GUT Course 22/23

Lecture XVI

20/12/2022

LMU

Fall 2023

SU(5) GUT (5)

Yukawa sector

- Comment on Lecture XV

$\begin{pmatrix} X \\ Y \end{pmatrix}$ gauge bosons

$\Delta M \propto M_W \iff$

scale of $SU(2)$ breaking $\sim M_W$

Decoupling of heavy particles

heavy = SM singlet mass

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \text{heavy} \Leftarrow$$

$$M_X \simeq M_Y \propto \underbrace{\langle \Sigma \rangle = \langle 24_H \rangle}_{\text{SM singlet}}$$

Example :

$$\Gamma(p \to \pi^0 e^+) \propto \frac{m_p^5}{M_X^4}$$

- matter: $(\bar{5}_F, 10_F)_L$

- Higgs: 5_H, 24_H

$$\Downarrow$$

$$5 \longrightarrow U\,5, \quad 10 \longrightarrow U\,10\,U^T$$

$$24 \longrightarrow U\,24\,U^+$$

SM: $\begin{pmatrix} u \\ d \end{pmatrix}_L$ $\quad u_R, \; d_R$

$$\Updownarrow$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad (u^c)_L, \quad (d^c)_L$$

$\boxed{CC^T = 1}$ $\qquad (f^c)_L \equiv C \bar{f}_R^{\,T} \qquad (1)$

mass term: $\bar{f}_R f_L = (f^c)_L^T C f_L$

$\Downarrow$

in $SU(5)$:

$10 \; 10 \;, \quad \bar{5} \; 10$

$$\bar{5} = 5^c = \begin{pmatrix} d^c \\ ---- \\ \begin{pmatrix} \nu \\ e \end{pmatrix} \end{pmatrix}_L \rightarrow U^* 5^c$$

$\wr$

5^*

$\Downarrow$

$$\mathcal{L}_Y = \bar{5}_F^{\,T}\, C\, 10_F\, 5_H^{*}\, Y_1 \qquad (a)$$

$$+ h.c.$$

$$\left(5^{*i}\, 10_{ij}\, 5^{*j} \right)$$

$$\left(\bar{5}_F^{\,T} U^{+} C\, U\, 10_F\, U^{T} U^{*} 5_H^{*} \right.$$

$$\left. = \bar{5}_F^{\,T} C \underbrace{U^{+}U}_{1}\, 10_F \underbrace{U^{T}U^{*}}_{1} 5_H^{*} = \text{inv.} \right)$$

$$+ Y_2\, 10_{F_{ij}}^{T}\, C\, 10_{F_{ke}}\, 5_{H\,m} \underbrace{\epsilon^{ijkem}}_{\text{AS tensor}}$$

(b)

$$+ h.c.$$

$\overset{\shortparallel}{\text{Anti-symmetric}}$

(b) <u>discussion</u>

- $SU(2)$: $D^T \epsilon D = \text{inv.} \quad (s=0)$

$$D = \begin{pmatrix} u \\ d \end{pmatrix} \to U D$$

- $SU(3)$: $3 \to U 3$

$$3_i \, 3_j \, 3_k \, \epsilon_{ijk} = \text{inv.}$$

$$\hookrightarrow U_{ii'} U_{jj'} U_{kk'} 3_{i'} 3_{j'} 3_{k'} \epsilon_{ijk}$$

$$= \epsilon_{i'j'k'} \underset{\substack{\| \\ 1}}{(\det U)} \xi_{i'} \xi_{j'} \xi_{k'}$$

$$= \epsilon_{ijk} \, \xi_i \xi_j \xi_k = \text{inv.}$$

- $SU(N)$

$$\underbrace{\underbrace{N_i \, N_j \, N_k \, \cdots}_{N \text{ times}} \quad \underbrace{\epsilon_{ijkl\ldots}\cdots}_{N}}_{\text{INV}}$$

$\Downarrow$ predictions

I. NO ν_R

$\nu_L^T C \nu_L \leftarrow$ breaks $SU(2)_L$ (twice)

$\Downarrow$

$$\boxed{m_\nu = 0}$$

II. relations between m_q and m_ℓ

$$(a)\ Y_1 \bar{5}_F{}^{i}\, C\, 10_{Fij} \langle 5_H{}^{*j} \rangle \longleftarrow \text{SSB}$$

$$\langle 5_H{}^{*j} \rangle = v_W \delta_{j5}$$

$$\Downarrow$$

$$Y_1 \bar{5}_F{}^{i}\, C\, 10_{Fi5}\, v_W$$

$$i = \underbrace{\alpha}_{SU(3)},\ \underbrace{a}_{SU(2)}$$

$$v_W Y_1 \Big(\underbrace{\bar{5}_F{}^{\alpha}\, C\, 10_{F\alpha 5}}_{\downarrow} + \overbrace{\bar{5}_F{}^{4}}^{(e_L)}\, C\, \overbrace{10_{F45}}^{(e^c)_L} + \cancel{\bar{5}_F{}^{5}\, C\, 10_{F55}} \Big)$$

