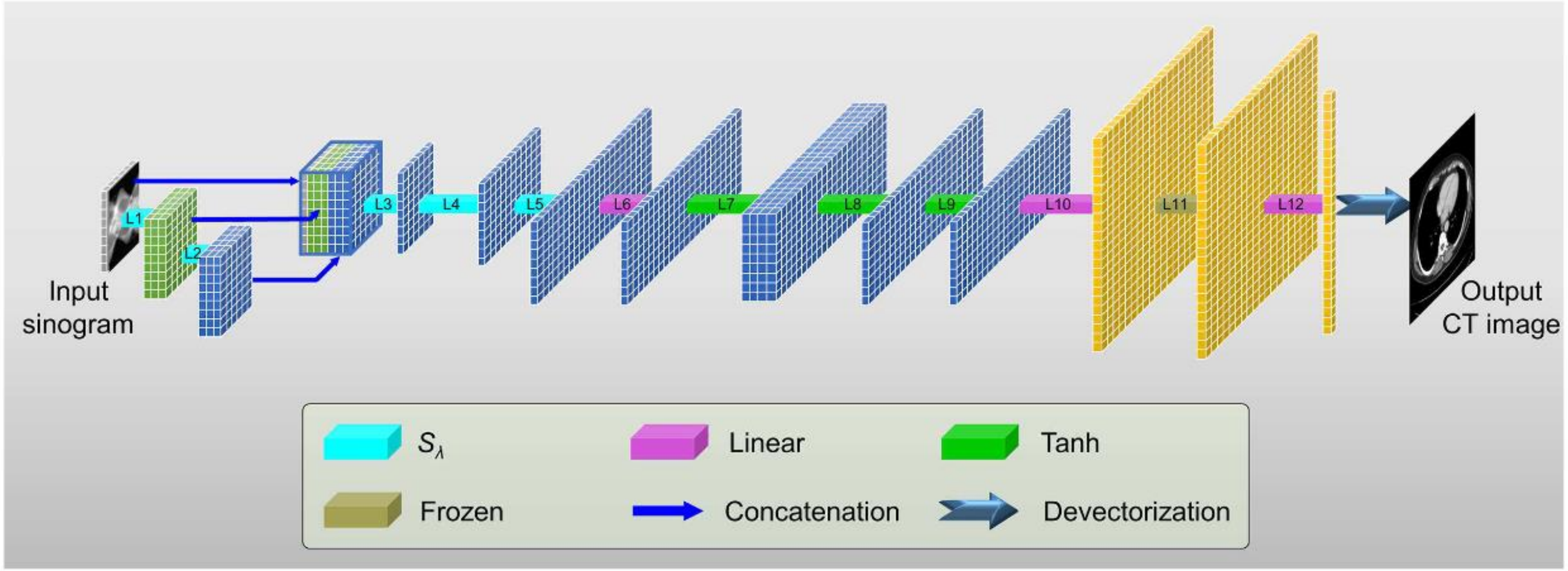
Direct reconstruction methods (iCT-Net)

- Five convolutional layers (L1-L5 in figure) to suppress noise and convert a sparse-view sinogram into a dense-view sinogram (i.e., manifold learning or pre-processing step)

- Four convolutional layers (L6-L9 in figure) to extract features (i.e., filtering)

- A fully connected layer (L10 in figure) to perform the domain transform from the extracted feature space to image space (i.e., back-projection step)

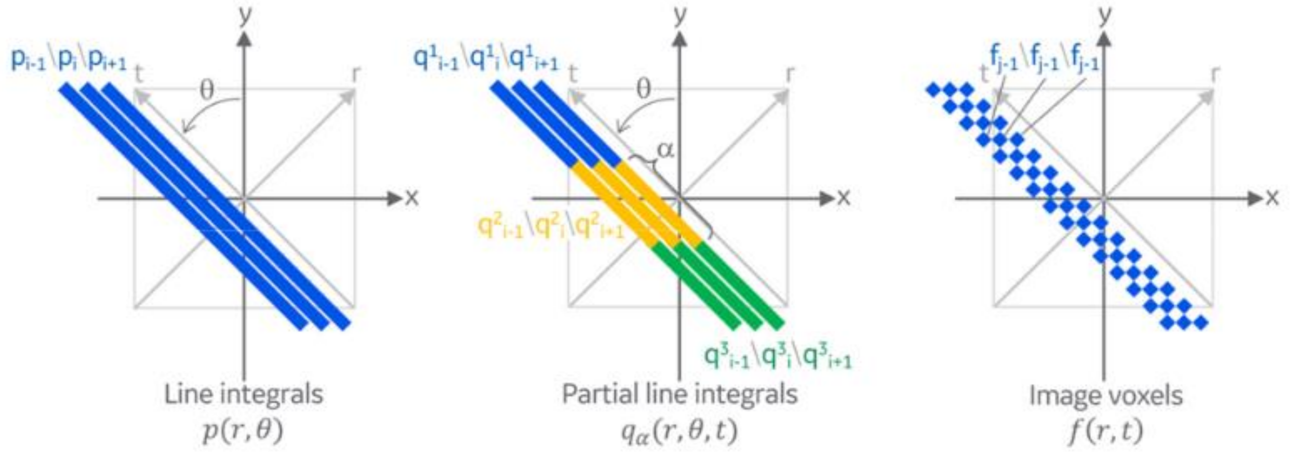
- Two convolutional layers (L11-L12 in figure) to learn a combination of the image from each view (i.e., summation step but with learnable summation weights)
  - The rotational symmetry of the back-projection is explicitly implemented to reduce the dimensionality of the network



