Neutrino GUT Course

Lecture XXII

27/11/2023

LMU

Winter 2023

Chiral anomalies

- Noether

global symmetry $\phi \to e^{i\alpha}\phi$

$$\Rightarrow \boxed{\partial_\mu j^\mu = 0}$$

$$\delta S = \alpha \int \partial^\mu j_\mu \, d^4x$$

$$\delta S = 0 \Rightarrow \partial^\mu j_\mu = 0$$

Example : chiral symmetry

$$\mathcal{L} = \bar{\Psi} i \gamma^\mu D_\mu \Psi - m \bar{\Psi} \Psi$$

(a) $\Psi \to e^{i\alpha} \Psi \Rightarrow$

$$\partial_\mu j^\mu = 0 \, , \; j^\mu = \bar{\Psi} \gamma^\mu \Psi$$

(b) $\Psi \to e^{i\beta \gamma_5} \Psi$

$$\partial_\mu k^\mu = 2m \, \bar{\Psi} \gamma_5 \Psi$$

$$k^\mu = \bar{\Psi} \gamma^\mu \gamma_5 \Psi$$

$m = 0 \Rightarrow$ chiral symmetry

$$\Rightarrow \boxed{\partial_\mu k^\mu = 0}$$

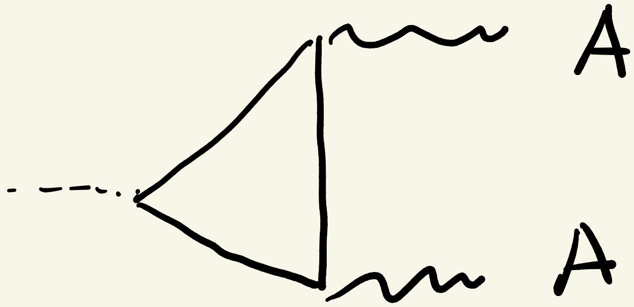
classical (tree)

<u>loop</u>

$F\tilde{F}^{\phi}$

|||

$$\boxed{\partial_\mu K^\mu = \frac{g^2}{32\pi^2} F_{\mu\nu} F_{\alpha\beta}\, \epsilon^{\mu\nu\alpha\beta}}$$



$$\Rightarrow \quad \delta S = \beta \int \partial^\mu K_\mu$$

$$= \beta \, \frac{g^2}{32\pi^2} F\tilde{F}^{\phi}$$

- if global transf.

$\Rightarrow$ who cares?

- local symmetry (gauge)

$\Rightarrow$ grave problem!

