Neutrino GUT Course

Lecture XXVI

10/2/2023

LMU

Winter 2023

Spinors of SO(2N) (2)

↳ SO(10)

$$SO(2N) \xrightarrow[\text{spinor}]{} Spin(2N)$$

· SO(2N)

$$[L_{ij}, L_{kl}] = i(----)$$

$$\begin{cases} L_{ij} = -L_{ji} \\ (L_{ij})_{kl} = -i(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) \end{cases}$$

$$O = e^{i\theta_{ij}L_{ij}} \qquad \theta_{ij} = -\theta_{ji}$$

↳ $\frac{1}{2} N(N-1)$ generators

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}$$

$$\Sigma_{ij} = \frac{1}{4i}[\Gamma_i, \Gamma_j]$$

$[\Sigma_{ij}, \Sigma_{kl}]$ ⟷ same as $[L_{ij}, L_{kl}]$

SO(2N) + spinors

Cartan = $\{\Sigma_{12}, \Sigma_{34}, \Sigma_{56}, \ldots, \Sigma_{2N-1,2N}\}$

rank SO(2N) = N

$$\Sigma_{12} = \frac{1}{2i} \Gamma_1 \Gamma_2$$

$$(2\Sigma_{12})^2 = +1$$

$$\Gamma_{FIVE} = (-i)^N \Gamma_1 \Gamma_2 \cdots \Gamma_{2N}$$

$$= (2\Sigma_{12})(2\Sigma_{34}) \cdots (2\Sigma_{2N-1,2N})$$

- ψ_+ ← our irreducible field

$$\therefore \quad \psi_+ \equiv P_+ \psi = \frac{1+\Gamma_{FIVE}}{2} \psi$$

$$\Rightarrow \Gamma_{FIVE} \psi_+ = \psi_+$$

$$\Leftrightarrow \Gamma_{FIVE} = 1$$

$$\underbrace{\left(2\,\Sigma_{2i-1,2i}\right)}_{\varepsilon_i}{}^2 = +1$$

$$\varepsilon_i \;:\; \varepsilon_i^2 = +1 \Rightarrow \boxed{\varepsilon_i = \pm 1}$$

$$\Rightarrow \boxed{\Gamma_{FIVE} = \varepsilon_1 \varepsilon_2 \cdots \varepsilon_N = +1}$$

$$\boxed{\Psi_+ = |\varepsilon_1 \varepsilon_2 \cdots \varepsilon_N\rangle \;:\; \prod_{i=1}^{N} \varepsilon_i = +1}$$

- $SO(2)$

$$\Sigma = \Sigma_{12}$$

$$\Psi_+ = |\varepsilon\rangle = |1\rangle \quad \text{single field}$$

$\Updownarrow$ $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{cases} \psi_+ = \begin{pmatrix} u \\ 0 \end{pmatrix} \\ \psi_- = \begin{pmatrix} 0 \\ d \end{pmatrix} \end{cases}$

$\Gamma_{FIVE} = \sigma_3 \nearrow$

- $SO(4) = SU(2)_L \times SU(2)_R$

$\psi_+ = |\varepsilon_1 \varepsilon_2\rangle \; : \; \varepsilon_1 \varepsilon_2 = +1$

$\Rightarrow \psi_+ = \{ |++\rangle, |--\rangle \}$

$\uparrow$

not chiral $\Leftrightarrow$ mass term

• <u>SO(6)</u> $r = 3$, # gen. $= \frac{6 \cdot 5}{2} = 15$

$$\psi_+ = |\varepsilon_1 \varepsilon_2 \varepsilon_3\rangle \quad : \quad \varepsilon_1 \varepsilon_2 \varepsilon_2 = +1$$

$$\Rightarrow \left.\begin{matrix} |{+}{+}{+}\rangle & (1) \\ \left.\begin{matrix} |{-}{-}{+}\rangle \\ |{-}{+}{-}\rangle \\ |{+}{-}{-}\rangle \end{matrix}\right\} & (3) \end{matrix}\right\} \text{4-component field}$$

$$\boxed{4 = 1 + 3}$$

SU(n) : $r = 3$, # gen. $= 15$

$$\Rightarrow n = 4$$

$$\boxed{SO(6) = SU(4)}$$

$$\psi_+ = 4\text{-dim}$$

$\uparrow$

Spinor

$$F = 4\text{-dim}$$

$\uparrow$

fundamental

$\Uparrow$

$$F = \begin{pmatrix} 1 \\ 2 \\ 3 \\ \hline 4 \end{pmatrix} \Big\} SU(3)$$

$$4 = 3 + 1$$

$$SU(4) \longrightarrow SU(4)_{\text{colour}} = SU(4)_{PS}$$

$\parallel$

1, 4, 6 ; v

Pati, Salam

'73 – '74

$\Downarrow$

$$\underbrace{\begin{pmatrix} u^r & u^y & u^b \\ d^r & d^y & d^b \end{pmatrix}}_{SU(3)_c} \begin{matrix} \nu \\ e \end{matrix}_L$$