Li, Y., Li, K., Zhang, C., Montoya, J., & Chen, G. H. (2019). Learning to reconstruct computed tomography images directly from sinogram data under a variety of data acquisition conditions. *IEEE transactions on medical imaging*, 38(10), 2469-2481.

Direct reconstruction methods (hierarchical deep learning)

- In the hierarchical deep learning, the image reconstruction is fully learnt by interpreting the domain transform as a continuum of intermediate representations between the input and output data
  - A partial line integral is proposed as an intermediate representation between line integral and voxel according to a hierarchical framework
  - The reconstructed image is made by voxels, which are essentially line integrals over the "length" of the voxel



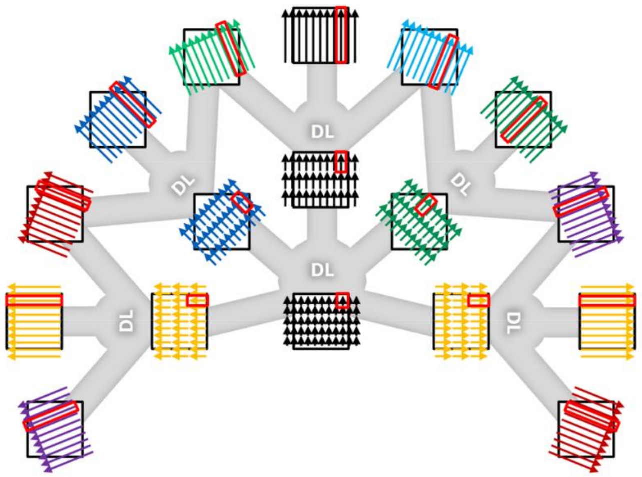
Fu, L., & De Man, B. (2019, May). A hierarchical approach to deep learning and its application to tomographic reconstruction. In *15th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine* (Vol. 11072, p. 1107202). SPIE.

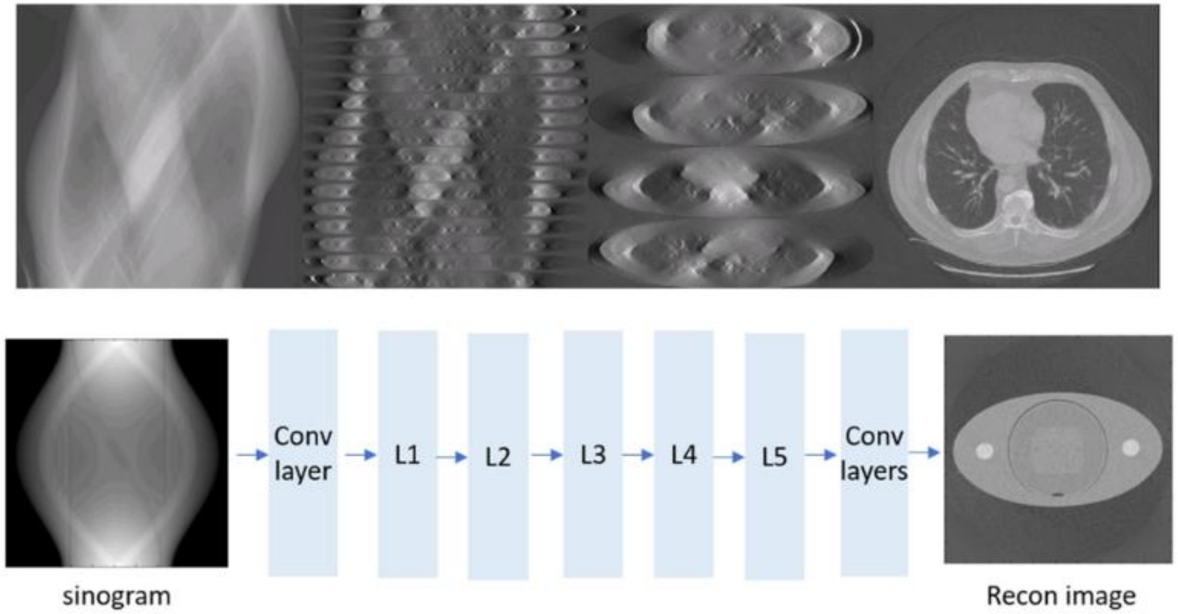
Direct reconstruction methods (hierarchical deep learning)

