GUT Course 22/23

Lecture XVII

23/12/2022

LMU
Winter 2022

$$SU(5)$$

$$\Big\downarrow \langle 24_H \rangle$$

$$G_{SM}$$

$$\Big\downarrow \langle 5_H \rangle$$

$$U(1)_{em} \times SU(3)_c$$

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but if there was only 5_H

$\Rightarrow$ $SU(4)$ at the end

$$\cancel{SU(3) \sim SU(2) \times U(1)}$$

$$\cancel{8 \text{ gen} \qquad (3+1) \text{ gen}}$$

$$\cancel{SU(N) \xrightarrow{\langle F_H \rangle = N_H} SU(N-1) \times U(1)}$$

$SU(5)$ $(r=4)$

maximal subgroups

$SU(3) \times SU(2) \times U(1)$
$(r=4)$

$SU(4) \times U(1)$
$(r=4)$

but

$$\langle 5_H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_W \end{pmatrix} \quad \} \; SU(4) \text{ good}$$

$+$

$$\mathcal{L}_Y = f(\bar{5}_F, 10_F, 5_H)$$

$\therefore$

$$\langle 5_H \rangle \neq 0 \Rightarrow$$

SU(4) inv. relations:

$$\boxed{m_d = m_e}$$

$\boxed{\text{add } d>4 \text{ terms}}$

$$\Delta \mathcal{L}_y = \bar{5}_F \, 10_F \, 5_H^* \frac{24_H}{\Lambda}$$

$$\Lambda \gg M_{GUT} = \langle 24_H \rangle$$

<u>check:</u>

$$(1) \quad \bar{5}_F{}^i \, 10_{F\,ij} \, 5_H^{*k} \frac{(24_H)^j_k}{\Lambda}$$

$$\to \bar{5}_F{}^i \, 10_{F\,ij} \, 5_H^{*k} \frac{\langle 24_H \rangle^j_k}{\Lambda}$$

$\uparrow$

breaks $SU(4)_c$

$$= \bar{5}_F{}^{i}\, 10_{F\,ij}\, 5_H^{*\,k}\, \frac{V_j\, \delta^j_k}{\Lambda}$$

$$\langle 24_h \rangle = \mathrm{diag}\, V_{GUT}\, (1, 1, 1, -3/2, -3/2)$$

$(\alpha = 1,2,3)$ $V_\alpha = V_{GUT}$

$a = 4,5$ $V_a = -3/2\, V_{GUT}$

$$\Downarrow$$

$$\Delta Y_Y = \bar{5}_F{}^{i}\, 10_{F\,i\alpha}\, 5_H^{*\,\beta}\, \frac{V_{GUT}\, \delta_\beta{}^\alpha}{\Lambda}$$

$$\langle 5_H^* \rangle^\beta = 0$$

$$+ \bar{5}_F{}^{i}\, 10_{Fia}\, 5_H^{*b}\, \frac{\left(-3/2\, v_{GUT}\right) \delta_b{}^{a}}{\Lambda}$$

$$\downarrow$$

$$\boxed{\bar{5}_F{}^{i}\, 10_{Fi5}\, \frac{\vartheta_W \left(-3/2\, v_{GUT}\right)}{\Lambda}}$$

$$\|$$

$$-\frac{3}{2}\, \frac{v_{GUT}}{\Lambda} \left\{ \bar{5}_F{}^{\alpha}\, 10_{F\alpha 5} + \bar{5}_F{}^{4}\, 10_{F45} + \cancel{\bar{5}_F{}^{5}\, 10_{F55}} \right\} v_W$$

$$= -\frac{3}{2} \left(\frac{v_{GUT}}{\Lambda}\right) v_W \left\{ d^{c\,T} c\, d + e^{T} c\, e^{c} \right\}$$

$$\boxed{\begin{array}{c}\text{keep our relations}\\ m_d = m_e\end{array}}$$

but also

(2) $\bar{5}_F{}^i \, 24_{H}{}_i^k \, 10_{F\,kj} \, 5_H^{*j} \dfrac{1}{\Lambda}$

$$\downarrow$$

$$\bar{5}_F{}^i \underbrace{\langle 24_H \rangle_i^k}_{} 10_{F\,kj} \, 5_H^{*j} \frac{1}{\Lambda}$$

breaks SU(4) accidental

transfer SU(4) breaking

to d^c and e in $\bar{5}_F$

$$\bar{5}_F^{\,i}\ \bar{V}_k\ \delta^k_i\ 10_{F kj}\ 5_H^{*j}\ \frac{1}{\Lambda}$$

