

Neutrino GUT Course

Lecture xxvi

10/2/2023

LMU

Winter 2023



Spinors of $SO(2N)$ (2)

$$\hookrightarrow SO(10)$$

$$SO(2N) \xrightarrow{\text{Spinor}} \text{Spin}(2N)$$

• $SO(2N)$

$$[L_{ij}, L_{ke}] = i(\dots)$$

$$\left\{ \begin{array}{l} L_{ij} = -L_{ji} \\ (L_{ij})_{ke} = -i(\delta_{ik}\delta_{je} - \delta_{ie}\delta_{jk}) \\ 0 = \ell \quad i\theta_{ij} L_{ij} \quad \theta_{ij} = -\theta_{ji} \end{array} \right.$$

$\hookrightarrow \frac{1}{2} N(N-1)$ generators

$$\{T_i, T_j\} = 2\delta_{ij}$$

$$\Sigma_{ij} = \frac{1}{4i} [T_i, T_j]$$

$$[\Sigma_{ij}, \Sigma_{kl}] \leftrightarrow \text{same as } [L_{ij}, L_{kl}]$$



$SO(2N) + \text{spinors}$

$$\text{Cartan} = \{ \Sigma_{12}, \Sigma_{34}, \Sigma_{56}, \dots, \Sigma_{2N-1, 2N} \}$$

$$\text{rank } SO(2N) = N$$

$$\Sigma_{12} = \frac{1}{2i} \Gamma_1 \Gamma_2$$

$$(2\Sigma_{12})^2 = +1$$

$$\Gamma_{\text{FIVE}} = (-i)^N \Gamma_1 \Gamma_2 \dots \Gamma_{2N}$$

$$= (2\Sigma_{12})(2\Sigma_{34}) \dots (2\Sigma_{2N-1,2N})$$

ψ_+ our irreducible field

$$\therefore \psi_+ \equiv P_+ \psi = \frac{1 + \Gamma_{\text{FIVE}}}{2} \psi$$

$$\Rightarrow \Gamma_{\text{FIVE}} \psi_+ = \psi_+$$

$$\Leftrightarrow \Gamma_{\text{FIVE}} = 1$$

$$\left(2 \sum z_{i-1, z_i} \right)^2 = +1$$

$$\varepsilon_i \therefore \varepsilon_i^2 = +1 \Rightarrow \boxed{\varepsilon_i = \pm 1}$$

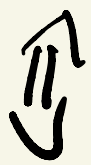
$$\Rightarrow \boxed{\Gamma_{\text{FIVE}} = \varepsilon_1 \varepsilon_2 \dots \varepsilon_N = +1}$$

$$\psi_+ = |\varepsilon_1 \varepsilon_2 \dots \varepsilon_N\rangle \therefore \prod_{i=1}^N \varepsilon_i = +1$$

• SO(2)

$$\bar{z} = \bar{z}_{12}$$

$$\psi_+ = |\varepsilon\rangle = |1\rangle \quad \text{single field}$$



$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

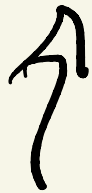
$$\left\{ \begin{array}{l} \psi_+ = \begin{pmatrix} u \\ 0 \end{pmatrix} \\ \psi_- = \begin{pmatrix} 0 \\ d \end{pmatrix} \end{array} \right.$$

$$\Gamma_{FIVE} = \sqrt{3} \rightarrow$$

• $SU(4) = SU(2)_L \times SU(2)_R$

$$\psi_+ = | \epsilon_1, \epsilon_2 \rangle \quad \therefore \quad \epsilon_1, \epsilon_2 = +1$$

$$\Rightarrow \psi_+ = \{ |++\rangle, |--\rangle \}$$



not chiral \Leftrightarrow mass term

• SO(6) $r=3, \# \text{ glu.} = \frac{6 \cdot 5}{2} = 15$

$\psi_+ = |\epsilon_1 \epsilon_2 \epsilon_3\rangle \therefore \epsilon_1 \epsilon_2 \epsilon_3 = +1$