- The estimation of the partial line integrals only requires the line integrals that are at nearby angular positions and at nearby radial positions

- Similarly, the estimation of the voxel values requires as inputs only the partial line integrals that are at nearby radial and depth positions

    - Sparse connections layers

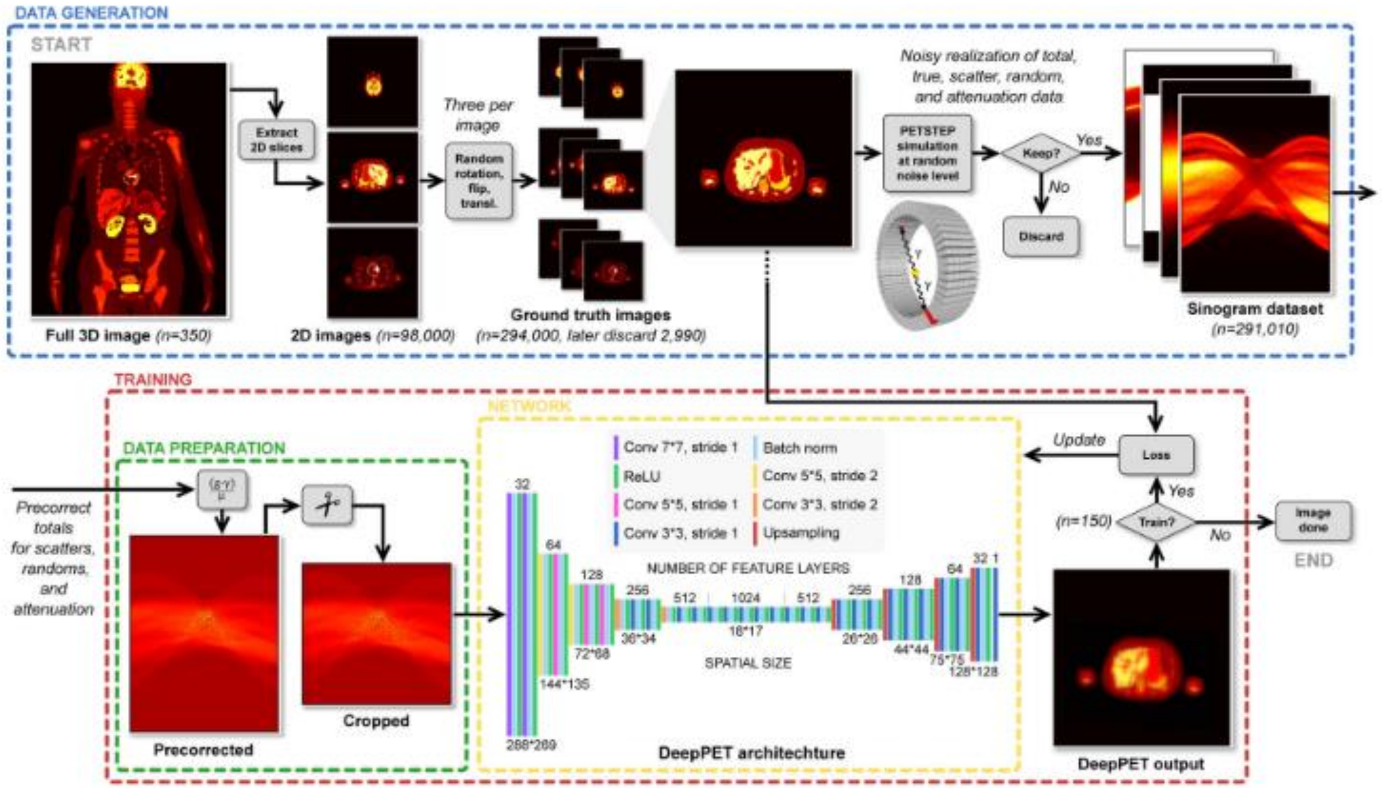




Fu, L., & De Man, B. (2019, May). A hierarchical approach to deep learning and its application to tomographic reconstruction. In *15th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine* (Vol. 11072, p. 1107202). SPIE.