$$\langle 5_H\rangle^{*j} = v_w\,\delta_{j5}$$

$$\to \bar{5}_F^{\,\underline{k}}\ \bar{V}_{\underline{k}}\ 10_{F k 5}\ v_W\ \frac{1}{\Lambda}$$

$$\Rightarrow\ d^c d\ \left(V_{GUT}/_{\Lambda}\ v_w\right) +$$

$$e\,e^c\ \left(-\tfrac{3}{2}\,V_{GUT}/_{\Lambda}\ v_w\right)$$

$$\Uparrow$$

splits m_d, m_e

$\Downarrow$

$\boxed{\text{\underline{no} relation between } m_d \text{ and } m_e}$

if $\Lambda = M_p \simeq 10^{19}\,\text{GeV}$

$$\Rightarrow \boxed{\frac{v_{GUT}}{\Lambda} \simeq 10^{-3}}$$

$$+ \; \Delta \mathcal{L}_Y = 10_F \, 10_F \, 5_V \, \frac{24_H}{\Lambda} \quad \checkmark$$

suggestion: stick in indices

step back

$$\mathcal{L}_Y = \bar{5}_F \, Y_1 \, 10_F \, 5_H^* \quad (d=4)$$

$$\Delta\mathcal{L}_Y = \bar{5}_F \, 10_F \, \underbrace{5_H^* \frac{24_H}{\Lambda}}$$

