Neutrino GUT Course

Lecture XXIII

LMU

Winter 2023

Anomalies (II)

chiral (γ_5)

$\Downarrow$

anomaly

$$\partial_\mu j^\mu_5 = \frac{g^2}{32\pi^2} F\tilde{F}^d$$

$$= \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$$

. <u>global symmetry</u> (continuous)

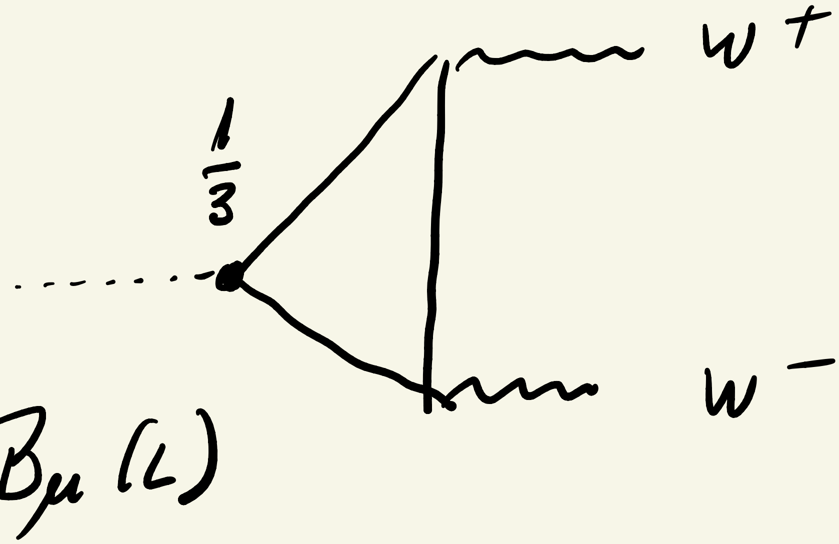
B (baryon number) $U(1)_B$

L (lepton —"—) $U(1)_L$

$\Rightarrow \quad U(1)_B \quad SU(2)^2 \qquad (a)$

$\quad U(1)_L \quad SU(2)^2 \qquad (b)$

(a)