Direct reconstruction methods (Deep PET)

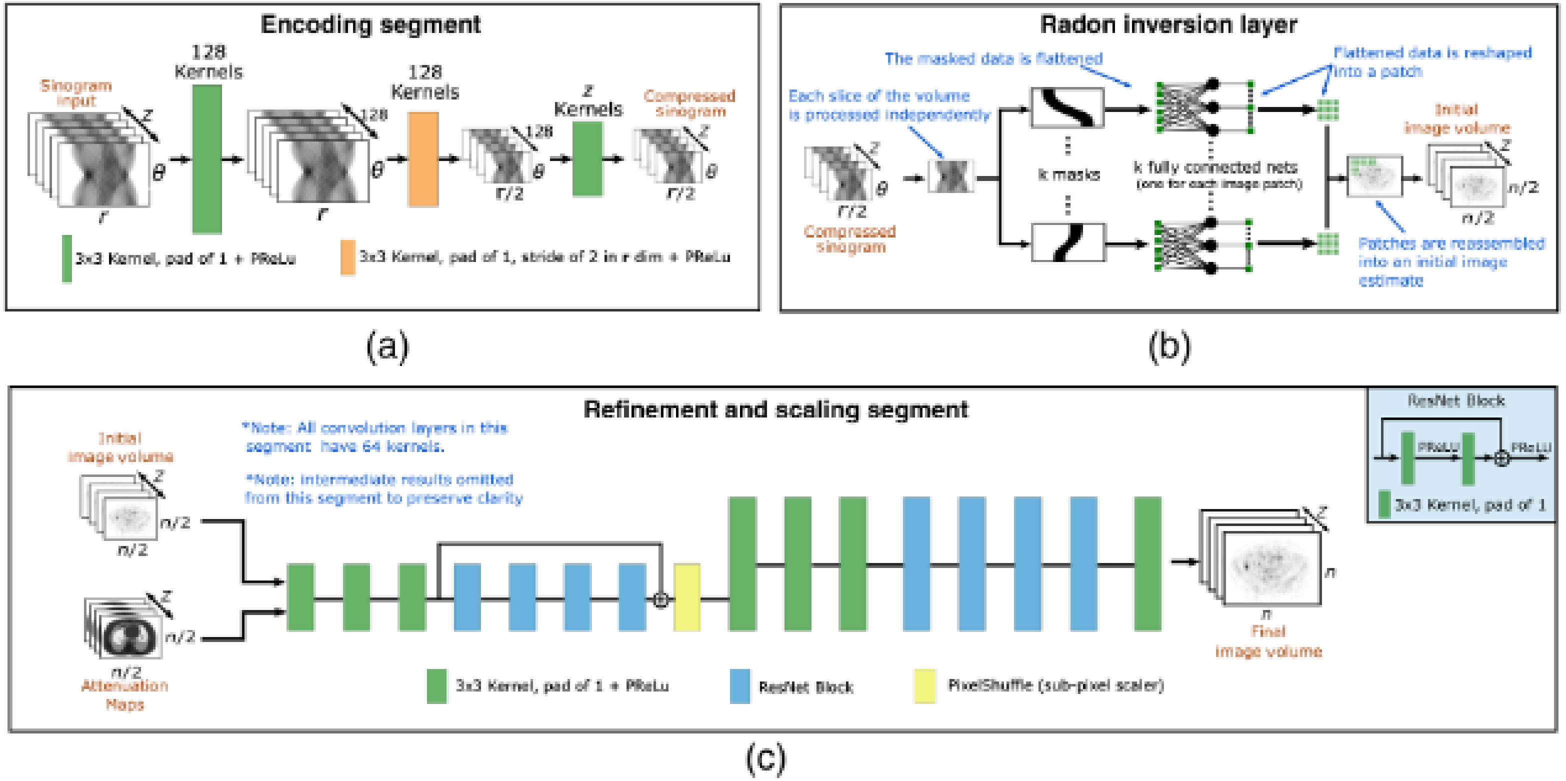
- The Deep PET is a convolutional encoder–decoder network without fully connected layers
  - Shift-invariant mapping of the convolution to encode sinogram data into feature maps (convolutional encoding)
    - Spatial down-sampling to introduce space variance (needed for domain transform)
    - Convolutional decoding with spatial up-sampling



Häggström, I., Schmidtlein, C. R., Campanella, G., & Fuchs, T. J. (2019). DeepPET: A deep encoder–decoder network for directly solving the PET image reconstruction inverse problem. *Medical image analysis*, 54, 253-262.