$$\underbrace{\boxed{5_H^* \, 24_H}}$$

$$5^{*i} \, 24^j_{\ u}$$

$\longrightarrow$ irreducible

$$(H_{new})^{ij}_{\ u}$$

$$\rightarrow (H_{new})^{[ij]}_{k} - Tr(H_{new})$$

$$\parallel$$

$$(H_{new}{}^{ij}{}_{i})(5)$$

$$[ij] = (AS)_{ij}$$

$$\Rightarrow [ij] = \frac{5\cdot 4}{2} = 10$$

$$\Rightarrow \boxed{50^{[ij]}_{u} - 5^{ij}{}_{i} = 45 \text{ components}}$$

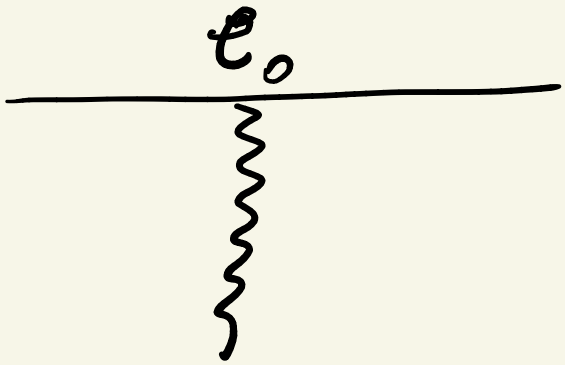
Instead, lets stick

good old minimal $SU(5)$

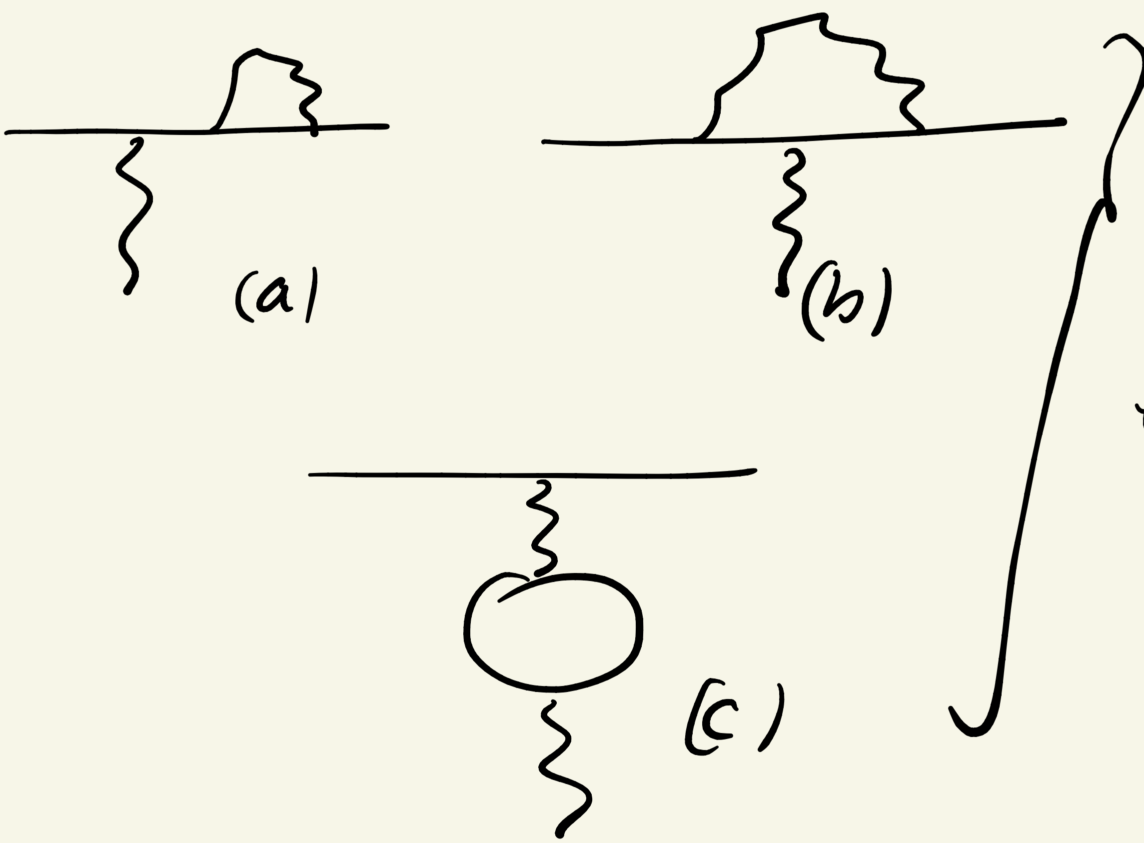
$(+\ d=5\ \ \Delta\mathcal{L}_Y)$

UNIFICATION of GAUGE COUPLINGS

QED



tree



loops

$$e(\Lambda) = e^0$$

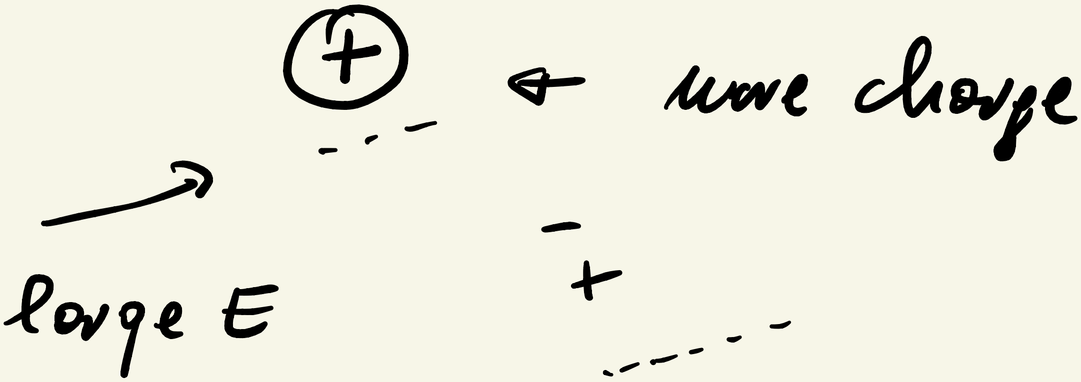
$$e(E) = e_0 - \frac{e_0^{\,3}}{16\pi^2} \ln \Lambda/E$$

(b)

$$\propto \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{\not k} \frac{1}{\not k}$$

$$\propto \int^{\Lambda} d^4k \, \frac{1}{k^4} \propto \ln \Lambda$$

$$\boxed{e(E_2) = e(E_1) + \frac{e^3}{16\pi^2} \ln \frac{E_2}{E_1}}$$



$$\Downarrow$$

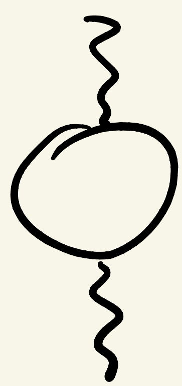
general formula

$$\frac{1}{\alpha(E_2)} = \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln \frac{E_2}{E_1}$$

$$b = \frac{11}{3} T_{GB} - \frac{2}{3} T_F - \frac{1}{3} T_S$$

gauge boson

fermion



scalar

gauge boson $\Longrightarrow$

Asymptotic Freedom (AF)