<u>why?</u> we need symmetry

$(\partial_\mu j^\mu = 0)$

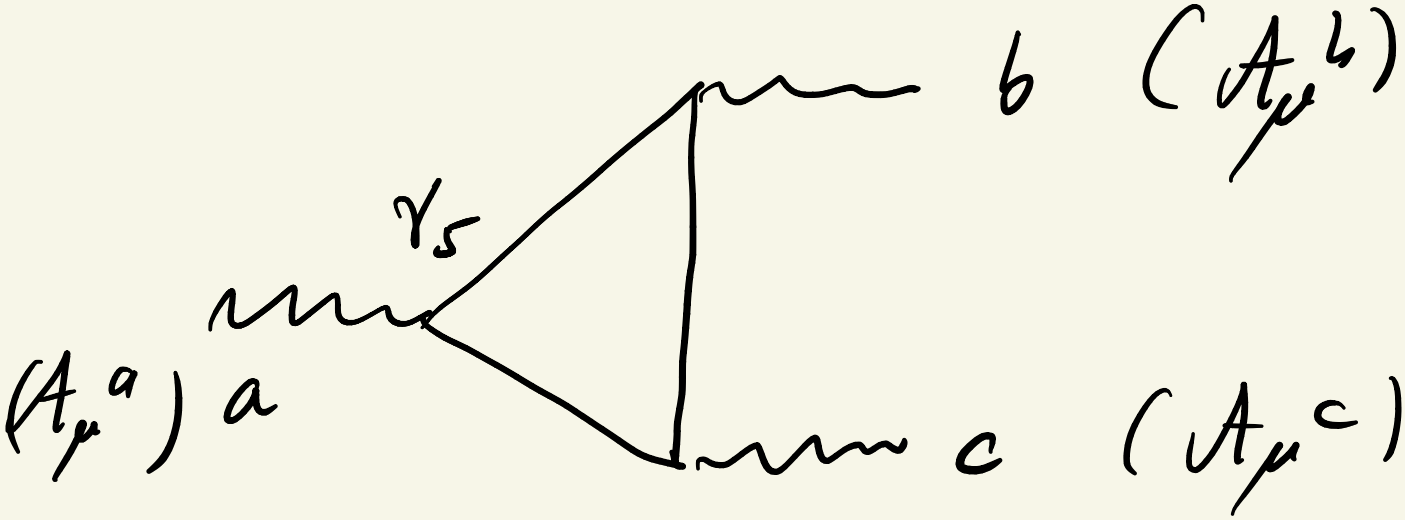
to prove renormalizability

$\Downarrow$

$\boxed{\text{kill anomalies!}}$

$$\Updownarrow$$

$\boxed{\text{anomalies = cancel}}$



$$A_{abc} \propto \mathrm{Tr}\, T_a T_b T_c + \text{perm.}$$

↳ coeff. of anomaly

$$\partial_\mu K_\mu \propto \mathbb{A}_{abc}$$

$\uparrow$

group repr. of my fermions

$$\boxed{\begin{aligned} \mathbb{A}_{abc} &= \mathrm{Tr}\{T_a, T_b\} T_c \, C_A \\ C_A(\text{fundamental } R) &= 1 \end{aligned}}$$

- $\underbrace{LH}_{\text{left-handed}} + \underbrace{RH}_{\text{right-handed}}$ fermions example of cancellation:

$$LH = RH$$

$$\Downarrow$$

$$SM \longrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\mathcal{L} = i\, \bar{\psi}_L \gamma^\mu D_\mu \psi_L + i\, \bar{\psi}_R \gamma^\mu D_\mu \psi_R$$

$$D_\mu \psi_L = (\partial_\mu - i g\, A_\mu^a T_a)\, \psi_L$$

$$D_\mu \psi_R = (\partial_\mu - i g\, A_\mu^a T_a)\, \psi_R$$

$$\Downarrow \qquad T_a^L = T_a^R \equiv T_a$$

$$\mathcal{L}_{int} = g\, A_\mu^a \left(\bar{\psi}_L \gamma^\mu T_a \psi_L + \bar{\psi}_R \gamma^\mu T_a \psi_R \right)$$

$$= g\, A_\mu^a\, \bar{\psi} \gamma^\mu T_a \psi$$

<u>no</u> γ_5

- LH + (LH anti) fermions

$$\underset{T_a^L}{\psi_L}\,, \quad \underset{(T_a^R)^*}{(\psi^c)_L} = C\,\bar{\psi}_R^{\,T} \propto \psi_R^*$$

$$\{T_a^*, T_b^*\}\, T_c^* = \{T_a^T, T_b^T\}\, T_c^T$$

$$T_a^* = T_a \Rightarrow \boxed{T_a^* = T_a^T}$$

$$= \{T_a, T_b\}^T T_c^T$$

$$\Rightarrow \mathrm{Tr}\{T_a^*, T_b^*\} T_c^* = \mathrm{Tr}\{T_a, T_b\}^T T_c^T$$

$$= \mathrm{Tr}\left(T_c \{T_a, T_b\}\right)^T$$

$$= \mathrm{Tr}\ T_c \{T_a, T_b\} = \mathrm{Tr}\{T_a, T_b\} T_c$$

<u>Check</u>

$$\{T_a^*, T_b^*\} T_c^* = \{T_a^T, T_b^T\} T_c^T$$

$$\stackrel{?}{=} \{T_a, T_b\}^T \, T_c^T \quad \text{(above)}$$

$$\{T_a, T_b\}^T = (T_a T_b + T_b T_a)^T$$

$$= T_b^T T_a^T + T_a^T T_b^T$$

$$\{T_a^T, T_b^T\} = T_a^T T_b^T + T_b^T T_a^T$$

check!