\Rightarrow

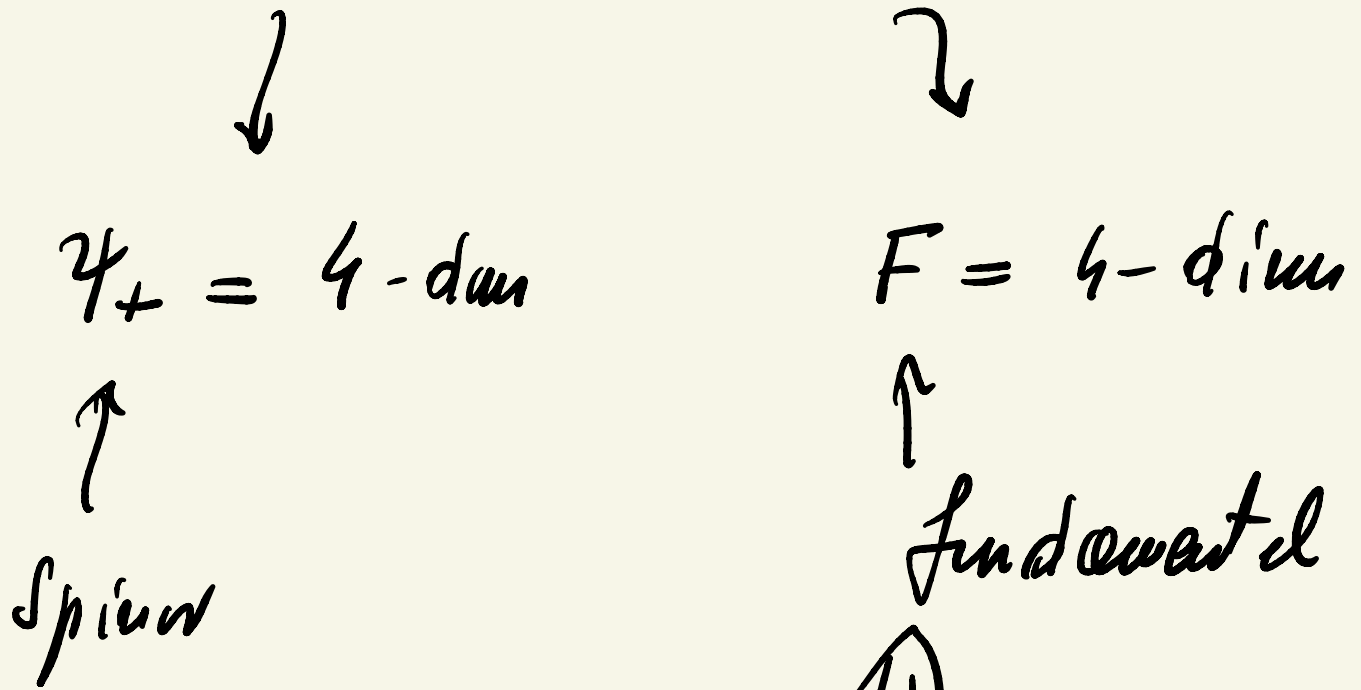
| | | | | |
|----------------|-------|---|----------------------|---|
| $ +++ \rangle$ | (1) | } | 4-component field | |
| $ --- \rangle$ | } | | | |
| $ +- \rangle$ | | | | } |
| $ +-- \rangle$ | | | | |
| (3) | | | | |

$4 = 1 + 3$

$SU(n) \therefore r=3, \# \text{ glu.} = 15$

$\Rightarrow n=4$

$SO(6) = SU(4)$



$$F = \left(\begin{array}{c|c} 1 & \\ \hline 2 & \\ 3 & \\ \hline 4 & \end{array} \right) \} SU(3)$$

$$4 = 3 + 1$$

$$SU(4) \longrightarrow SU(4)_{\text{color}} = SU(4)_{\text{PS}}$$

\parallel
 $7, 4, 6; \nu$ Pati; Selam

$$\begin{array}{c} \Downarrow \\ \left(\begin{array}{cccc} u^r & u^y & u^b & \vdots & \nu \\ d^r & d^y & d^b & \vdots & e \end{array} \right)_L \\ \underbrace{\hspace{10em}} \\ SU(3)_c \end{array}$$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix} = \text{violet} \\ \text{(4th color)}$$