$$\ell = \begin{pmatrix} \nu \\ e \end{pmatrix} = \text{violet}$$

(4th color)

$$SU(4)_{PS} \xrightarrow[M_{PS}]{} SU(3)_c \times U(1)$$

$$\boxed{M_{PS} \gg M_W \quad (\text{exp})}$$

($M_{PS} \gtrsim 10^5$ GeV) theory

SU(3)

$$T_3 = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

$$T_8 \propto \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

<u>SU(4)</u>

$$T_3 = \text{diag}\ \frac{1}{2}(1,-1,0,0)$$

$$T_8 \propto \text{diag}\ (1,1,-2,0)$$

$$\boxed{T_{15} \propto \text{diag}\,(1,1,1,-3) \propto B-L}$$

$B_q = \frac{1}{3}, \quad L_q = 0$

$B_e = 0, \quad L_e = 1$

$$\Rightarrow \boxed{\begin{aligned} (B-L)_q &= 1/3 \\ (B-L)_e &= -1 \end{aligned}}$$

- ~~SO(8)~~ not chiral

- SO(10) $\quad r = 5, \quad n_g = 45$ (# gen)

$\downarrow$

$$10 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \hline 7 \\ 8 \\ 9 \\ 10 \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \\ \\ \end{matrix}\right\} SO(6) = SU(4)_C \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \begin{matrix} SO(4) = \\ = SU(2)_L \times SU(2)_R \end{matrix} \end{matrix}$$

$$SU(4) \supseteq SU(3)_C \times U(1)_{B-L}$$

$$SO(4) \supseteq SU(2)_L \times U(1)_R$$

$$(T_{3R})$$

$$\boxed{\frac{Y}{2} = \frac{B-L}{2} + T_{3R}}$$

(Max. subgroup)

PATI-SALAM MODEL

$$SO(10) \supseteq SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$\cup|$$

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

LR SYM. MODEL

↑ SSB

$$M_{W_R} \gg M_{W_L}$$

- $SO(10) \supseteq \underbrace{SU(5) \times U(1)}_{\text{Max. subgroup}}$

$SO(10)$

$$O_{10} O_{10}^T = O_{10}^T O_{10} = 1$$

$$\det O_{10} = 1$$

$$O_{10} = \begin{pmatrix} O_6 & 0 \\ 0 & 1_4 \end{pmatrix} \therefore$$

$$O O_6^T = O_6^T O = 1$$

$$\det O_6 = 1$$

$$O_{10} = \begin{pmatrix} 1_6 & 0 \\ 0 & O_4 \end{pmatrix} \therefore$$

$$O_4^T O_4 = O_4 O_4^T = 1$$

$$\det O_4 = 1$$

$$SO(10) \supseteq SO(6) \times SO(4)$$

trivial

$$SO(10) \supseteq SU(5) \times U(1)$$

not-trivial

(harder)

- spinor ψ_+ of $SO(10)$

$$\psi_+ = |\varepsilon_1 \varepsilon_2 \cdots \varepsilon_5\rangle$$

$$\because \quad \varepsilon_1 \varepsilon_2 \cdots \varepsilon_5 = +1$$

(a) $|+++++\rangle$ $\quad (1)$

(b) $|----+\rangle$ $\quad ?$

$|---+-\rangle$

$|{-}{-}{+}{-}{-}\rangle$ ⌋ (5)

$- - - - -$

(c) $|{-}{-}{+}{+}{+}\rangle$

$|{-}{+}{-}{+}{+}\rangle$

$\cdots$

$\underbrace{\hphantom{xxxxxxxxxx}}_{(10)}$

$\binom{5}{2} = \frac{5!}{2!\,3!}$

$\binom{5}{3} = \frac{5!}{3!\,2!}$

$$16_L = (1 + \bar{5} + 10)_L$$

$\downarrow$

$$\psi_+ = \underbrace{(\bar{5} + 10)}_{(SU(5) = SM)\ \text{fermions}} + 1\ (\nu_R)$$

$\Downarrow$

- $SO(10) \Rightarrow$ unifies a family of fermions

- $SO(10) \Rightarrow \exists\ \nu_R$

  $\Rightarrow$ neutrino mass

Pati – Salam

$$G_{PS} = SU(4)_C \times \underbrace{SU(2)_L \times SU(2)_R}_{P}$$

$$\vec{W}_L, \vec{W}_R \downarrow \begin{pmatrix} \overbrace{u^\alpha \quad\ \ \nu}^{X_{PS}} \\ d^\alpha \quad\ \ e \end{pmatrix}_{L,R} \qquad (L \leftrightarrow R)$$

gluons $\longrightarrow$

$\Downarrow$

$\exists\ \nu_R$

$$\psi_+ = |\varepsilon_1 \varepsilon_2 \varepsilon_3 \quad \varepsilon_4 \varepsilon_5\rangle$$