Direct reconstruction methods (DirectPET)

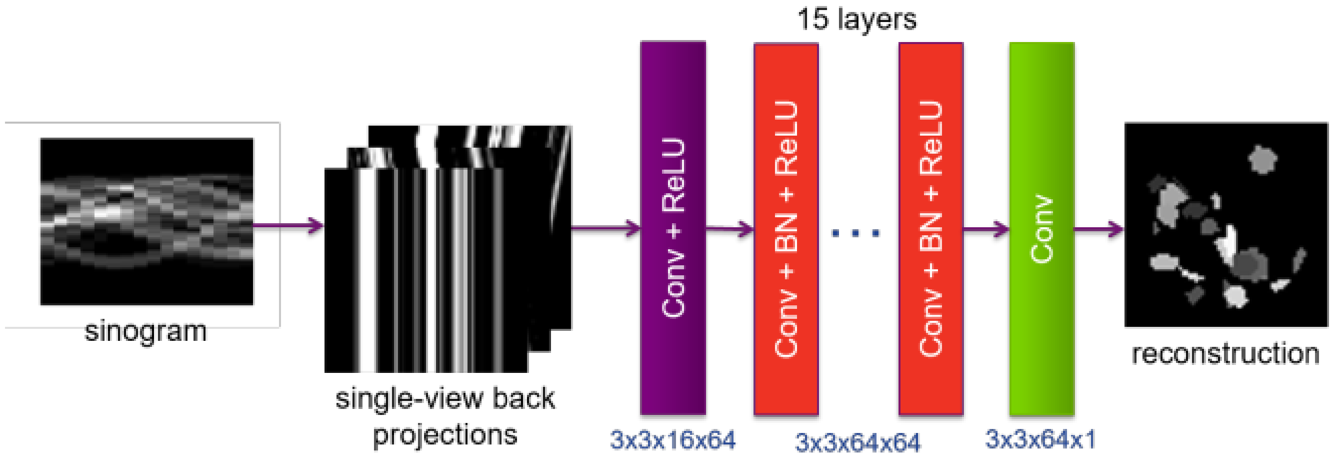
- The DirectPET is a large-scale direct neural network that performs image reconstruction by introducing a Radon inversion layer
  - An encoding segment compressing the sinogram data into a lower dimensional space
  - A domain transformation segment (i.e., Radon inversion) using sinogram data masking along with fully connected layers
  - A refinement and scaling segment enhancing and up-sampling the reconstructed image



Whiteley, W., Luk, W. K., & Gregor, J. (2020). DirectPET: full-size neural network PET reconstruction from sinogram data. *Journal of Medical Imaging*, 7(3), 032503.

Direct reconstruction methods (DBP)

- With the Deep Back-Projection (DBP) the geometrical relationship between the projection domain and the image domain is encoded in single-view back-projections that are stacked and then fed as input into the convolutional neural network
  - Typically, image reconstruction needs non-linear and shift-variant mapping, as introduced by fully connected layers, down-sampling (encoding) and then up-sampling (decoding)
  - In this case, the spatial invariance of the purely convolutional neural network is retained



Ye, D. H., Buzzard, G. T., Ruby, M., & Bouman, C. A. (2018, November). Deep back projection for sparse-view CT reconstruction. *In 2018 IEEE Global Conference on Signal and Information Processing (GlobalSIP)* (pp. 1-5). IEEE.