$$\mathrm{Tr}\, \{T_a, T_b\}\, T_c = \mathrm{Tr}\, \{T_a^*, T_b^*\}\, T_c^*$$

$$\boxed{\mathrm{Tr}\, M = \mathrm{Tr}\, M^T}$$

$$\boxed{\operatorname{Tr} M \neq \operatorname{Tr} M^* \quad \text{in } \underline{\text{general}}}$$

group = Algebra

$$\{T_a, T_b\} = i f_{abc} T_c$$

$$\Downarrow$$

$$\{T_a, T_b\}^* = -i f_{abc} T_c^*$$

$$\boxed{\{-T_a^*, -T_b^*\} = i f_{abc} (-T_c^*)}$$

$$\Uparrow$$

$(f_{abc} \in \mathbb{R})$

$$\boxed{T_a(\bar{R}) = -T_a^*}$$

$$\boxed{\mathcal{A}(\bar{R}) = -\mathcal{A}(R)}$$

SM chiral Anomalies

$$\underbrace{SU(3)_c}_{V = L + R} \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{chiral } L \neq R}$$

$V = L + R$ $\Updownarrow$ No anomaly

chiral $L \neq R$ $\updownarrow$ $[T_a, Y] = 0$

$\Downarrow$

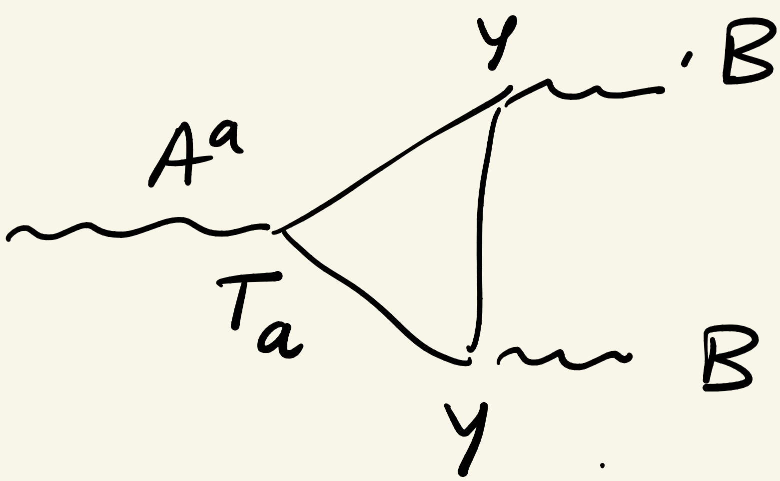
$SU(2)_L \times U(1)_Y$ anomaly

- pure $SU(2)_L$

$$\mathrm{Tr}\{T_a, T_b\} T_c \propto \mathrm{Tr}\{\sigma_a, \sigma_b\}\sigma_c$$

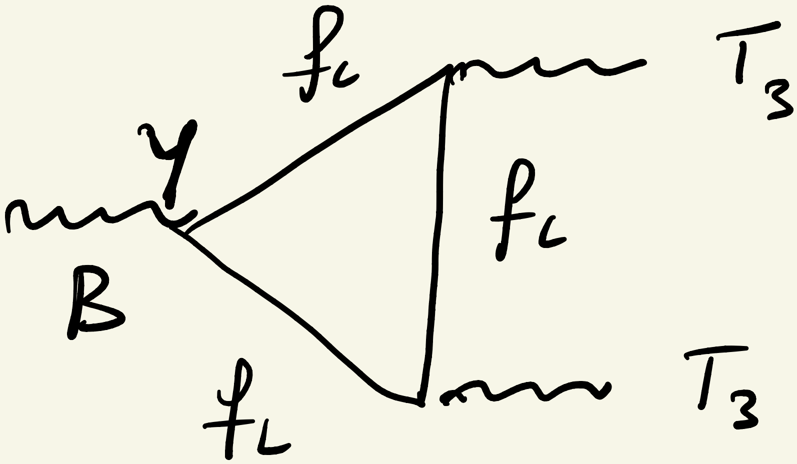
$$\propto \delta_{ab}\, \mathrm{Tr}\, \sigma_c = 0$$