$$\therefore \underbrace{\prod_{i=1}^{3} \varepsilon_i}_{\pm 1} \underbrace{\prod_{i=4}^{5} \varepsilon}_{\pm 1} = +1$$

$$\psi_+ = |+++\rangle \begin{cases} |++\rangle \\ |--\rangle \end{cases}$$

lepton

quarks

$$\begin{cases} |--+\rangle \begin{cases} |++\rangle \\ |--\rangle \end{cases} \\ \\ \left.\begin{matrix} |-+-\rangle \\ |+--\rangle \end{matrix}\right\} \begin{matrix} |++\rangle \\ |--\rangle \end{matrix} \end{cases}$$

$+ \nearrow$

$|---\rangle \quad \begin{matrix} |+-\rangle \\ |\pm-\rangle \end{matrix}$

lepton

$\left.\begin{matrix} |-++\rangle \\ |+-+\rangle \\ |++-\rangle \end{matrix}\right\} \begin{matrix} |+-\rangle \\ |-+\rangle \end{matrix}$

quarks

$$\psi_{16} = \begin{pmatrix} u^\alpha \\ d^\alpha \\ \nu \\ e \\ u^{c\alpha} \\ d^{c\alpha} \\ \nu^c \\ e^c \end{pmatrix} \qquad \alpha = r, y, b$$

charge conjugation C

$$f_R \longrightarrow (f^c)_L \equiv C \bar{f}_R^{\,T}$$

$$\boxed{C \subseteq SO(10)}$$

$$\Leftrightarrow SU(2) \qquad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\left(\begin{aligned} \psi \xrightarrow{D} -\psi &= e^{i\pi\sigma_3}\psi \\ &= e^{i2\pi\sigma_3/2}\psi \\ &= e^{i2\pi T_3}\psi \end{aligned} \right)$$

- <u>gauge bosons</u>

$$45 = \underbrace{24 + 10 + \overline{10} + 1}_{SU(5)}$$

$$45 = \underbrace{15}_{SU(4)} + \underbrace{(15_{PS}, 1_L, 1_R)}_{SU(4)_C \times SU(2)_L \times SU(2)_R}$$

$$\underbrace{15}_{SU(4)} = \underbrace{8 + \underbrace{3 + \bar{3}} + 1}_{SU(3)}$$

8 → gluons

$3 + \bar{3}$ → Pati – Salam X_{PS}

$$X_{PS}^{\mu} \left[\bar{u} \nu + \bar{d} e \right]_{L,R}$$

γ_μ

⑥ $\int + (1_{PS}, 3_L, 1_R) + \} \vec{W}_L,$

$$(\; + (1_{PS}, 1_L, 3_R) \quad \int \vec{W}_R$$

$$\textcircled{24} \; + (6, 2_L, 2_R) \quad \begin{pmatrix} X \\ Y \end{pmatrix}, \begin{pmatrix} X' \\ Y' \end{pmatrix}$$

$$X_\mu \left[\overline{u^c} \gamma^\mu u + \cdots \right] + \bar{X}$$

$SU(2)_L$

$$Y_\mu \left[\overline{u^c} \gamma^\mu d + \cdots \right] + \bar{Y}_\mu$$

$SU(2)_R$ $\updownarrow$

$$X'_\mu \left[\overline{d^c} \gamma^\mu u + \cdots \right]$$

$$Y'_\mu \left[\overline{d^c} \gamma^\mu d + \cdots \right]$$

$X = 3_c, \quad \bar{X} = 3_c^*$

$X' = 3_c, \quad \bar{X}' = 3_c^*$

$$\Downarrow$$

$$45 = \underbrace{15} + \underbrace{24} + 6$$

8_c, $X_{\mu s} + \bar{X}_{\mu s}$, $+1$ gluons

$\Delta B \neq 0$ $(L, R = W_L, W_R)$

(X, Y, X', Y')

$B-L$

$SO(10)$

$$\Downarrow$$

$$
10_H = \begin{pmatrix} 1 \\ 2 \\ \vdots \\ 6 \\ \hline 7 \\ \vdots \\ 10 \end{pmatrix}
\begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} SO(6) \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} SO(4) \end{matrix}
$$

$$
= SU(2)_L \times SU(2)_R
$$

$$
10_H = \cdots + \boxed{2_L} \text{ in } (7, 8, 9, 10)
$$

$$
16_H = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix} \;] \text{ doublet} \Leftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}
$$

Fermions $\subset 16_F$

$$\mathcal{L}_Y = 16_F\, 16_F\, \cancel{16_H}$$

<u>not</u> inv.

$$\boxed{16_F\, 16_F\, 10_H}$$

$$\downarrow$$

$$16_F^T\, \Gamma_i\, 16_F\, (10_H)_i$$

$$i = 1, \ldots, 10$$