Unrolled iterative reconstruction methods

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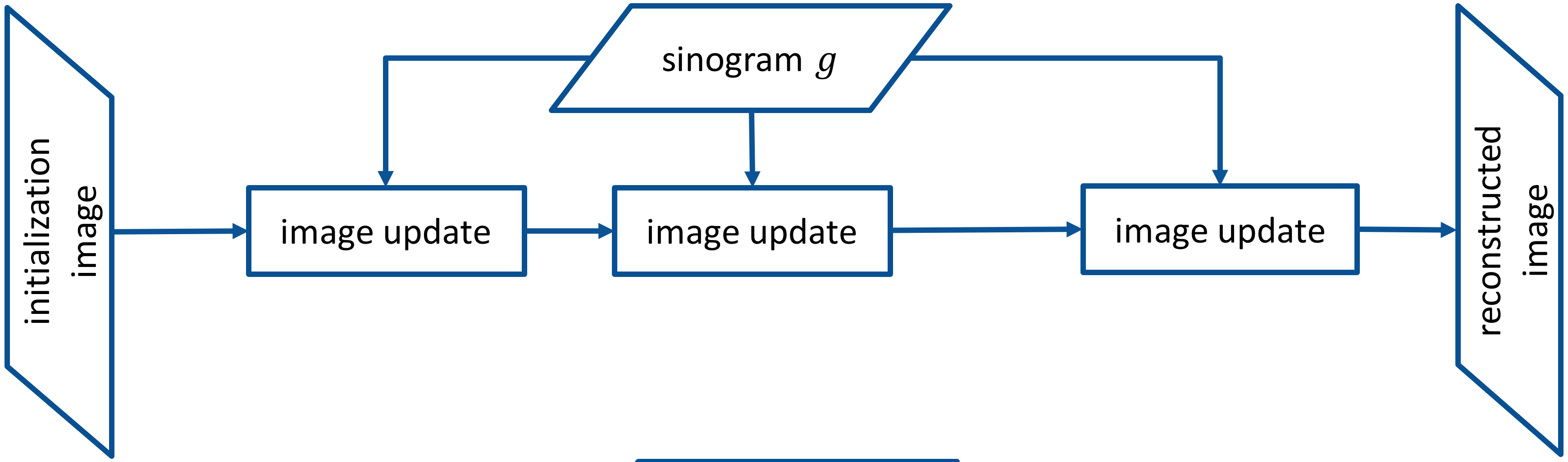
- Interpretability and generalizability in medical applications play fundamental roles but deep neural networks are usually difficult to interpret because of the huge number of parameters
  - Algorithm unrolling or unfolding is proposed to improve the interpretability and the generalizability (i.e., overfitting) of the deep neural network
    - Explicitly connected to the iterative algorithms used in imaging (and signal processing)
    - Explicitly based on domain knowledge as in imaging (and signal processing)
  - If the normal operator of the forward-projection model is a convolution (i.e., denoising and deblurring in the back-projection model), convolutional neural networks take part of unrolled iterative reconstruction methods

$$\begin{bmatrix} t_1 \\ \vdots \\ t_J \end{bmatrix} \approx \begin{bmatrix} \boxed{a_{11}} & & a_{1J} \\ & \boxed{} & \\ a_{J1} & & \boxed{a_{JJ}} \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_J \end{bmatrix}$$

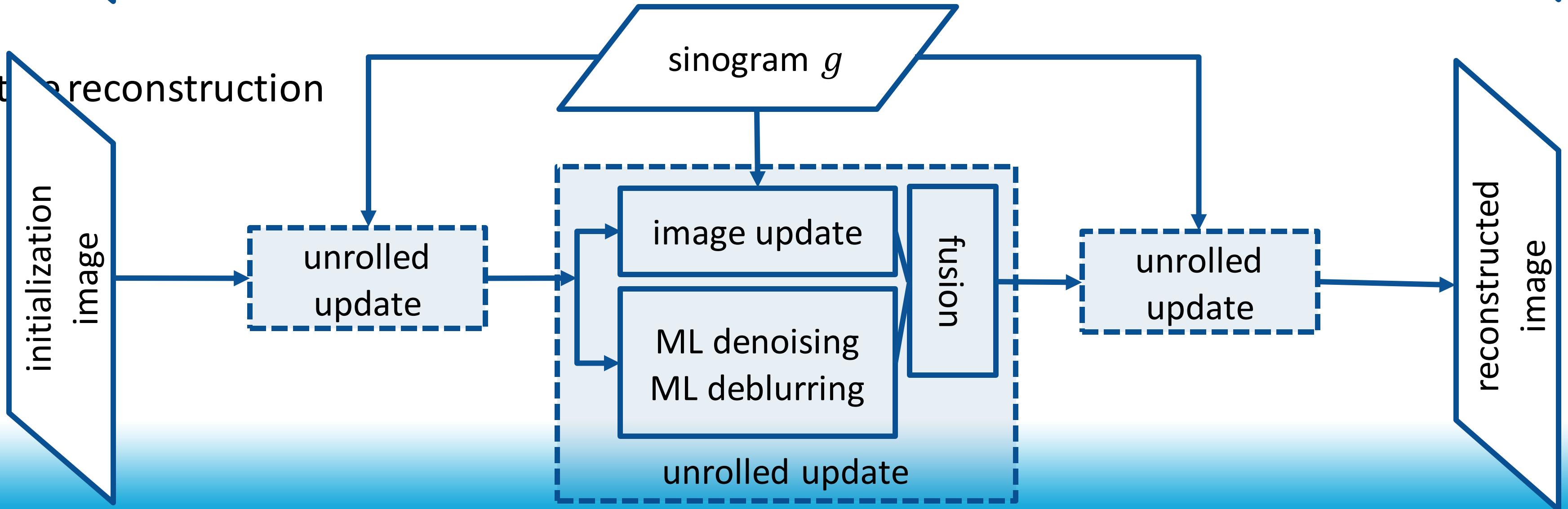
    - Denoising and deblurring can be described by convolutional layers

