Neutrino GUT Course

Lecture XX

20/11/2023

LMU

Winter 2023

SM and neutrino mass

$\downarrow$

go beyond

$\boxed{\text{Generic seesaw}}$ (Type I)

- $\exists\ \nu_R$ or $N_L = C\bar{\nu}_R^{\,T}$

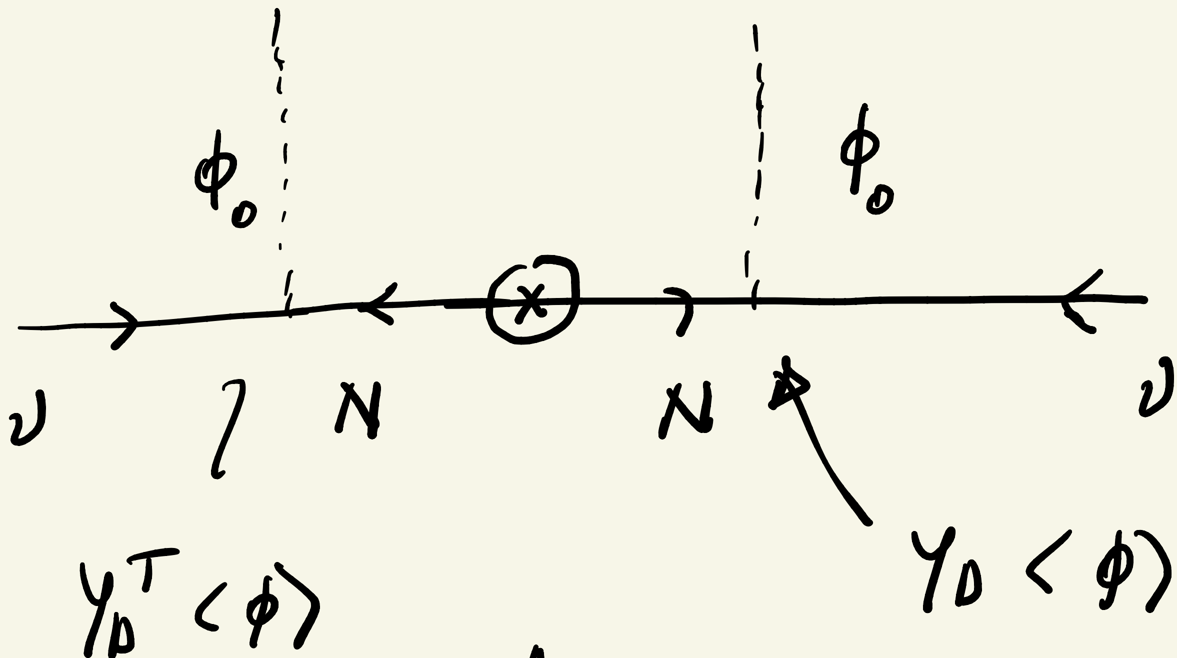
$\Downarrow \quad M_N \gg M_D$

$$\underline{M}_\nu = -M_D^T \frac{1}{\underline{M}_N} M_D$$

Majorana

diagrammatically

$\downarrow$



$$\Downarrow$$

$$\mathcal{L}_Y(\mathrm{eff}) =$$

$$Y_D^T \frac{1}{\not{p} - M_N} Y_D \, \phi_0 \phi_0$$

seesaw limit: $M_N \gg \cancel{\phi}$

$$\Downarrow \quad (\phi = 0)$$

$$\underline{M}_\nu = M_D{}^T \frac{1}{-M_N} M_D \quad \therefore$$

$$M_D = Y_D \langle \phi \rangle$$

$$\underline{M}_\nu \propto \frac{\langle \phi \rangle^2}{\Lambda_{new}}$$

$$\frac{1}{\Lambda_{new}} \simeq \frac{Y_D{}^2}{m_N}$$

• $N \longrightarrow T_F^0$ (Type III)