$$SU(4)_{PS} \xrightarrow{M_{PS}} SU(3)_c \times U(1)$$

$$M_{PS} \gg M_W \quad (\text{exp})$$

($M_{ps} \approx 10^5 \text{ GeV}$) theory

$$SU(3) \quad T_3 = \frac{1}{2} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

$$T_8 \propto \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$\underline{SU(4)} \quad T_3 = \text{diag } \frac{1}{2} (1, -1, 0, 0)$$

$$T_8 \propto \text{diag } (1, 1, -2, 0)$$

$$T_{15} \propto \text{diag } (1, 1, 1, -3)$$

$$\propto B-L$$

$$B_q = \frac{1}{3}, \quad L_q = 0$$

$$B_e = 0, \quad L_e = 1$$

$$\Rightarrow \left(\begin{array}{l} (B-L)_e = 1/3 \\ (B-L)_e = -1 \end{array} \right)$$

~~SO(8)~~ not chiral

• SO(10) $r = 5, \quad \overset{\# \text{ gen}}{\nearrow} \mu_f = 45$



$$10 = \left. \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \hline 7 \\ 8 \\ 9 \\ 10 \end{array} \right\} \begin{array}{l} SO(6) = SU(4)_C \\ \\ \\ \\ \\ \\ \\ SO(4) = \\ = SU(2)_L \times SU(2)_R \end{array}$$

$$SU(4) \supseteq SU(3)_C \times U(1)_{B-L}$$

$$SO(4) \supseteq SU(2)_L \times U(1)_R \quad (T_{3R})$$

$$\frac{Y}{2} = \frac{B-L}{2} + T_{3R}$$

(Max. subgroup)

$$SO(10) \cong \left(\begin{array}{c} \text{Pati-Salam Model} \\ SU(4)_C \times SU(2)_L \times SU(2)_R \\ \hline U(1) \end{array} \right)$$

$$\left| \begin{array}{c} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ \hline \text{LR Sym. Model} \end{array} \right.$$

SSB

$$\boxed{M_{WR} \gg M_{WL}}$$

- $SO(10) \cong SU(5) \times U(1)$

⏟

Max. subgroup

SO(10)

$$O_{10} O_{10}^T = O_{10}^T O_{10} = 1$$

$$\det O_{10} = 1$$

$$O_{10} = \begin{pmatrix} O_6 & 0 \\ 0 & \mathbb{1}_4 \end{pmatrix} \therefore$$

$$O O_6^T = O_6^T O = 1$$

$$\det O_6 = 1$$

$$O_{10} = \begin{pmatrix} \mathbb{1}_6 & 0 \\ 0 & O_4 \end{pmatrix} \therefore$$

$$O_4^T O_4 = O_4 O_4^T = 1$$

$$\det O_4 = 1$$

$$SO(10) \cong SO(6) \times SO(4)$$

trivial

$$SO(10) \cong SO(5) \times U(1)$$

(harder) not-trivial

• Spinors ψ_+ of $SO(10)$

$$\psi_+ = | \varepsilon_1 \varepsilon_2 \dots \varepsilon_5 \rangle$$

$$\therefore \varepsilon_1 \varepsilon_2 \dots \varepsilon_5 = +1$$

(a) $| + + + + + \rangle$ (1)

(b) $| - - - - + \rangle$?
 $| - - - + - \rangle$

$$1 \text{---} + \text{---} > \int (5)$$

$$\text{---}$$

(c)

$$\left. \begin{array}{l}
 1 \text{---} + ++ > \\
 1 - + - ++ > \\
 \text{---} \text{---} \text{---} \text{---}
 \end{array} \right\} \begin{array}{l}
 \binom{5}{2} = \frac{5!}{2! 3!} \\
 \binom{5}{3} = \frac{5!}{3! 2!}
 \end{array}$$