$$\Downarrow \qquad e_L^c \equiv C \bar{e}_R^T, \quad C^2 = -1$$

$$v_W \left(d_L^{c\,T} C\, Y_1\, d_L + e_L^T C\, Y_1 (e^c)_L \right) + h.c.$$

$$= v_W \left(\bar{d}_R\, Y_1\, d_L + e_L^T\, Y_1 (-\bar{e}_R)^T \right) + h.c.$$

$$= v_W (\underbrace{\bar{d}_R\, Y_1\, d_L}_{\text{down quark masses}} + \underbrace{\bar{e}_R\, Y_1^T\, e_L}_{\text{lepton masses}}) + h.c.$$

$$\boxed{M_d = Y_1 v_W}$$

$\Downarrow$

$$\boxed{M_e = M_d^T}$$

down quarks

charged leptons

$\Updownarrow$

$$\boxed{\text{WRONG!}}$$

$\Downarrow$

$$\left.\begin{aligned} m_e/m_\mu &= m_d/m_s \\ m_\mu/m_\tau &= m_s/m_b \end{aligned}\right\} \text{NOT True}$$

• <u>comment</u>

$$\underbrace{\overline{f_R}\, f_L}_{\text{Dirac notation}} = \underbrace{(f^c)_L^T\, C\, f_L}_{\text{Majorana notation}}$$

Dirac notation

$\downarrow$

(a) $Y_1\, \overline{5_R}\, 10_L\, 5_H^*$

$$5_R = \begin{pmatrix} d \\ \hline e^c \\ -\nu^c \end{pmatrix}_R$$

<u>equivalent</u>

(b) $Y_2 \, 10^T_{F\,ij} \, C \, 10_{F\,kl} \, 5^*_m \, \varepsilon_{ijklm}$

$$\Big\downarrow \text{ SSB}$$

$$Y_2 \, 10^T_{F\,ij} \, C \, 10_{F\,kl} \, v_w \, \varepsilon_{ijkl5}$$

$i, j = \alpha, \beta$: $\boxed{(u^c_L)^T C \, Y_2 \, u_L \, v_w}$

$$10 = \left(\begin{array}{c|cc} u^c & u & d \\ \hline & 0 & e^c \\ & & 0 \end{array}\right)_L$$

$\boxed{M_u = Y_2 \, v_w}$ (factor = ?)

• from $\epsilon = -\epsilon^T$, $C = -C^T$

$$\Downarrow$$

$$\boxed{Y_2 = Y_2^T}$$ * Prove! *

Why $\underline{M}_e = \underline{M}_d^T$?

Is there a symmetry?

$SU(5)$

$$\langle 24_H \rangle \longrightarrow \quad SM = G_{SM}$$

$$\downarrow \langle 5_H \rangle$$

$$U(1) \times SU(3)_c$$

$\Downarrow$

No symmetry ! ?

- $m_f \leftarrow \langle 5_H \rangle = \left.\begin{pmatrix} 0 \\ 0 \\ 0 \\ \hdashline 0 \\ v_W \end{pmatrix}\right\}$

$$SU(5) \longrightarrow SU(4)$$
$$\langle 5_H \rangle$$

$$5_F = \begin{pmatrix} d \\ \hdashline e^c \\ \hline -\nu^c \end{pmatrix}_R \Big\} \; SU(4)_c$$

$$\boxed{\text{reason for } \quad M_d = M_e^T}$$

$\Downarrow$ in order to save minimal SU(5)

new Higgs repr. (45_H)

$\left(45_{H\,k}^{[ij]} \right) - \mathrm{Tr}(\,)$ AS

bring in SU(4) breaking

$\boxed{\text{bring in } 24_H}$

$\Downarrow$

Effective interactions

SM: $\mathcal{L}_{SM}$ (renormalizable)

$$\Downarrow \qquad\qquad \downarrow$$

$$A = A_{SM}\left(1 + \left(\frac{M_W}{\Lambda_{new}}\right) + \dots\right)$$

+ new physics $\leftarrow \Lambda_{new}$

Expected!