SU(2)$_L$ triplet

$$N^T M_D \nu \leftarrow N^T Y_D \phi \nu$$

$$\downarrow$$

$$T_F^{0T} M_D \nu \leftarrow T_F^{0T} Y_D \phi \nu$$

doublet (D) doublet (D)

triplet $\sim (D \times D)_S$

singlet $\sim (D \times D)_{AS}$

- <u>Type II</u>

add scalar triplet Δ :

$$\mathcal{L}(\Delta) = \overset{(1)}{l_L^T \, i\sigma_2 \, Y_\Delta \, \Delta \, l_L} +$$

$$+ \mu \, \phi^T i\sigma_2 \, \Delta^* \phi + h.c.$$
(2)

$$\therefore \boxed{\Delta \to U \Delta U^+}$$

$$\boxed{Y(\Delta) = +2 \quad (Y(\phi) = 1)}$$

$$\Downarrow$$

$$\langle \Delta^0 \rangle = \mu \frac{\langle \phi \rangle^2}{m_\Delta^2} \Leftarrow (1)$$

$$\Downarrow$$

$$M_\nu = Y_\Delta \langle \Delta^0 \rangle \Leftarrow (2)$$

$$M_\nu = Y_\Delta \, \mu \frac{\langle \phi \rangle^2}{m_\Delta^{\,2}}$$

$$\propto \frac{\langle \phi \rangle^2}{\Lambda_{new}}$$

$$\frac{1}{\Lambda_{new}} = Y_\Delta \frac{\mu}{m_\Delta^{\,2}}$$

$$\simeq \frac{Y_\Delta}{m_\Delta} \quad (\mu \sim m_\Delta)$$

Weinberg 1979

<u>Effective Field Theory</u>

of

($\cancel{L}$) Lepton Number Violation (LNV)

($\cancel{B}$) Baryon Number Violation (BNV)

$\boxed{\bullet \text{ LNV}}$

$$l_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$\frac{1}{\Lambda_{new}} \underbrace{l_L^T C \, i\sigma_2 \, l_L}_{\substack{Y=-2 \\ (d=3)}} \; \underbrace{\phi^T i\sigma_2 \, \phi}_{\substack{Y=2 \\ (d=2)}}$$

$$\left(\Lambda_{\text{new}} = \Lambda_{\not{L}} = \Lambda\right)$$

$$\phi^T \underbrace{i\,\sigma_2}_{\varepsilon} \phi = \phi_i\, \varepsilon_{ij}\, \phi_j = 0$$

$$\varepsilon_{ij} = -\varepsilon_{ji}$$

$$\left(l_L^{\,T} C\, i\sigma_2\, l = 0 \quad \text{also true} \right)$$

$$\boxed{\text{How to write } d=5 \\ \not{L} \text{ operator?}}$$

$$\frac{1}{\Lambda_{new}} \underbrace{\left(l_L^T\, i\sigma_2\, \phi\right)}_{\substack{SU(2)\ \text{singlet} \\ Y=0}} C \left(\phi^T\, i\sigma_2\, l_L\right) \qquad (3)$$

$$\mathcal{L}_{Y\,\not{L}}^{eff} = \mathcal{L}_Y\,(d=5)_{\not{L}}$$

<u>Exercise</u> <u>Write 3 different ways of $\mathcal{L}_Y(\not{L})$.</u>

Are they related?
(<u>mathematically</u>)

$$l_L^T \, i\sigma_2 \, \phi_{un} = \left(\nu_L^T \; e_L^T \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}$$

$$= \nu_L^T \phi_0$$

$$\Downarrow$$

$$\mathcal{L}_Y^{eff} = \underbrace{\nu_L^T C \nu_L}_{\Delta L = 2} \frac{\phi_0 \phi_0}{\Lambda_{new}}$$

$$\Rightarrow \boxed{M_\nu \simeq \frac{\langle \phi_0 \rangle^2}{\Lambda_{new}}}$$

$$\boxed{\Lambda_{new} \to \infty \iff \text{SM limit}}$$

$$\phi_0 = h + v$$

$$\Downarrow$$

$$\mathcal{L}^{eff}_{Y} = (\nu_L{}^T C \nu_L) \frac{(h+v)^2}{\Lambda_{new}}$$

$$= (\nu_L{}^T C \nu_L) \frac{v^2 + 2vh + \cdots}{\Lambda_{new}}$$

$$\Rightarrow m_\nu \cong \frac{v^2}{\Lambda_{new}}$$

$$\Rightarrow y_\nu \cong \frac{v}{\Lambda_{new}} \qquad (y_\nu h \, \nu^T C \nu)$$

