Neutrino GUT Course

Lecture XXI

24/11/2022

LMU
Winter 2023

BNV: Effective Theory

- LNV

$$\mathcal{H}_{eff} = \frac{\ell \, \ell \, \phi \, \phi}{\Lambda_{\not{L}}} \qquad (d=5) \qquad (1)$$

$$\Downarrow$$

(D × D)

$$\underbrace{\nu \, \nu}_{SU(2)\ \text{triplet}\ (T)} \frac{\langle \phi \rangle^2}{\Lambda_{\not{L}}}$$

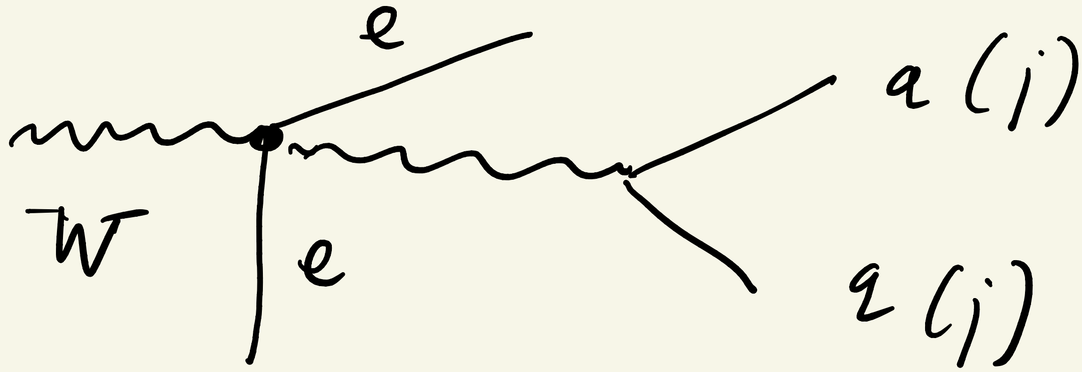
$$m_\nu = \frac{\langle \phi \rangle^2}{\Lambda_{\not{L}}}$$

$$\Rightarrow \; y_\nu = \frac{\langle \phi \rangle}{\Lambda_{\not{L}}} \; \Rightarrow$$

$$\Gamma(h \to \nu\nu) \propto y_\nu^2 = \frac{m_\nu}{\Lambda_{\not{L}}} \to 0$$

From (1):

$$e\,e\,\frac{\phi^+\phi^+}{\Lambda_{\not{L}}} \longrightarrow e\,e\,\frac{WW}{\Lambda_{\not{L}}}$$



(negligible)

BNV in SU(5)

B - L: $\quad 2/3 \qquad -1/3+1 = 2/3$

$$X_\mu \left[\bar{u}^{c\,0} \gamma^\mu u^0 + \bar{e}^0 \gamma^\mu d^{0\,c} + \right.$$

$$\left. + \bar{d}^0 \gamma^\mu e^{0\,c} \right] \qquad (2)$$

$-1/3+1 = 2/3$

$$\bar{u}_L^0 \underline{M}_u u_R^0$$

. - - - -

$$\bar{d}_L^0 M_d d_R^0$$

$\underline{M}_u, M_d, M_e \longrightarrow$ diagonal

$$f^0_{L,R} \longrightarrow F_{L,R} \quad f_{L,R}$$

physical

$$F_L^+ F_L = F_R^+ F_R = 1$$

(mass states)

$$f_L^c = C \bar{f}_R^T = C \gamma_0 f_R^*$$

$$\Downarrow$$

$$X_\mu \Big[\overline{u^c}\, U_R^T U_L \gamma^\mu u + \\ + \bar{e}\, E_L^\dagger D_R^* \gamma^\mu d^c + \\ + \bar{d}\, D_L^\dagger E_R^* \gamma^\mu e^c \Big]$$

Minimal ren. $(d=4)$ model

$$M_d^T = M_e, \quad M_U^T = M_U$$

Mohapatra 1979

$$\Downarrow$$

$$\left.\begin{aligned} D_L^{+} E_R^{*} &= 1 \\ E_L^{+} D_R^{*} &= 1 \\ V_R^{T} V_L &= 1 \end{aligned}\right\} \Rightarrow \boxed{\text{all fixed}}$$

$$Y_u \left[\overline{u^c}\, V_R^{T} D_L\, d \;+\; - - \; - - - \right]$$

$$\|$$

$$\overline{u^c}\; \underbrace{V_R^{T} V_L}_{1}\; \underbrace{V_L^{+} D_L}_{V_{CKM}}\; d .$$

$$\Rightarrow \boxed{\text{all } p \text{ decay BR are predicted}}$$

$\Downarrow$ cure by $d>4$

M_u, M_d, M_e = free

$\Rightarrow$ unknown, arbitrary mixings

Weinberg 1979

$\underline{d>4}$ $\leftarrow$ generic to GUT

$\Updownarrow$

generic to large scale BNL

$(\Lambda_{\cancel{B}} \equiv \Lambda)$

$\Downarrow$

<u>leading operators</u>

expansion in $\frac{M_W}{\Lambda_{\cancel{B}}} \ll 1$

$\Downarrow$

leading operator symmetric under G_{SM}

(d>4) Lorentz, $SU(2) \times SU(3) \times U(1)$ symmetric

$\uparrow$

| Why?