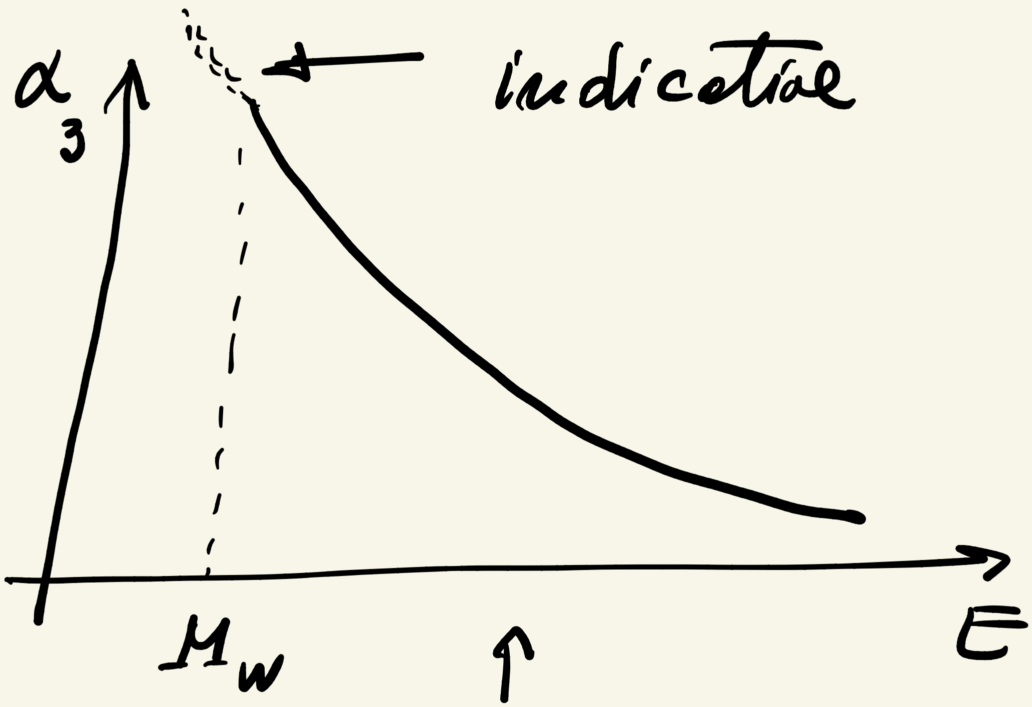
$$\boxed{T\,\delta_{ab} = \mathrm{Tr}\, T_a T_b}$$

recall: $T_{\text{fund}} = T_f$ (fundamental repr.)

$$\boxed{\mathrm{Tr}\, T_f^a\, T_f^b = \frac{1}{2}\,\delta^{ab}}$$

- $b > 0$ (g.b. there)

$\Rightarrow$ AF



example : QCD $\leftrightarrow$ SU(3)

$$\alpha_3(M_W) \simeq 1/10 \qquad SU(3)_c$$

$$\alpha_2(M_W) \simeq 1/30 \qquad SU(2)_L$$

$$\alpha_{em}(M_W) \simeq 1/130 \qquad U(1)_{em}$$

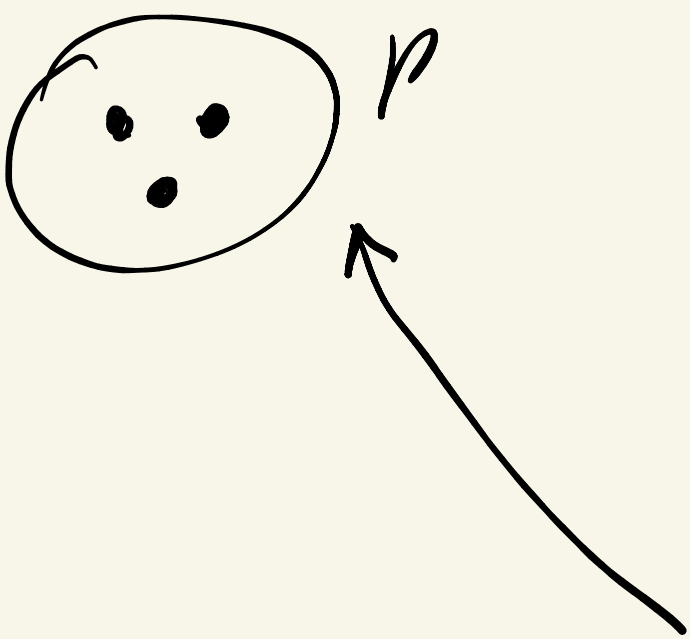
$\boxed{\text{Deep Inelastic Scattering}}$ $\boxed{(1969)}$

quarks $\longleftarrow$ Gell-Mann 1964

(aces) $\longleftarrow$ *** Zweig 1964

1967 – 1969 Bjorken

$\Downarrow$ Rutherford picture



$(E \gg m_p)$

e

$$e + p \longrightarrow e + X$$

(scaling)