$\underbrace{\hspace{10em}}_{(10)}$

$$16_L = (1 + \bar{5} + 10)_L$$

↓

$$4_+ = (\bar{5} + 10) + 1 (\nu_R)$$

⏟

(SU(5) = SM) fermions

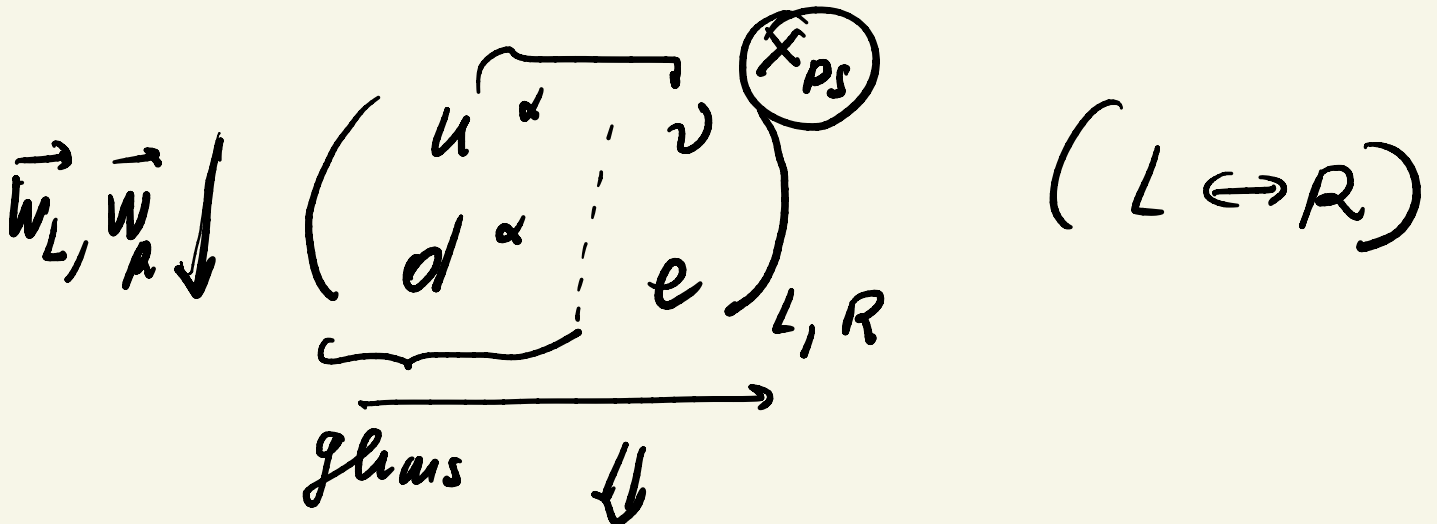


• $SO(10) \Rightarrow$ unifies a family of fermions

• $SO(10) \Rightarrow \exists \nu_R$
 \Rightarrow neutrino mass

Pati - Salam

$$G_{PS} = SU(4)_C \times \overset{\underline{P}}{\swarrow} SU(2)_L \times \searrow SU(2)_R$$



$$\exists \nu_R$$

$$\psi_+ = |\varepsilon_1 \varepsilon_2 \varepsilon_3 \varepsilon_4 \varepsilon_5\rangle$$