$E \gg M_W \Rightarrow$

$$\cancel{SU(2) \times U(1)} \propto \frac{M_W}{E}$$

- $E \gtrsim \Lambda_{\cancel{B}} \simeq 10^{15}\,\mathrm{GeV} \Rightarrow$

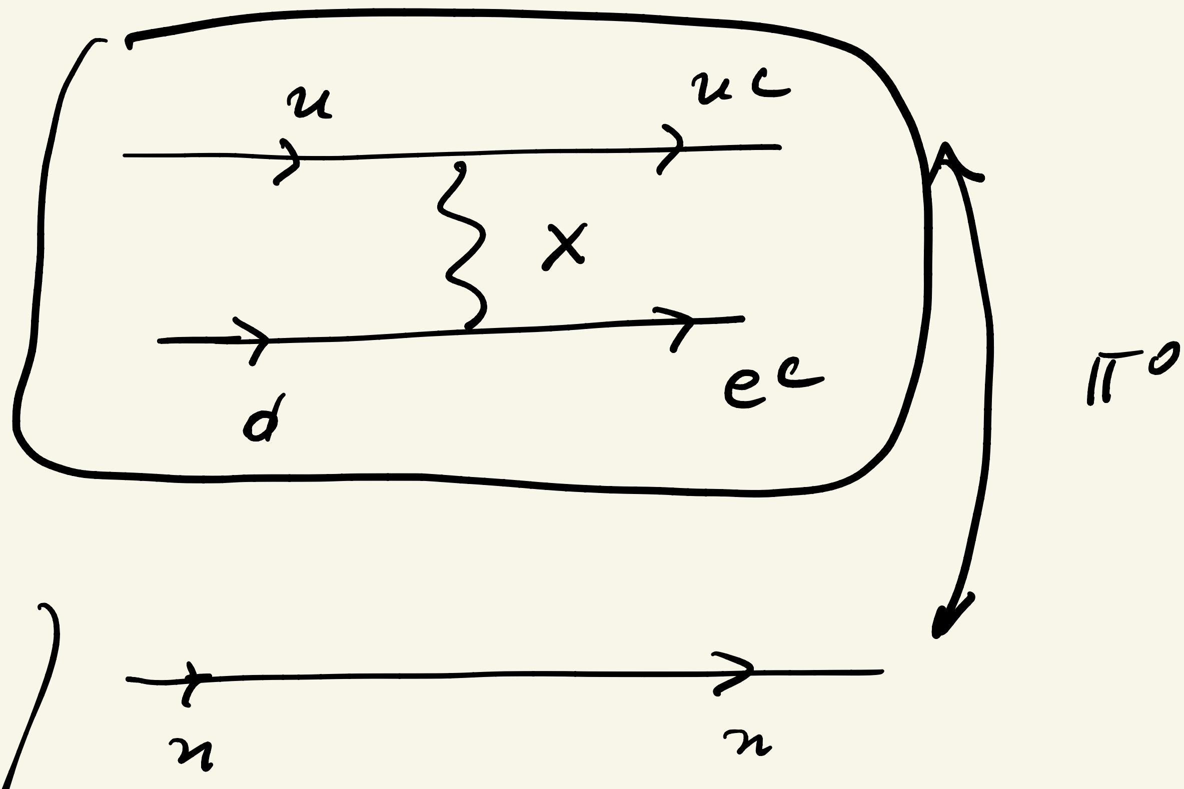
$$\frac{M_W}{E} \lesssim \frac{100}{10^{15}} \simeq 10^{-13}$$

- $E \simeq 10^{3} M_W \Rightarrow$

$$M_W/E \simeq 10^{-3} \longrightarrow 0$$

Digression SU(5)

$$X \left[\overline{u^c}\, u + \bar{d}\, e^c + \bar{e}\, d^c \right] + h.c.$$



$$\boxed{u\, d\, \overline{u^c}\, \overline{e^c}\, \frac{1}{M_X^2}}$$

$$\Downarrow \quad (d > 4)$$

$$\cancel{q\bar{q}} \qquad \Delta B = 0$$

$q\,q \leftarrow$ not good $\cancel{SU(3)_c}$

$\downarrow$

$$\underbrace{\epsilon_{\alpha\beta\gamma}\, q^\alpha q^\beta q^\gamma}_{\Delta B = 1} = SU(3)_c \text{ singlet}$$