Unrolled iterative reconstruction methods

- “Unfolded” iterative reconstruction



- Unrolled iterative reconstruction

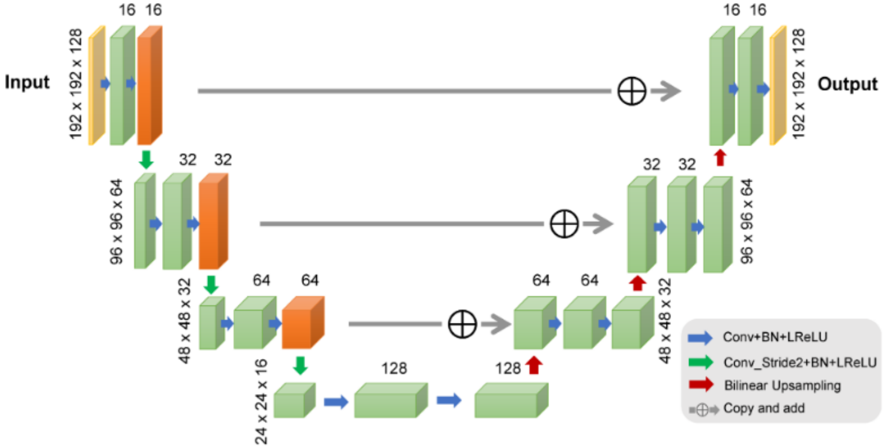


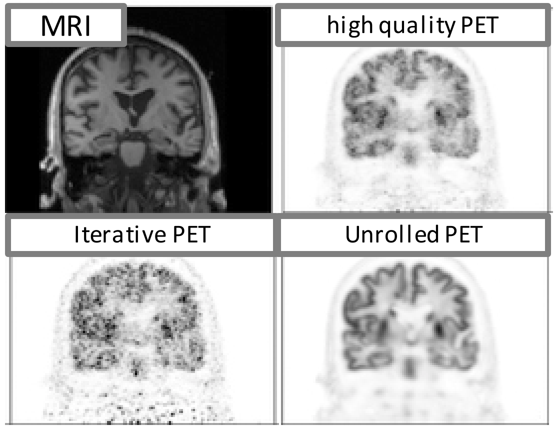
Unrolled iterative reconstruction methods

- Each iteration of the algorithm is represented as one layer of the network
  - Concatenating these layers forms a deep neural network
  - The number of layers in a deep network is typically much smaller than the number of iterations required in an iterative algorithm
- The network can be trained through all layers or layer per layer
- Unrolled iterative reconstruction methods are typically image domain networks intended to reduce the image noise due to low count statistics PET sinograms based on prior information from high quality imaging
  - The network is trained using patient-specific prior information (i.e., MRI image) and the measured data (i.e., PET sinogram)

Unrolled iterative reconstruction methods

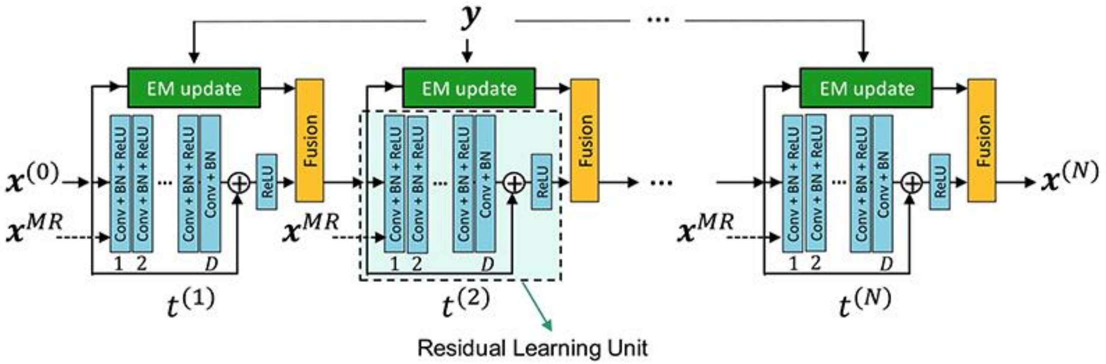
- Modified fully convolutional network (U-net)

