GUT Course 22/23

Lecture X

25/11/2022

LMU

2022

Monopoles in Cosmology

$$\partial_\mu F^{\mu\nu} = j^\nu \Rightarrow \partial_\mu j^\mu = 0$$

$$\partial_\mu \tilde{F}^{\mu\nu} = k^\nu \Rightarrow \partial_\mu j^\mu = 0$$

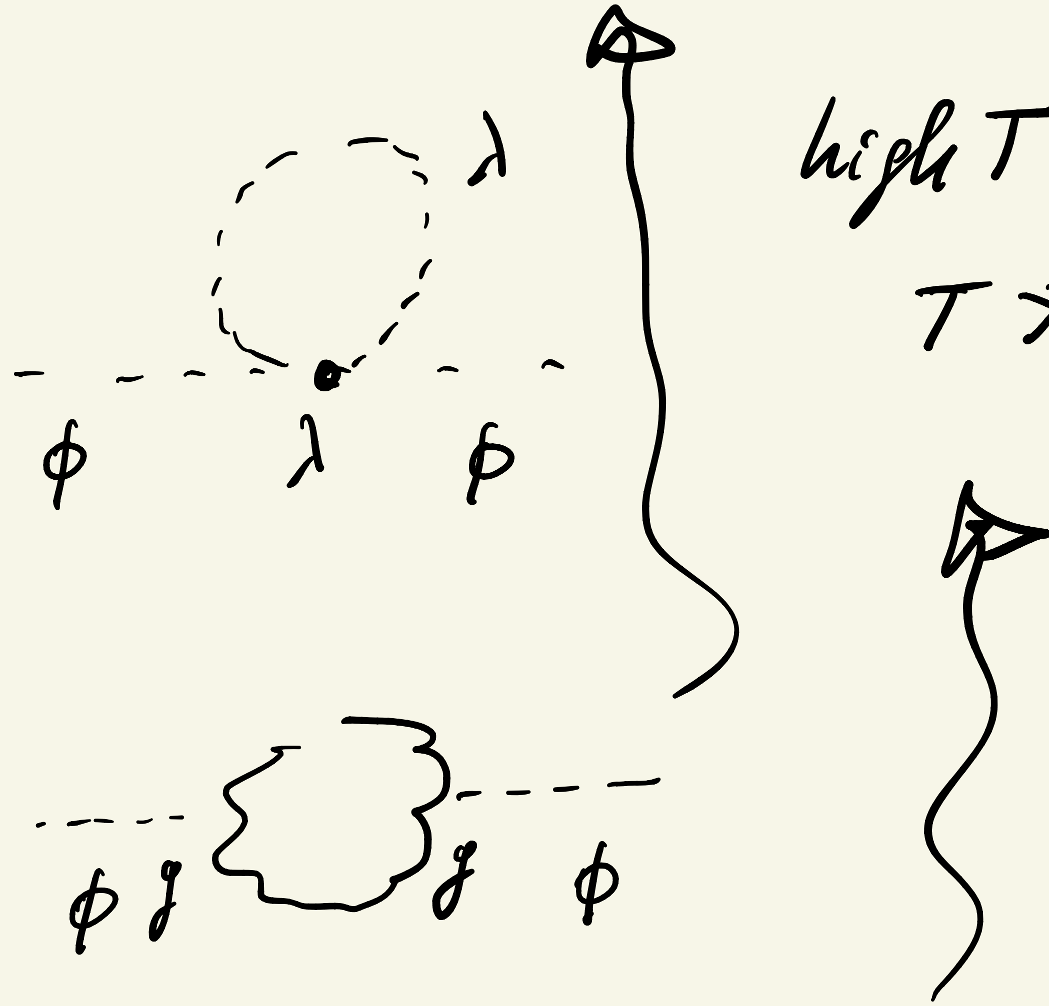
magnetic current

- at high $T \Rightarrow \langle \phi \rangle = 0$

$\Uparrow$

$$V(T) = V(0) + a T^2 \phi^\top \phi$$

$$a = \lambda + g^2 > 0 \quad (\lambda > 0)$$



high T :'.