(BNV)

$\Downarrow$ Lorentz

$$\boxed{q\,q\,q\,\ell \;;\; q\,q\,q\,\bar{\ell}}$$

$\uparrow$

$f_L,\ f_R \quad (q_L, q_R)$

Possibilites:

(a) all R $\boxed{\Delta(B-L)=0}$

- $(u_R{}^T C\, u_R)(d_R{}^T C\, e_R)\dfrac{1}{\Lambda_B^2}$

~~$(u_R{}^T C\, d_R)(d_R{}^T C\, \nu_R)$~~

~~$(d_R{}^T C\, d_R)(d_R{}^T C\, (e^c)_R)$~~

$$(e^c)_R = C\bar{e}_L{}^T$$

by $SU(2)_L$!!

$$(d_R{}^T C\, d_R)\; d_R{}^T C\, (e^c)_R\, \frac{\langle\phi^0\rangle}{\Lambda_B^3}$$

Fimy

$$\phi_0 \, (e^c)_R \sim \bar{e}_L \, \phi_0$$

$$T_3 \quad \tfrac{1}{2} \; - \tfrac{1}{2} = 0$$

<u>but:</u> $$\frac{\langle \phi^0 \rangle}{\Lambda_{\cancel{B}}} \simeq \frac{M_W}{\Lambda_{\cancel{B}}} \ll 1$$

- $\cancel{\underbrace{u_R^T C u_R \, u_R^T}_{Q_{em}:\ 2} \, C \, \underset{\text{nothing}}{\uparrow} \, ?}$

$$\Downarrow$$

$$\boxed{\text{one } RR \text{ operator}}$$

- purely $L = (L)(L)$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L , \quad q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\cancel{l_L^T C i\sigma_2 l_L} \qquad q_L^T i\sigma_2 C q_L$$

$$l_L^T C\, i\sigma_2\, q_L$$

$$\Downarrow$$

$$\boxed{\Delta(B-L)=0}$$

$$(q_L^T c\, i\sigma_2 q_L)(l_L^T c\, i\sigma_2 q_L)$$

$$\|$$

$$(u_L^T c d_L)(\nu_L^T c\, d_L - e_L^T c\, u_L)$$

Q_{em}: $\quad 1/3 \qquad -1/3 \qquad -1/3$ ✓

$$Q_{em} = T_3 + Y/2$$

$$\Delta Q_{em} = \underset{\substack{\shortparallel \\ 0}}{\Delta T_3} + \Delta (Y/2)$$

$$\Delta(B-L)=0$$

- $\underbrace{\left(q_L{}^T C\, i\sigma_2\, q_L\right)}_{(u_L{}^T C d_L)} \left(u_R{}^T C e_R\right)$

Q_{em}: $\quad 1/3 \qquad\qquad -1/3$

$$\boxed{\Delta(B-L)=0}$$

- $\underbrace{\left(l_L{}^T C\, i\sigma_2\, q_L\right)}_{\nu_L{}^T C d_L - e_L{}^T C u_L} \underbrace{\left(u_R{}^T C d_R\right)}_{1/3}$

$$\underbrace{\nu_L{}^T C d_L - e_L{}^T C u_L}_{Q_{em} = -1/3}$$

$$\Downarrow$$

$$p \to \pi^0 + e^+ \,, \quad \cancel{\pi^+ + \pi^+ + e^-}$$

$$\cancel{n \to \pi^+ + e^-} \,, \quad n \to \pi^- + e^+$$

$$\uparrow \text{ this } \Longrightarrow \Lambda_{\not B} \simeq O(M_W)$$

$$\boxed{\begin{array}{l} \text{Weinberg } d=6 \\ \dfrac{1}{\Lambda_{\not B}^2}\left(qqql + \cancel{\dfrac{M_W}{\Lambda_{\not B}}\, qqq\bar{l}} \right) \\ \text{only iff } \Lambda_{\not B} \gg M_W \end{array}}$$

$qqql \Rightarrow qqsl$

$\Downarrow$

$\bar{s}$ comes out!