(E, q^2 = exchanged momentum)

$\Longleftrightarrow$ "free" quarks

$\Updownarrow$

$$\boxed{\alpha_3 = \alpha_3(E)}$$

high precision

- $T(\text{fund}) = \frac{1}{2}$

  $T(\text{Adjoint}) = ?$

$$A = F \times \overline{F}$$

$$\Leftrightarrow \quad F \to UF$$

$$A \to U A U^{+}$$

<u>SU(2)</u> $F = f = D \Rightarrow T(D) = \frac{1}{2}$

$$\Updownarrow$$

$$T_a = \sigma_a/2$$

$$\therefore \ \mathrm{Tr}\left(\sigma_3/2\right)^2 = 1/2$$

$$A = \text{triplet}$$

$$T_3 = \mathrm{diag}(1, -1, 0)$$

$$\Rightarrow \mathrm{Tr}\, T_3^2 = 2 = T_{GB}$$

$$= T(\text{Adjoint})$$

SU(3) $\qquad F = 3 \Rightarrow T(F) = 1/2$

$$T_3 = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

$$A = \text{octet} = 8$$

$$8 = \underbrace{3 + 2 + 2 + 1}_{SU(2)}$$

SU(3)

"pions" (π^+, π^-, π^0)

"kaons" (K^+, K^0) SU(2) doublet

"anti-kaons" $(\bar{K}^0, K^0)$

η' singlet

$$\underset{SU(3)}{3} = \underbrace{2 + 1}_{SU(2)}$$

$$8 = 3 \times \bar{3} = (\bar{2} + 1) \times (2 + 1)$$

$$= \underbrace{3 + 2 + \bar{2} + 1} + 1$$

$$\underbrace{T(8)}_{SU(3)} = \underbrace{T(3) + T(2) + T(\bar{2}) + T(1)}_{SU(2)}$$

$$= 2 + \frac{1}{2} + \frac{1}{2} + 0 = 3$$

$$T(A)_{SU(2)} = 2$$

$$T(A)_{SU(3)} = 3$$

$$\boxed{T(A)_{SU(N)} = N}$$

Proof Try during holiday

$$\frac{1}{\alpha(E_2)} = \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln E_2/E_1$$

particles (p) that run:

$m_p \leq E_1$ (enter b)

$\left(\ln E_2/E_1 \gg 1 \right)$

SSB:

$M_X = M_Y = v_{GUT}$

$M_W = M_Z = M_A = 0$

$(\leq M_W)$

run

$\boxed{(X, Y) \text{ don't ``run''}}$

$b \Leftarrow$ contribution only from "light" particles

$(m \leq E_1)$

$E \gg v_{ewt}$

$\Rightarrow$ X, Y "run" (ewk, b)

Kaons: $\begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$, $\begin{pmatrix} \overline{K^0} \\ K^- \end{pmatrix}$

$$\underbrace{\qquad\qquad\qquad}_{K^0 \neq \overline{K^0}}$$

(CP viol., Δm_K)

$$\boxed{\Delta m_K = \frac{m_{K^0} - m_{\overline{K^0}}}{m_{K^0} + m_{\overline{K^0}}} \simeq 10^{-14}}$$

Computed in SM

$\Longleftrightarrow$ GIM mechanism

$K^+ = u\bar{s}$ $\qquad$ $\bar{K}^0 = \bar{d}s$

$K^0 = d\bar{s}$ $\qquad$ $K^- = \bar{u}s$

$$\pi^0 = (\bar{u}u - \bar{d}d)\frac{1}{\sqrt{2}}$$

$$K^0 \underset{CP}{\longleftrightarrow} \bar{K}^0$$

CP: $K_+ = \dfrac{K_0 + \bar{K}_0}{\sqrt{2}}$, $K_- = \dfrac{K_0 - \bar{K}^0}{\sqrt{2}}$

$$K_S = K_+ + \varepsilon K_-$$

$$\left(\varepsilon \simeq 1/10^3\right)$$

$$K_L = K_- - \varepsilon K_+$$

$$\left.\begin{array}{l} K_+ \longrightarrow \pi\pi \\ \\ K_- \longrightarrow \pi\pi\pi \end{array}\right\} CP$$

$$\boxed{\Gamma_+ \gg \Gamma_- \Leftrightarrow \tau_+ \ll \tau_-}$$