- $SU(2) \times U(1)^2$ anomaly



$$\mathrm{Tr}\, T_a Y^2 \propto y\, \mathrm{Tr}\, T_a = 0$$

- $SU(2)^2 \times U(1)$ anomaly



$$T_3^2 = t_3^2 = \frac{1}{4}$$

$$\Downarrow$$

$$\boxed{A_{A^2B} \propto \mathrm{Tr}\, Y_L = 0}$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$Y = \frac{1}{3} \qquad Y = -1$$

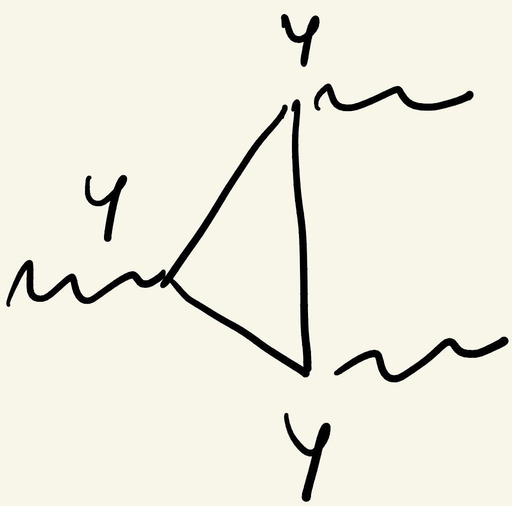
$$\Downarrow$$

$$A_{A^2B} \propto \underset{Y}{\frac{1}{3}} \cdot \underset{(u,d)}{2} \cdot \underset{\text{color}}{3} + \underset{Y}{(-1)} \cdot \underset{(\nu,e)}{2} = 0$$

a must

2nd $\begin{pmatrix} c \\ s \end{pmatrix}$ $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$

- $U(1)^3$ anomaly



$\Rightarrow \mathrm{Tr}\, Y^3 = 0$

$\Updownarrow$

$$\boxed{\mathrm{Tr}\, Y_L^3 = \mathrm{Tr}\, Y_R^3}$$

$$\left[T_r\left(Y_f^3 + Y_{\bar{f}}^3\right) = 0 \right]$$

LH LH

$\boxed{\text{Check !!!}}$

$$\boxed{Q = T_3 + \frac{Y}{2}}$$

Charge quantization in SM

$\Uparrow$

Anomaly cancellation

$$\begin{array}{cc} (Y_Q) & (Y_\ell) \\ \begin{pmatrix} u \\ d \end{pmatrix}_L & \begin{pmatrix} \nu \\ e \end{pmatrix}_L \end{array}$$

$$\begin{array}{ccc} u_R, & d_R, & e_R \\ Y_U & Y_D & Y_E \end{array}$$

$$Q = T_3 + \frac{Y}{2}$$

$$\boxed{\bullet\ \text{QED} = \text{vector-like} \iff Q_L = Q_R}$$

$$Q_{u_L} = Q_{u_R}$$

$$\frac{1}{2} + \frac{Y_q}{2} = \frac{Y_U}{2}$$

$$\Rightarrow \boxed{\begin{aligned} Y_U &= 1 + Y_q \quad (1) \\ Y_D &= -1 + Y_q \quad (2) \\ Y_E &= -1 + Y_\ell \quad (3) \end{aligned}}$$

$$\bullet\ \mathcal{L}_Y = \bar{Q}_L\, \phi\, d_R + \bar{Q}_L\, i\sigma_2\, \phi^* u_R + \bar{\ell}_L\, \phi\, e_R + h.c.$$

$$\boxed{Y(\phi) = +1}$$ normalization

$$\Rightarrow \begin{cases} -Y_Q + 1 + Y_D = 0 \\ -Y_Q - 1 + Y_U = 0 \\ -Y_\ell + 1 + Y_E = 0 \end{cases}$$