$\Downarrow$

2 body neutron decay

(1) $\cancel{n \rightarrow K^+ e^-}$ $\Leftarrow$ $\Delta(B-L)=0$

(2) $\cancel{n \rightarrow K^- e^+}$ $\Leftarrow$ $\bar{s}$ comes out

$\uparrow$

$K^- = \bar{u} s \quad , \quad K^+ = u\bar{s}$

$$\left.\begin{array}{l} p \rightarrow \pi^0 e^+ \\ n \rightarrow \pi^- e^+ \end{array}\right\} \text{ related}$$

$$\pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

$$\pi^+ = u\bar{d} \quad , \quad \pi^- = d\bar{u}$$

$$p \to \pi^+ + (\bar{\nu})_R \qquad p \to \pi^0 + e^+$$

$$n \to \pi^0 + \bar{\nu}$$

$$\boxed{\Gamma\left(p \to \pi^+ + (\bar{\nu})_R\right) = 2\Gamma\left(p \to \pi^0 + (e^+)_R\right)}$$

polarization

Summary

- $(q_L \, q_L)(q_L \, l_L)$
- $(q_L \, q_L)(u_R \, e_R)$
- $(q_L \, l_L)(u_R \, d_R)$
- $(u_R \, u_R)(d_R \, e_R)$

$\Delta(B-L) = 0$

$\dfrac{1}{\Lambda_B^2}$

$\Uparrow$

$\boxed{\text{only SM states}}$

leading $d = 6$

$$+ \ (d=7) \rightarrow q\,q\,q\,\bar{l}\ \frac{\langle\phi\rangle}{\Lambda_{\not B}}\ \frac{1}{\Lambda_{\not B}^2}$$

Conclusion:

$u \not\to k\,l$ of any type from $d=6$

- no GUT model that predicts BR of p decay

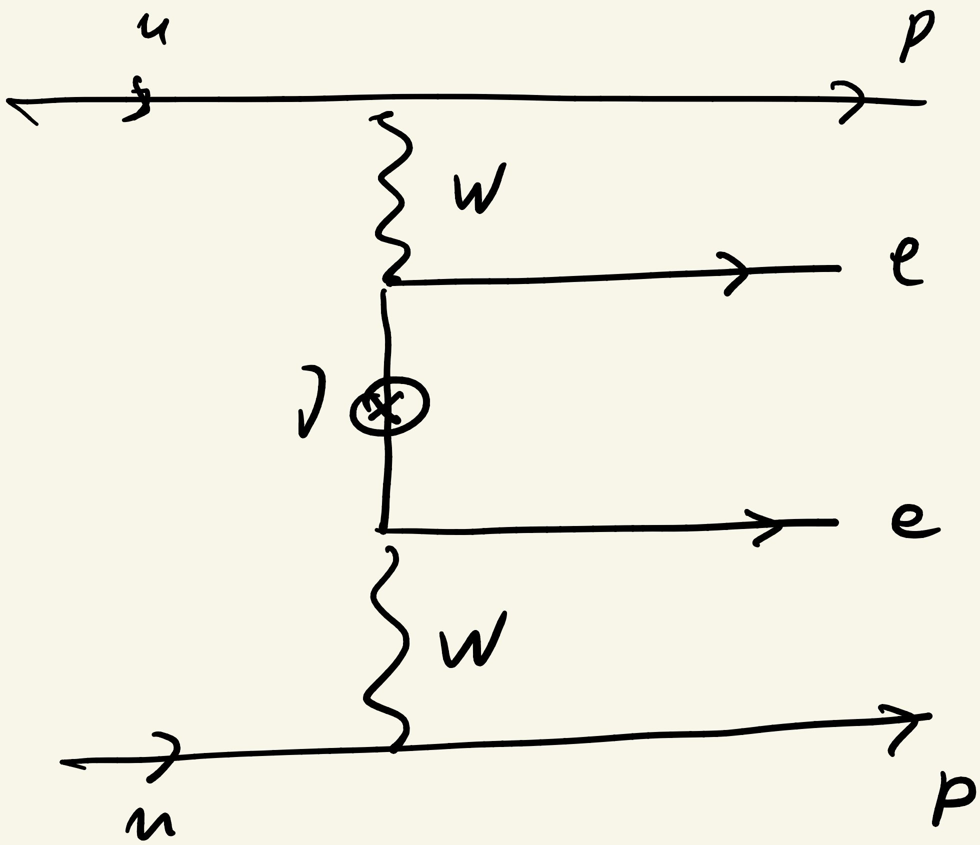
$$BR\,(p \to \mu^+ \pi^0) = ?$$

$$BR\,(p \to \pi^+ \bar{\nu}) = ?$$

$$\boxed{\Delta L = 2}$$

$0\nu 2\beta$

Neutrinoless double beta



$(\bar{p}p)(nn)(\bar{e}\bar{e})$ $\quad d=9$

$$\frac{\bar{u}\bar{u}\, dd\, \bar{e}\bar{e}}{\Lambda_{\not{L}}^5}$$

- $\boxed{\Lambda_{\not{B}} \gtrsim 10^{15}\,\text{GeV}} \Leftrightarrow \tau_p \gtrsim 10^{34}\,\text{yr}$

- $\tau_{0\nu2\beta} \gtrsim 10^{26}\,\text{yr} \Rightarrow \boxed{\Lambda_{\not{L}} \gtrsim 3\,\text{TeV}}$