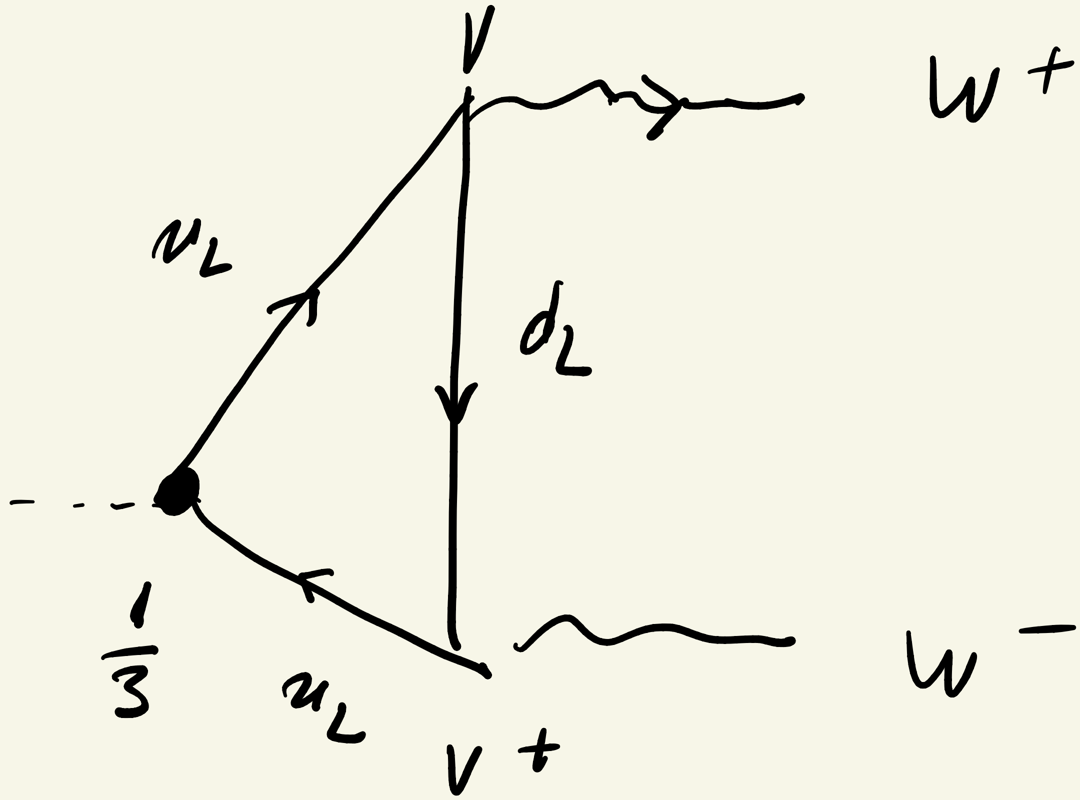


$$\Rightarrow \partial_\mu B^\mu \propto F^a_{\mu\nu} F^a_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta}$$

baryon current

/

$\downarrow$ magnify



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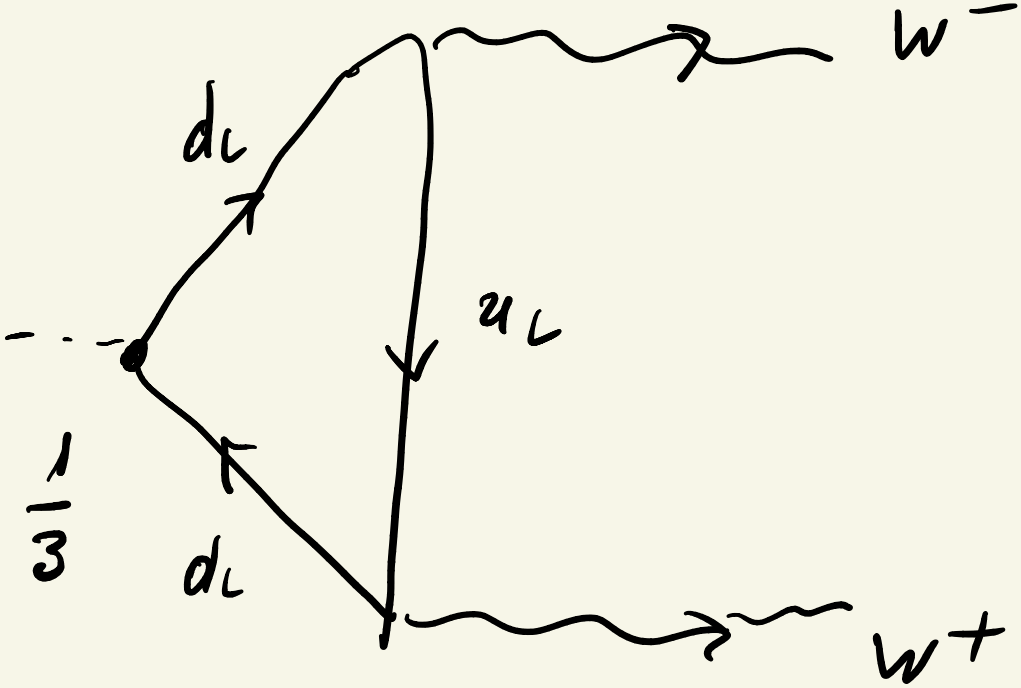


$$\Downarrow$$

$$\partial_\mu B^\mu = c\,\frac{1}{3}\cdot 2\cdot 3 \cdots \quad (\text{anomaly}) \neq 0$$

$B(q)$ $\quad (u_L + d_L)$ $\quad$ color

$$\Downarrow$$

$\Delta B \neq 0$ in SM

$p \to \cdots$

$$F F^d = \partial_\mu K^\mu$$

$$\boxed{K^\mu = \epsilon^{\mu\nu\alpha\beta}\left[\epsilon^{abc} A_\nu^a A_\alpha^b A_\beta^c + A_\nu^a F_{\alpha\beta}^a\right]}$$

$$\underbrace{\int F \tilde{F}^d \propto \int dS_\mu K^\mu \neq 0}_{\text{instanton solutions}}$$

$$\Downarrow$$

$$\boxed{\text{non-perturbative phenomenon}}$$

digression

$a = 1, \dots, 8$

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a}$$

$$\underbrace{+ \frac{g^2}{32\pi^2}\theta\, G^a_{\mu\nu} G^a_{\alpha\beta}\, \epsilon^{\mu\nu\alpha\beta}}$$