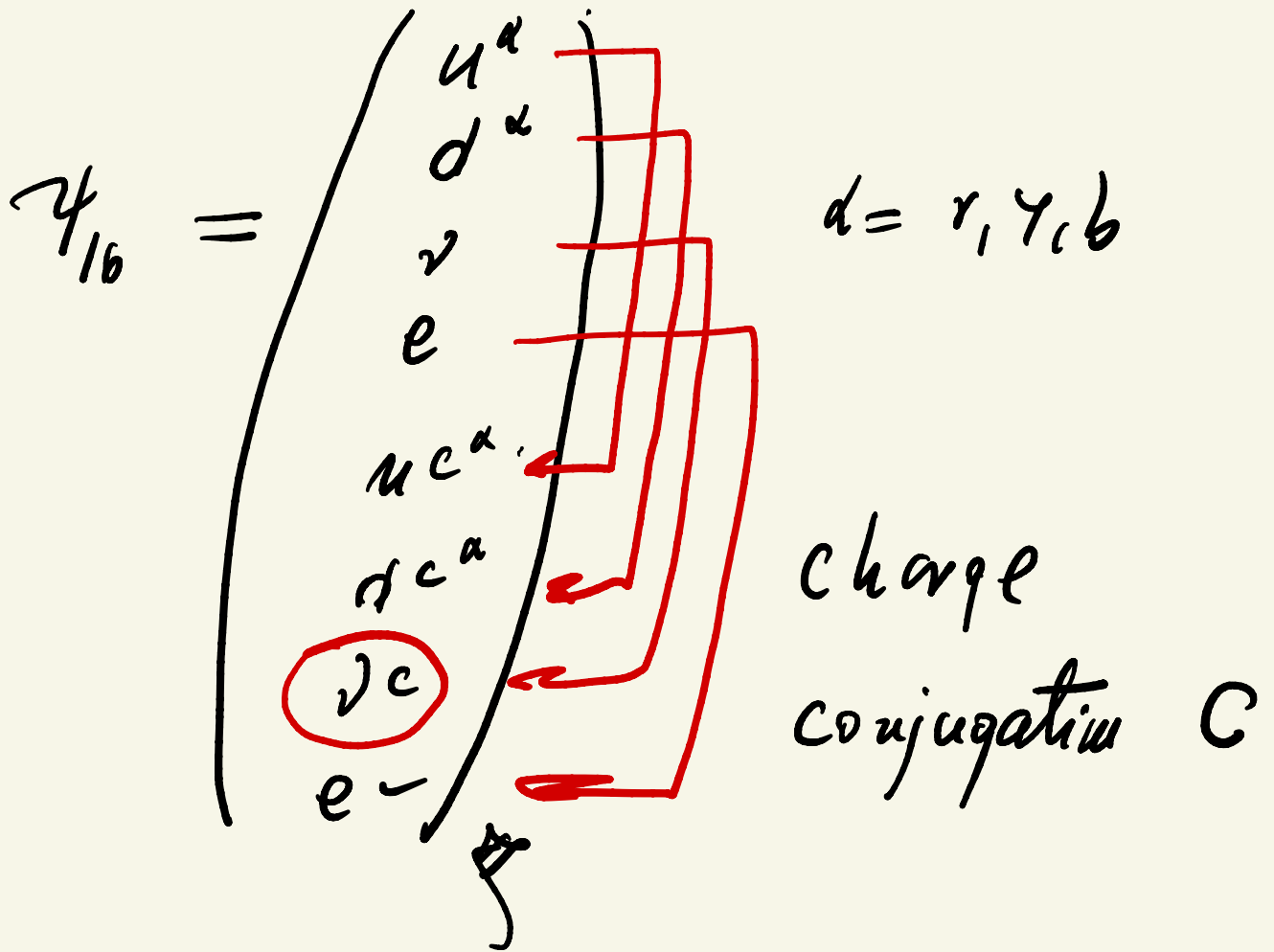
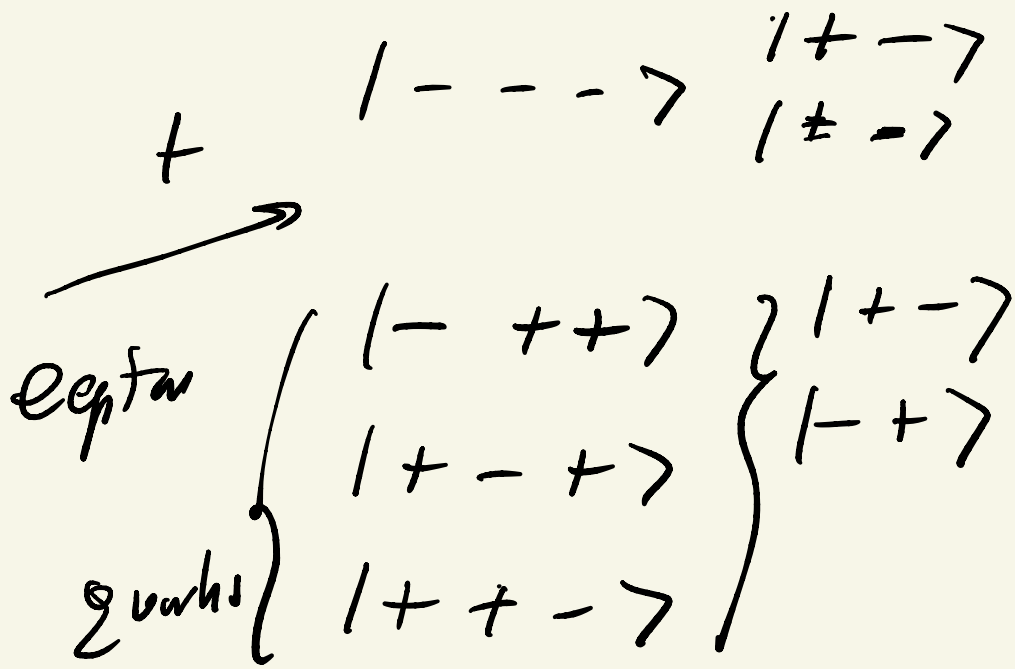
$$\therefore \underbrace{\prod_{i=1}^3 \varepsilon_i}_{\pm 1} \underbrace{\prod_{i=4}^5 \varepsilon_i}_{\pm 1} = +1$$