$T \gg \langle\phi\rangle$

$\Rightarrow \mu_\phi^2 = \cancel{\mu_0^2} + aT^2 > 0$

correlation length ≃ horizon

⇓

~1 monopole / horizon

- $$d_H = t = \frac{M_{pl}}{T^2}$$

a) $T \to 0$, $t \to \infty$

b) $M_{pl} \to \infty$ $(G_N \to 0) \Rightarrow$
$t \to \infty$

$$H \equiv \dot{R}/R \Rightarrow H^2 = G_N \rho$$

$$T \gg \langle \phi \rangle \Rightarrow \rho \sim T^4$$

$$\Rightarrow H = T^2/M_{pl}$$

$$H = 1/t \Rightarrow \boxed{t = \frac{M_{pl}}{T^2}}$$

early universe (confirmed)

$T \sim 10 - 100\ \mathrm{MeV}$

Black Holes:

$$R_{BH} = G_N M_{BH}$$

Hawking:

$$G_N \to 0, \quad M_{BH} \to \infty$$

$$\therefore \quad R_{BH} = \text{finite}$$

$\Downarrow$

$$n_M = \left(1/d_H\right)^3$$

density of monopoles

Kibble mechanism

$$n_M = \frac{T^6}{M_p^3} \quad \therefore \quad T \sim \langle \phi \rangle$$

$$n_\gamma = T^3 \qquad n_B - n_{\bar{B}} = 10^{-10}\, n_\gamma$$

UNIVERSE

$$N_\gamma = \text{const}$$

$$N_B - N_{\bar{B}} = \text{const}$$

$$R \equiv R_U$$

$$\Rightarrow \quad N_\gamma = n_\gamma R^3$$

$$\Rightarrow \quad RT = \text{const}$$

$\Updownarrow$ big-bang

total # is conserved

$\Downarrow$

$$\frac{n_M}{n_B} \simeq 10^{10} \frac{T^3}{M_P^3}$$

$v = \langle \phi \rangle$

$m_M \sim \frac{1}{g} v$

$$\Rightarrow \quad \frac{n_M}{n_B} = v/\mathrm{GeV} \;\; 10^{10} \frac{T^3}{M_p^3}$$

universe $= v^4 (\mathrm{GeV}) \; 10^{10} / M_p^3 \; \mathrm{GeV}^3$

big-bang

$M_p \simeq 10^{19}$ GeV

$$M_M / M_B \leq O(1)$$

$$\Downarrow$$

$$10^{10}\, v^4 (\text{GeV}) \leq 10^{57}$$

$$\boxed{v \leq 10^{12}\ \text{GeV}}$$

$$\text{GUT} \Leftrightarrow v_{GUT} \simeq 10^{16}\ \text{GeV}$$

$$\Rightarrow \boxed{\frac{M_M^{GUT}}{M_B} \simeq 10^{16}}$$

monopole problem in GUT

However, if $v \lesssim 10^{12}$ GeV

$\nexists$ DM ?

Pati - Salam

q - l unification

v_{PS} could be $\leq 10^{12}$ GeV

$$G_{PS} = SU(2)_L \times SU(2)_R \times SU(4)_c$$

$$\Downarrow$$

monopoles

GUT = Grand Unification

$\Updownarrow$

single gauge non-Abelian group

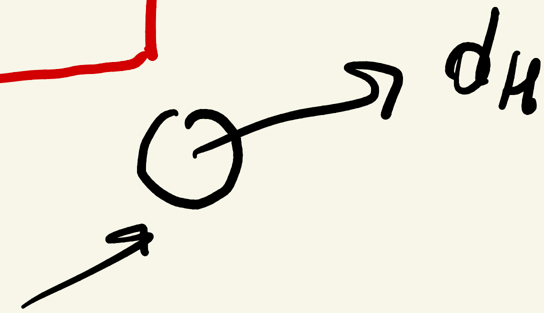
$\Uparrow$

$G_{SM} \subseteq SU(5), SO(10)$

$E(6)$

Ways out of monopole problem:

• inflation

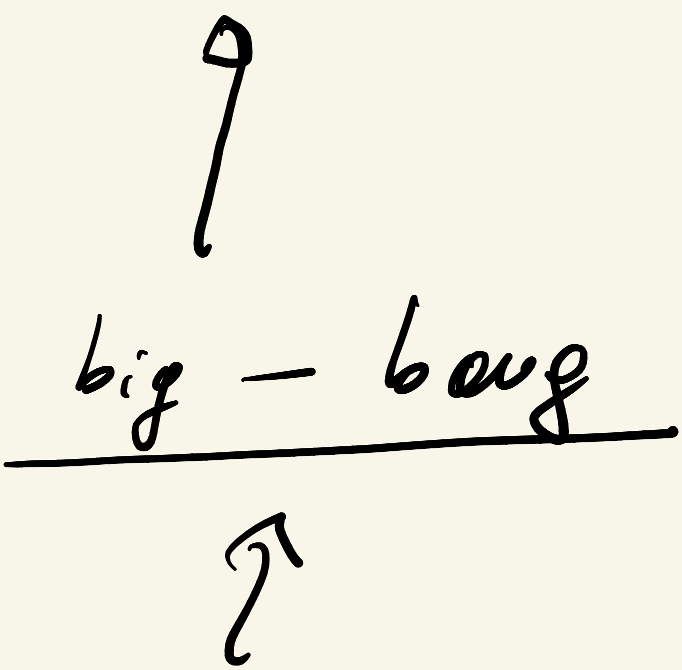


T°, t° : from a horizon

$\Downarrow$

\# of monopoles $\sim O(1)$

in our universe



$$d_H = \frac{M_p}{T^2} \qquad R = \frac{c}{T}$$

today: $T_0 \simeq 10^{-13}\,\text{GeV}$

$R_0 \simeq 10^{29}\,\text{cm}$

$\text{cm GeV} \simeq 10^{14}$

$$\Rightarrow \boxed{c_0 = c = 10^{30}}$$

$$\boxed{d_H^0 = R_0}$$

$$\Downarrow$$

$$\left(\frac{R}{d_H}\right)^3 = \left(10^{30}\, T/M_p\right)^3$$

$T \simeq \mathrm{MeV} \Rightarrow \left(R/d_H\right)^3 \simeq 10^{24}$

nucleosynthesis

HORIZON PROBLEM

SSB at high T

$\phi_1, \ \phi_2 \leftarrow$ real scalar

$$V(0) = -\frac{\mu_i^2}{2}\phi_i^2 + \frac{\lambda_i}{4}\phi_i^4 + \frac{\lambda_3}{2}\phi_1^2\phi_2^2$$

$(\mu_i^2 > 0)$

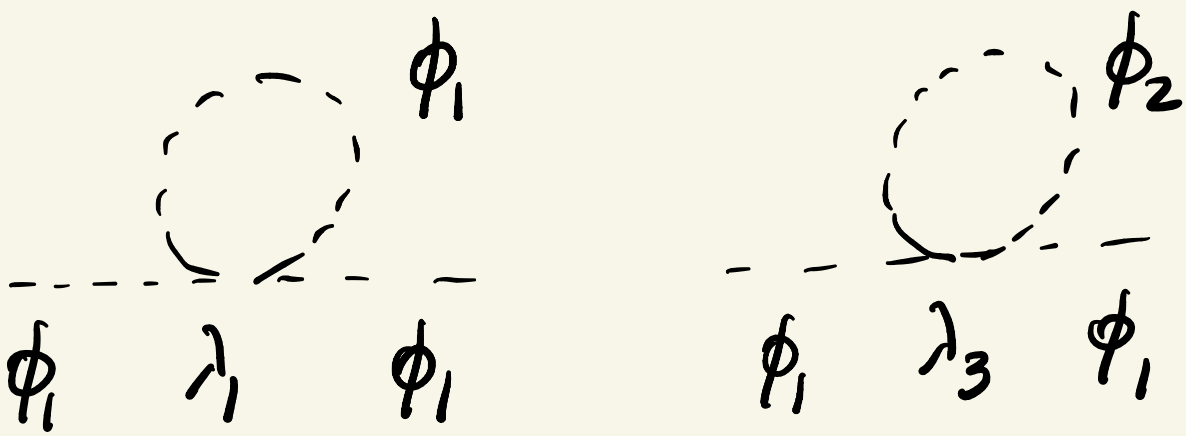
Symmetry(ies)?

$D_1: \ \phi_1 \to -\phi_1, \ \phi_2 \to \phi_2$

$D_2: \quad \phi_1 \to \phi_1, \quad \phi_2 \to -\phi_2$

$T=0: \quad \langle \phi_1 \rangle \neq 0 \neq \langle \phi_2 \rangle$

high T $\therefore$ $T \gg \langle \phi_i \rangle$



$+ \quad 1 \leftrightarrow 2$ diagrams

$$V(T) = V(0) + a_i T^2 \phi_i^2$$

$$a_1 = \lambda_1 + \lambda_3, \quad a_2 = \lambda_2 + \lambda_3$$

$V(\phi) =$ bounded

$\Downarrow$

$\lambda_i \ (\lambda_1, \lambda_2) > 0$

$\lambda_1 \lambda_2 > \lambda_3^2$ ← DERIVE

$\Downarrow$

λ_3 can be negative

$a_1 = \lambda_1 + \lambda_3 < 0$ possible

$\Rightarrow a_2 = \lambda_2 + \lambda_3 > 0$

$$-|a_1|\,T^2\phi_1^2 \Rightarrow$$

$$\langle\phi\rangle_T \neq 0$$

$$\Downarrow$$

$$\boxed{\langle\phi\rangle_T \simeq T}$$

Mohapatra, G.S. '78

Weinberg '74
(told by Coleman)

Rochelle salt

$T_{1c} \to$ more crystalized

$\to$ T_{2c} ($> T_{1c}$) $\Rightarrow$ melts

QFT systems $\sim$

Dvali, Melfo, G.S
'95

$\Downarrow$

rid of monopole problem

Dvali, G.S. '94

rid of domain walls