$$\cancel{CP}, \; \cancel{P}$$

$$\updownarrow$$

$$\cancel{T}$$

QED

$$F\tilde{F} \propto \vec{E}\cdot\vec{B}$$

$$F^2 \propto \vec{E}^2 - \vec{B}^2$$

P:
$$\vec{E} = \vec{V}, \quad \vec{B} = \vec{A}$$

$$\vec{E}\cdot\vec{B} \xrightarrow{P} -\vec{E}\cdot\vec{B}$$

T:
$$\vec{F}(\text{Lorentz}) = q\underbrace{(\vec{E} + \vec{v}\times\vec{B})}$$

$$\boxed{\begin{array}{l}\vec{v} \to \\ -\vec{v}\end{array}}$$

$$\Rightarrow \quad \vec{E}\cdot\vec{B} \xrightarrow[T]{} -\vec{E}\cdot\vec{B}$$

CP

$$\Rightarrow \quad \boxed{\begin{array}{l}\theta \; G\, G^{d} \; (\text{QCD}) \\ \text{violates both P and CP}\end{array}}$$

SU(2) $k^\mu \propto \epsilon^{\mu\nu\alpha\beta} \left[\epsilon A^3 + + A F \right]$

in non-Abelian

$\Downarrow$

QED

$$k^\mu = \epsilon^{\mu\nu\alpha\beta} \left[\cancel{A_\nu A_\alpha A_\beta} + A_\nu F^{\alpha\beta} \right]$$

$\uparrow$

U(1)

$$\int F F^d = \int_{\text{surface at } \infty} k^\mu \, dS_\mu = 0$$

$\Uparrow$

but

$$\underbrace{\int F^2} = \int F_{\mu\nu} F^{\mu\nu} = \text{finite}$$

$$\Downarrow$$

$$F_{\mu\nu} \to 0 \ \text{at} \ \infty$$

$$\boxed{\int F_{\mu\nu} F^{\mu\nu} \, d^4x = \text{Maxwell action}}$$

$$\Downarrow$$

$$\int_{\infty} K_\mu \, dS^\mu \propto \int_{\infty} F = 0$$

$$\Downarrow$$

In QED **NO** effects from $F\tilde{F}$

QCD

$$K^\mu \propto A^3 + \cancel{AG}$$

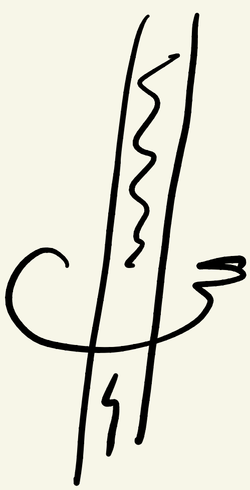
but: $F = 0 \not\Rightarrow A = 0$

$$\Downarrow$$

$$\boxed{A = \text{pure gauge}}$$

example

Strings (cosmological)



$$\int \bar{B} \cdot d\bar{S} = \oint A_\mu dx^\mu$$

$$A_\mu \propto \partial_\mu \theta$$

$$\oint A_\mu dx^\mu = \Delta\theta = 2\pi$$

$$\Downarrow$$

Non-Abelian

$$\exists \int F\tilde{F} \neq 0$$

non-perturbative

result

$$\Delta B\,(SM) \propto e^{-\frac{4\pi}{\alpha}} \ldots$$

or better

$$\Gamma(p \to \pi^0 e^+) \propto e^{-\frac{4\pi}{\alpha_w}} M_w$$

$$1/\alpha_w = 30$$

$$e^{-300} = 0$$

$$\Updownarrow$$

$$\boxed{\tau_p\,(SM) \simeq 10^{130}\ yr}$$

QCD

$$e^{-\frac{4\pi}{\alpha_s}} \simeq O(1)$$

Must be kept!