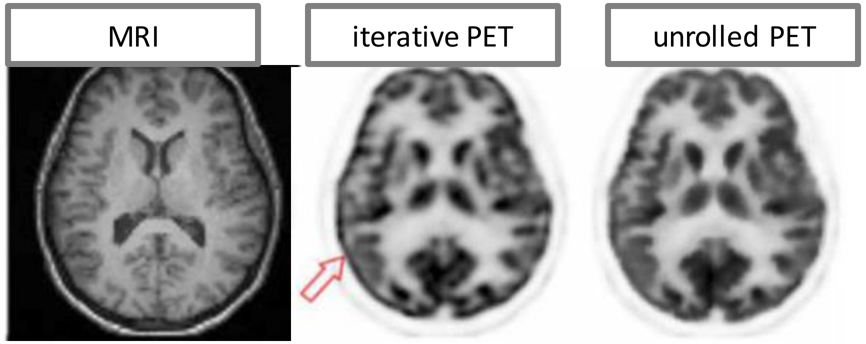




Gong, K., Catana, C., Qi, J., & Li, Q. (2018). PET image reconstruction using deep image prior. *IEEE transactions on medical imaging*, 38(7), 1655-1665.

- Residual network based on convolutional layers

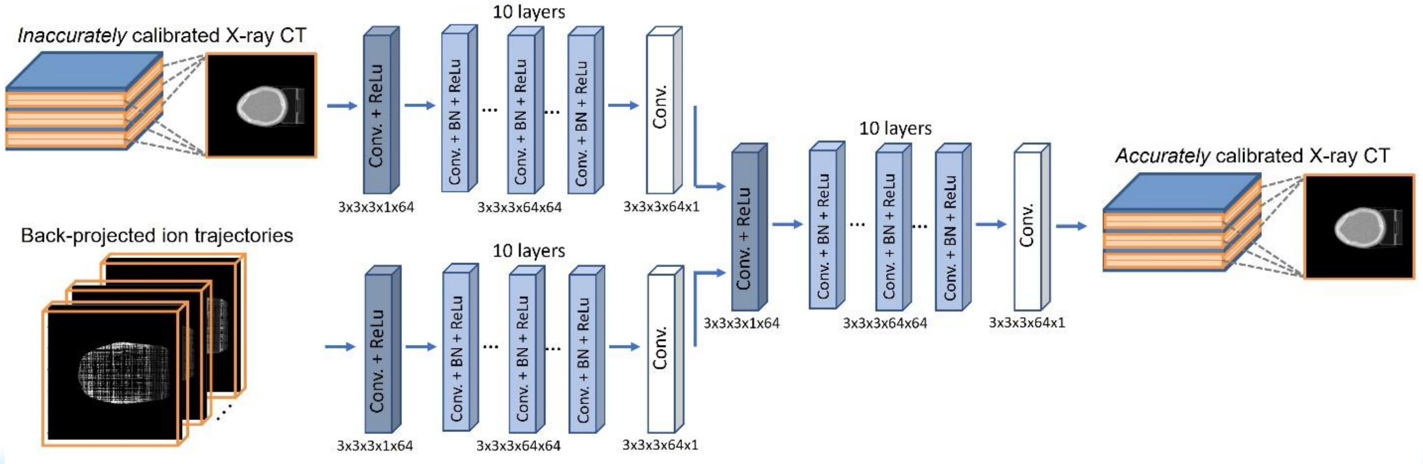




Mehranian, A., & Reader, A. J. (2020). Model-based deep learning PET image reconstruction using forward–backward splitting expectation–maximization. *IEEE transactions on radiation and plasma medical sciences*, 5(1), 54-64.

Deep reconstruction in ion imaging

- Inspired by the ML literature about tomographic image reconstruction but extended toward hybrid X-ray and ion imaging
  - With the "deep back-projection", the geometrical relationship between the projection domain and the image domain is encoded in single-view back-projections that are stacked and fed as input to the convolutional neural network
- The network is designed as a two input branches (one for the X-ray CT image and the other for the ion radiographies) followed by an integration branch
  - Two ion radiographies (two projection angles) are clustered according to the scattering angles of the ion trajectories (ten clusters for each projection angles)



Courtesy of Ines Butz

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