Example: $\Lambda_{new} = M_{GUT}$

SU(5):

$$A = A_{SU(5)} \left(1 + \frac{M_{GUT}}{\Lambda_{new}} \right)$$

$$\left.\begin{aligned} M_{GUT} &\simeq 10^{16}\ \mathrm{GeV} \\ \Lambda \lesssim M_P &\simeq 10^{19}\ \mathrm{GeV} \end{aligned}\right\} \; 10^{-3}$$

$$\boxed{\text{argument: essential in } \mathcal{L}_Y, \text{ since } Y \ll 1}$$

$\Downarrow$

$$\mathcal{L}_Y = \mathcal{L}_Y^{(SU(5))} + \qquad (\Lambda \equiv \Lambda_{new})$$

$$\bar{5}_F \, 10_F \, 5_H^* \frac{24_H}{\Lambda} \qquad (a)$$

$$+ \epsilon \, 10_F \, 10_F \, 5_H \, \frac{24_H}{\Lambda}$$

$$\underbrace{\bar{5}_F \, 10_F \, 5_H^* \frac{\langle 24_H \rangle}{\Lambda}}$$

indices to have $M_d \neq M_e^T$

$$\langle 24_H \rangle \propto \mathrm{diag}\,(\underbrace{1,\ 1,\ 1,\ -3/2,\ -3/2}_{\boxed{\text{breaks } SU(5)}})$$

$$(5_H = ?\ )$$

$$5_H = \begin{pmatrix} T^\alpha \\ \hline \phi \end{pmatrix} \begin{matrix} \leftarrow \text{color triplet} \\ \\ \rightarrow \text{Higgs doublet} \end{matrix}$$

$$\mathcal{L}_Y = \bar{5}_F\, Y_1\, 10_F\, 5_H^{*} + 10_F 10_F\, 5_H\, Y_2$$

$$\downarrow$$

$$\bar{5}_F^{\,i}\, Y_1\, 10_{Fi\alpha}\, T^{*\alpha}$$

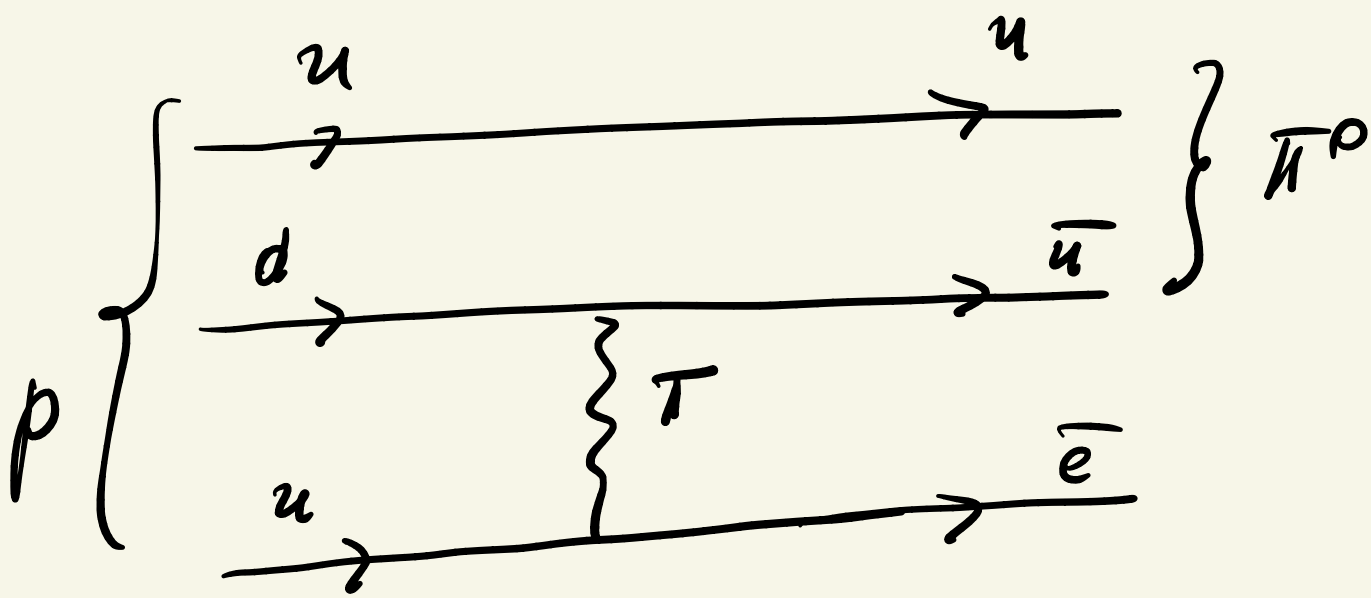
$$= \Big(\underbrace{d^c_L\, Y_1\, u^c_L}_{Q:\ -1/3} + \underbrace{e_L\, u_L}_{-1/3} \Big)\, T^*$$

$$\boxed{\Delta B \neq 0}$$

$$\Downarrow$$

$$\boxed{Y_1 :\ e_L\, u_L\, T^* + T\, u\, d}$$

$$\Downarrow$$



$$p \to \pi^0 + \bar{e}\,(e^c)$$

$$A_T = \frac{y_1^2}{m_T^2} \longleftrightarrow A_{(X,Y)} = \frac{g^2}{M_X^2}$$

$$y_1 \simeq y_d \simeq m_d / M_W \simeq 10^{-3}$$

$$\Downarrow$$

$$\boxed{m_T \gtrsim 10^{12}\ \mathrm{GeV}}$$