$$\left[\frac{g_s^2}{32\pi^2}\,\theta\, G\, G^d\right] \quad \not P \quad \not{CP}$$

$$d_n^e = \theta / \Lambda_{QCD} \times ?$$ ⊛

electric dipole moment of neutron

⊛ $m_q = 0$ (u, d)

$$q \to e^{i\beta \gamma_5} q$$

$$\Downarrow \qquad j_\mu^5 = \bar{q}\, \gamma_\mu \gamma^5 q$$

$$\delta S = \beta \int \partial^\mu j_\mu^5$$

$$= \beta / \frac{g^2}{32\pi^2} \int G \tilde{G}\, d$$

$$\boxed{\begin{gathered}\theta \longrightarrow \theta + \beta = 0 \\ \text{for } \beta = -\theta\end{gathered}}$$

$\circledast$

$$\Downarrow$$

$$d_n^e = \frac{\theta}{\Lambda_{QCD}} \frac{m_q}{\Lambda_{QCD}} \leq 10^{-26}\,\text{cm}$$

$$\Lambda_{QCD} = \text{GeV}$$

$$\text{GeV}^{-1} = 10^{-14}\,\text{cm}$$

$$\frac{m_q}{\Lambda_{QCD}} \simeq 10^{-2} \quad (m_q \sim 10 \text{ MeV})$$

$$\Downarrow$$

$$d_n^e = \theta \times 10^{-16} \text{ cm} \leq 10^{-26} \text{ cm}$$

$$\Downarrow$$

$$\boxed{\theta \leq 10^{-10}}$$

Strong CP "problem"!

Global anomalies

- B :

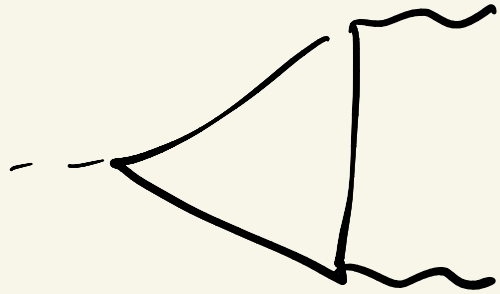
$$\partial_\mu B^\mu \,(\text{current}) \;\propto\; F\tilde{F}^d$$