(1), (2), (3)

$$\Downarrow$$

- $SU(2)^3$ anomaly $= 0$

- $SU(2) \times U(1)^2$ —''— $= 0$

- $SU(2)^2 \times U(1)$ —''— $\propto \mathrm{Tr}\, Y_L = 0$

$$\Rightarrow \boxed{3Y_q + Y_e = 0}$$

- $U(1)^3$ —''— $=$ messy

$$\Downarrow$$

$$(Y_e + 1)^3 = 0$$

$$\Rightarrow Y_e = -1, \; Y_q = \frac{1}{3}$$

charge quantization

$Q_e = -1, \quad Q_\nu = 0$

$Q_u = 2/3, \quad Q_d = -1/3$

Instead: $Q_L = Q_R$

$\Downarrow$

$Y_U = 1 + Y_q$

$Y_D = -1 + Y_q$

$Y_E = -1 + Y_e$

$\mathrm{Tr}\, Q_L = \mathrm{Tr}\left(T_3 + \frac{Y_L}{2}\right) = 0$

$\mathrm{Tr}\, Q_R = \mathrm{Tr}\, \frac{Y_R}{2}$

but: $\mathrm{Tr}\, Q_R = \mathrm{Tr}\, Q_L = 0$

$$\Rightarrow \boxed{\mathrm{Tr}\, Y_R = 0}$$

$$\Downarrow$$

$$\boxed{(Y_U + Y_D)\, \underset{\text{col}}{3} + Y_E = 0}$$

$$\Downarrow$$

$$(Y_q \cancel{+1} + Y_q \cancel{-1}) \cdot 3 + Y_e - 1 = 0$$

$$6 Y_q + Y_e - 1 = 0 \qquad (R)$$

$$3 Y_\ell + Y_e = 0 \qquad (L)$$

$$\Downarrow$$

$$-2Y_e + Y_e - 1 = 0$$

$$\Rightarrow \boxed{Y_e = -1}$$

. if instead:

$$\mathrm{Tr}\, Y_L^3 = \mathrm{Tr}\, Y_R^3$$

$$Y_q^3 \cdot \underset{u,d}{2} \cdot \underset{\text{color}}{3} + Y_e^3 \cdot \underset{\nu,e}{2} = \mathrm{Tr}\, Y_L^3$$

$$\boxed{\mathrm{Tr}\, Y_L^3 = 2\left(3Y_q^3 + Y_e^3\right)}$$

$$Tr\, Y_A^3 = [(Y_Q + 1)^3 + (Y_Q - 1)^3]\, 3$$

$$+ (Y_E - 1)^3$$

$$= 6 Y_Q^3 + (3 Y_Q (1)^2 + 3 Y_Q (-1)^2$$

$$+ \cancel{3 Y_Q^2 (1)} + \cancel{3 Y_Q^2 (-1)} + \cancel{1 - 1})\, 3$$

$$+ (Y_e^3 + 3 Y_e - 3 Y_e^2 - 1)$$

$$Tr\, Y_A^3 = 6 Y_Q^3 + 18 Y_Q + Y_e^3 + 3 Y_e - 3 Y_e^2 - 1$$

$$= Tr\, Y_L^3 = 6 Y_e^3 + 2 Y_e^3$$

$$+$$

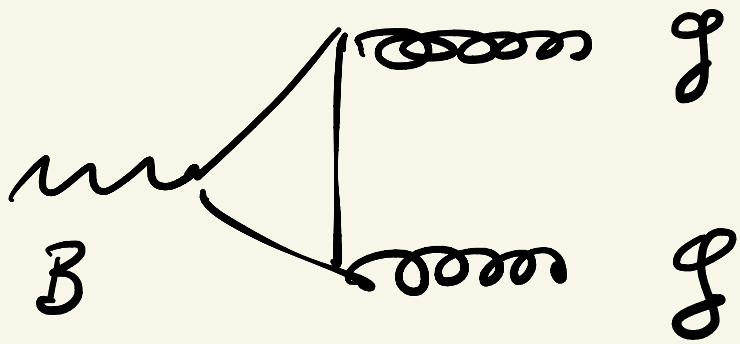
$$\boxed{3 Y_e + Y_e = 0}$$

$$\Downarrow$$

$$\boxed{Y_e = -1}$$

Prove

+ gravity



$\propto \mathrm{Tr}\, Y$

$\mathrm{Tr}\, Y_L = 0 \Rightarrow \mathrm{Tr}\, Y_R = 0$