Yukawa

$$\Gamma(h \to \nu\nu) \propto y_\nu^2 = \frac{v^2}{\Lambda_{new}^2}$$

$$\Downarrow$$

$$\Gamma(h \to \nu\nu) \propto \frac{m_\nu}{\Lambda_{new}} \leq \frac{m_\nu}{M_W}$$

since $\Lambda_{new} > M_W$

$$m_\nu \leq 10^{-9}\,\text{GeV}, \quad M_W = 100\,\text{GeV}$$

$$\Rightarrow \boxed{\Gamma(h \to \nu\nu) \leq 10^{-11}}$$

(negligible)

$\boxed{\text{Minimal } SU(5)}$

Double failure:

$$\boxed{\begin{array}{ll} 1. & \text{NO unification} \\ 2. & m_\nu = 0 \end{array}}$$

(a)

(b)

$\Downarrow$

Minimal theory ruled out

$\Downarrow$

What is a minimal

realistic SU(5)?

$\boxed{\text{add: } 24_F}$

Bajc, G.S.
2006

$24_F \longleftrightarrow 24_V$

$\underbrace{8 \text{ gluons} \quad 3(W, Z) \quad A}$ $+ (X, Y)$

$8_c^F, \quad 3_W^F, \quad 1^F + \text{----}$

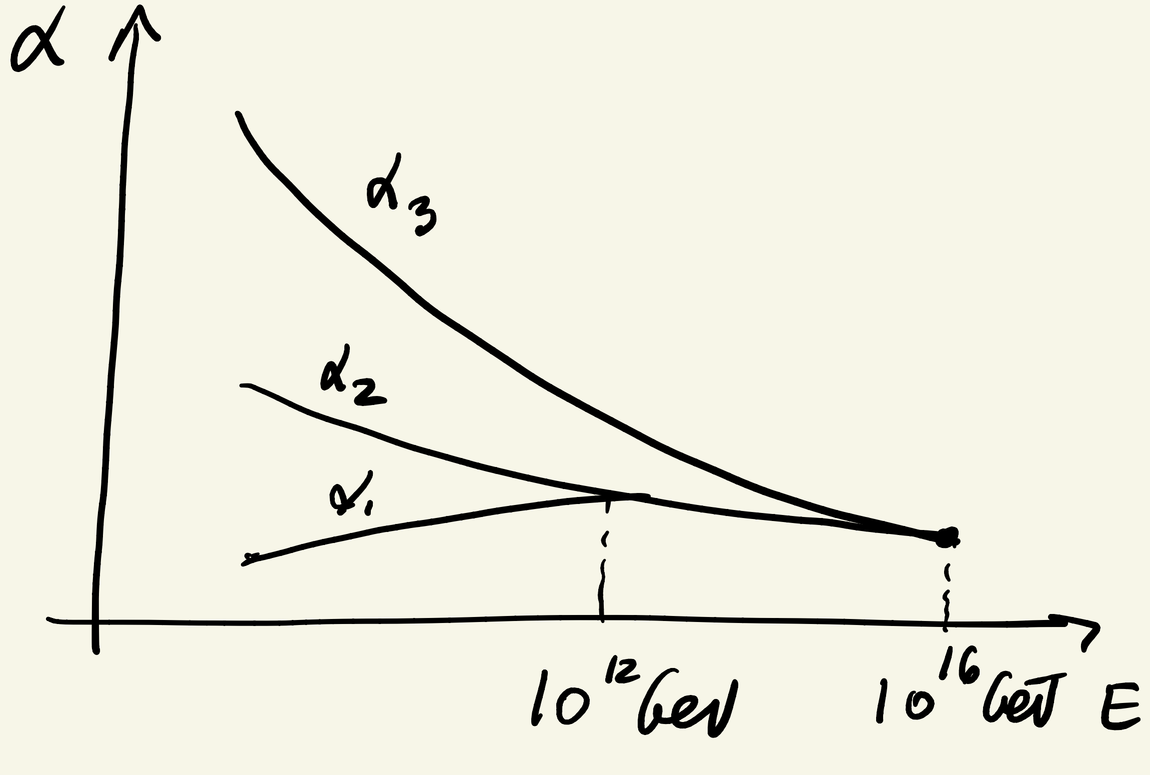
$3_W^F = T_F, \quad 1^F = N$

Type III + Type I seesaw

$\Downarrow$

(b) $m_\nu \neq 0$ (to be checked)

(a) unif. failure in minimal



$$\Downarrow$$

slow down α_2 !

$$\underbrace{T_F = 3^F_W}\quad (Y=0)$$

does <u>not</u> affect α_1

slows down α_2

$$b = \frac{11}{3} T_G - \frac{2}{3} T_F - \frac{1}{3} T_S$$

$$\Downarrow$$

$$\frac{1}{\alpha_2(M_{GUT})} = \frac{1}{\alpha_2(M_W)} + \frac{1}{2\pi} b_2^{SM} \ln \frac{m_T}{M_W}$$

$$+ \frac{1}{2\pi} b_2' \ln \frac{M_{GUT}}{m_T}$$

$$b_2' = b_2^{SM} + \delta b_2(T)$$

$$\boxed{\delta b_2 = -\frac{2}{3} \cdot 2 = -4/3}$$