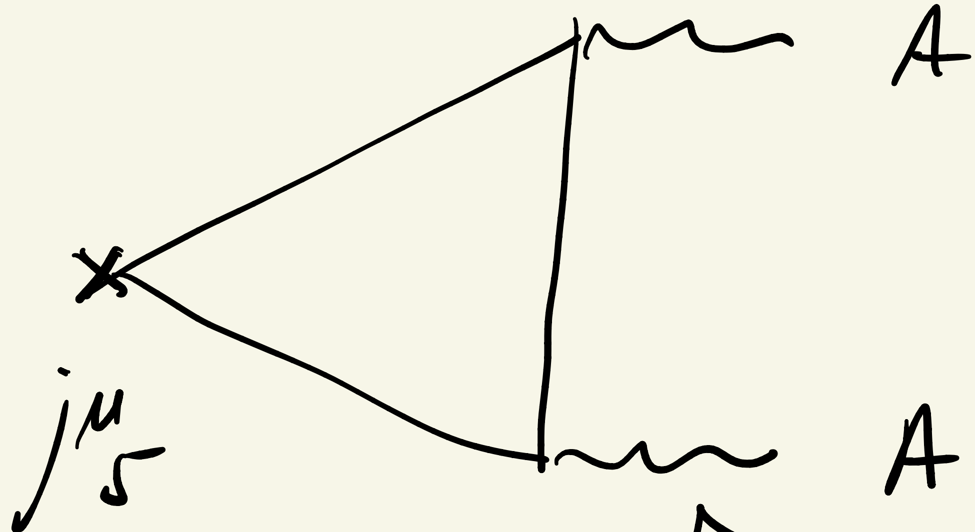
$$\Downarrow$$

$$\Gamma(p \to \pi^0 e^+) \simeq e^{-\frac{4\pi}{\alpha}} M_W$$

largest possible scale

$$\Rightarrow \boxed{\tau(p \to \pi^0 e^+) \simeq 10^{+130}\ \text{yr}}$$

$\partial_\mu j^\mu_5 \propto$ 



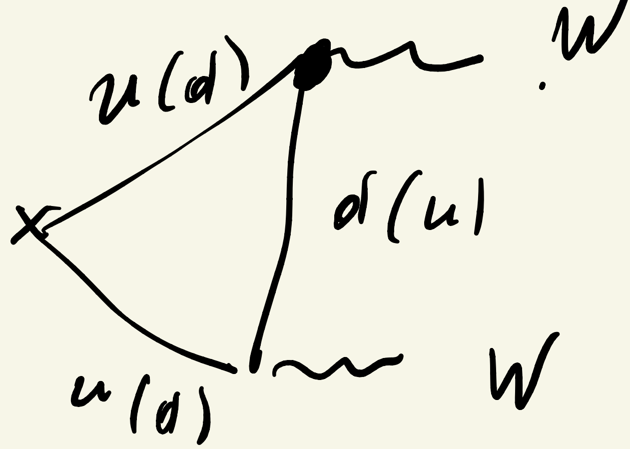
$$\Rightarrow \partial^\mu j_\mu^5 \alpha \propto F\tilde{F}$$

$$\Downarrow$$

$$\boxed{\Gamma(\Delta B \neq 0) \neq 0}$$

$$\underbrace{e^{-\frac{4\pi}{\alpha}} \simeq 0}_{\boxed{\text{in weak coupling regime}}}$$

• B



$B_Q = 1/3$

$(u + d)$

$$\partial_\mu B^\mu = C \cdot \frac{1}{3} \cdot 2 \cdot 3$$

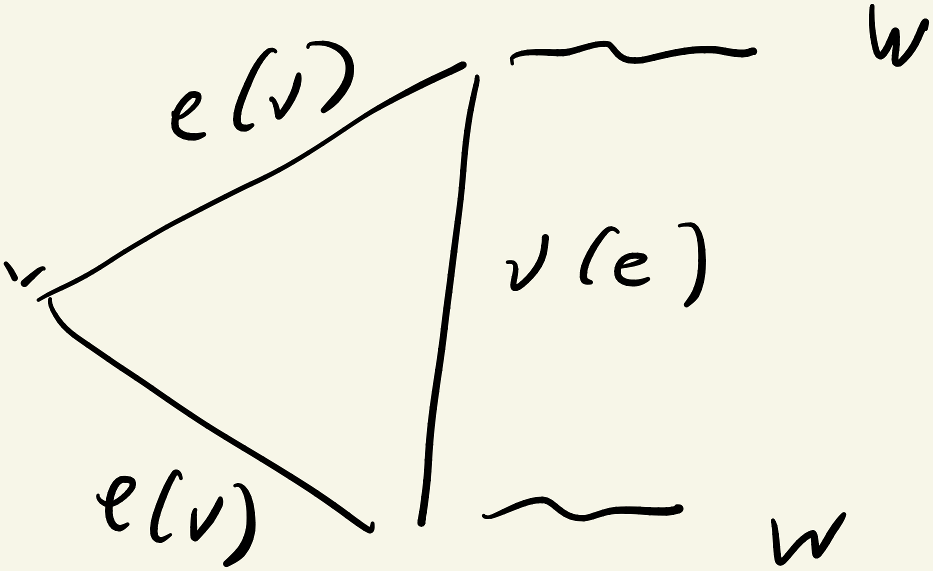
B_q $\quad u_L + d_L \quad$ color

• L

$$\partial^\mu L_\mu = C \cdot 1 \cdot 2 \cdot 1 \quad \text{(no color)}$$

L_e $\quad (\nu_L + e_L)$

$L_e = 1$



$\Downarrow$

$$\boxed{\partial_\mu \left(B^\mu - L^\mu\right) = 0}$$

$\boxed{\text{anomaly-free}}$

- $SU(5)$: $\Delta(B-L) = 0$

- effective

$$\frac{1}{\Lambda_B^2}\, qqq\ell$$

$$\Downarrow$$

$$\Delta(B-L) = 0$$

$$\boxed{\text{NO QCD anomaly for } B, L}$$

$$\Downarrow$$

$$\boxed{\text{proton = stable (effectively)}}$$

Early Universe?

$\Delta B \neq 0$?

$\Delta L \neq 0$?

$T \gg M_W \Rightarrow$

$e^{-4\pi/\alpha}$ suppression

goes away

$$\Rightarrow \quad \Delta(B+L) \neq 0 \quad \text{at } T \gg M_W$$

$$\Delta(B-L) = 0 \quad \text{— " —}$$