$$\psi_+ = \left\{ \begin{array}{l} |+++ \rangle \\ |--- \rangle \end{array} \right\} \parallel \begin{array}{l} |++ \rangle \\ |-- \rangle \end{array}$$

lepton

quark

$$\left\{ \begin{array}{l} |---+ \rangle \\ | - + - \rangle \\ | + - - \rangle \end{array} \right\} \parallel \left\{ \begin{array}{l} |++ \rangle \\ |-- \rangle \end{array} \right.$$



$$f_L \rightarrow (f^c)_L \equiv C \bar{f}_R^T$$

$$\boxed{C \subseteq SO(10)}$$

$$\Leftrightarrow SU(2) \quad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\left(\begin{array}{l} \psi \xrightarrow{D} -\psi = e^{i\pi T_3} \psi \\ = e^{i2\pi T_3/2} \psi \\ = e^{i2\pi T_3} \psi \end{array} \right)$$

• gauge bosons

$$45 = 24 + 10 + \bar{10} + 1$$

$$\underbrace{\hspace{15em}}_{SU(5)}$$

$$45 = \underbrace{15}_{SU(4)} + \underbrace{(15_{PS}, 1_L, 1_R)}_{SU(4)_C \times SU(2)_L \times SU(2)_R}$$

$$15 = \underbrace{8}_{SU(4)} + \underbrace{3 + \bar{3} + 1}_{SU(3)}$$

gluons

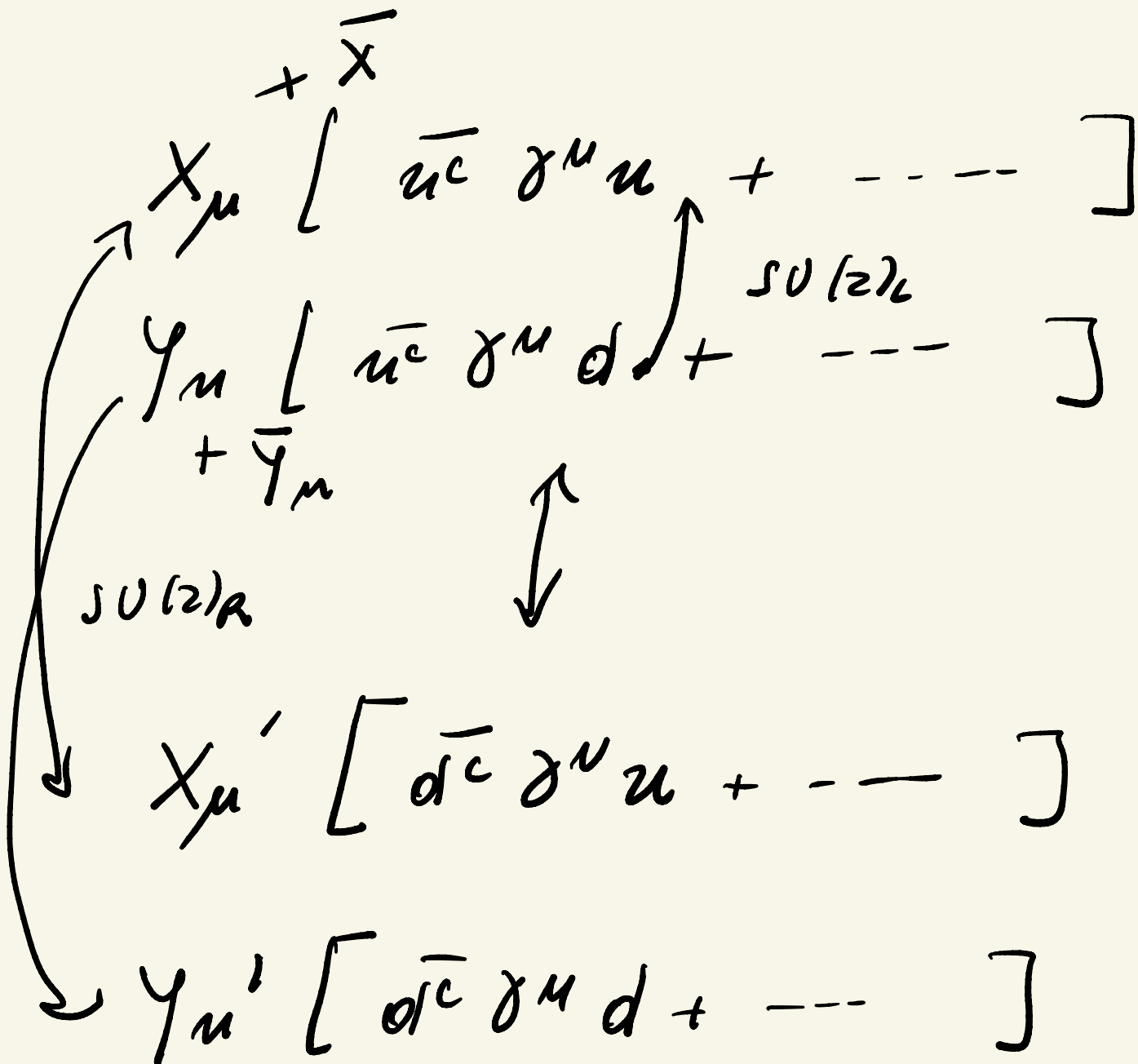
Pati-Salam X_{PS}

$$X_{PS}^\mu \left[\underbrace{\bar{u} \nu + \bar{d} e}_{\gamma_\mu} \right]_{L,R}$$