$$\boxed{T \delta_{ab} = T_r T_a T_b}$$

$$\Downarrow$$

$$T(F) = T(3_W) =$$

$$T_3 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}$$

$$\Rightarrow \quad T(3) = \mathrm{Tr}\, T_3^{\,2} = 2$$

$$\Downarrow$$

determine m_T :

coupling unity

$$\Downarrow \quad (m_g = ?)$$

$$\boxed{m_T \simeq O\,(\mathrm{TeV})}$$ (prediction)

$$\boxed{m_T^{\,exp} \gtrsim \mathrm{TeV}}$$ (LHC)

experiment

In order to have seesaw

$\Downarrow$

$$\boxed{\text{couple } 24_F \; (3^F_W, 1^F) \text{ to leptons}}$$

Check.

$$\boxed{24_F \to U \, 24_F \, U^+}$$

$$\mathcal{L}_Y(24_F) = \bar{5}_F C \, 24_F \, 5_H +$$

$$\downarrow$$

$$\ell \, (1^F, 3^F_W) \, \phi$$

$$+ \mathrm{Tr}\, 2\psi_F C 2\psi_F \, m_{24}$$

$$U 2\psi_F U^\dagger U 2\psi_F U^\dagger$$

$$= U 2\psi_F 2\psi_F U^\dagger$$

$$m_N \, N C N$$

- Minimal $SU(5)$

$\Downarrow$ at $d=4$

$$\boxed{m_d = m_e}$$ not good

$\Downarrow$

$\boxed{\text{add } d > 4 \quad \Rightarrow \text{ realistic theory}}$

$$\mathcal{L}(x,y) = X_\mu \Big[\underbrace{\bar{u}^c \gamma^\mu u}_{\bar{10}_F \times 10_F} + \underbrace{\bar{d}\, \gamma^\mu e^c}_{\bar{10}_F \times 10_F}$$

$$+ \underbrace{\bar{e}\, \gamma^\mu d^c}_{\bar{5}_F \times 5_F} \Big]$$

new mixing angles

$\Downarrow$

<u>no</u> clear predictions of p decay

$$B(p \to \pi^0 e^+) = ?$$

$$B(p \to \pi^0 \mu^+) = ?$$

$$B(p \to K^0 e^+) = ?$$

$$B(p \to K^+ \bar{\nu}) = ?$$

NO way of knowing