$$\textcircled{6} \int + (1_{PS}, 3_L, 1_R) + \int \bar{W}_L,$$

$$\left(+ (1_{ps}, 1_L, 3_R) \right) \int \vec{W}_R$$

$$(24) + (6, 2_L, 2_R) \begin{pmatrix} X \\ Y \end{pmatrix}, \begin{pmatrix} X' \\ Y' \end{pmatrix}$$



$$X = 3c, \quad \bar{X} = 3c^*$$

$$X' = 3c, \quad \bar{X}' = 3c^*$$



$$45 = \underbrace{15} + \underbrace{24} + 6$$

$p_c, x_{ps} + \bar{x}_{ps}, +1$ $\Delta B \neq 0$ $(L, R = w_L, w_R)$
plus (x, y, x', y')

B-1

SO(10)



$$\begin{aligned}
10_H &= \left(\begin{array}{c} 1 \\ 2 \\ \vdots \\ 6 \\ \hline 7 \\ \vdots \\ 10 \end{array} \right) \begin{array}{l} \} \\ \} \end{array} \begin{array}{l} SO(6) \\ SO(4) \end{array} \\
&= SU(2)_L \times SU(2)_R
\end{aligned}$$

$$\begin{aligned}
10_H &= \dots + \textcircled{2}_L \nu_H \\
&\quad (7, 8, 9, 10)
\end{aligned}$$

$$16_H = \left(\begin{array}{c} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{array} \right) \text{doublet} \Leftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$\text{Fermions} \subseteq 16_F$$

$$\mathcal{L}_Y = 16_F 16_F$$

~~16_H~~
not Dim.

$$\boxed{16_F 16_F 10_H}$$

$$16_F^T \Gamma_i 16_F (10_H)_i$$

$$i = 1, \dots, 10$$

