# GUT Course 22/23

## Lecture XV

16/12/2022

LMU

Fall 2022

# SU(5) GUT (4)

- SU(5) = minimal GUT
- matter:

$$\underbrace{\bar{5}_F \; (5_F^c), \; 10_F}_{15 \text{ SM } q, \ell}$$

- Higgs:

$$24_H, \quad 5_H \quad \ldots$$

$$\langle 24_H \rangle \propto v_{GUT}$$
$$\langle 5_H \rangle \propto v_W$$

(1) gauge interactions

$$\bar{5}_F = \begin{pmatrix} d^c \\ \hdashline l \end{pmatrix}_L \quad \begin{matrix} \} \text{ gluons} \\ \} \text{ } W \end{matrix}$$

(new)

$$10_F = \left(\begin{array}{c:cc} u^c & u & d \\ \hdashline & 0 & e^c \\ & & 0 \end{array}\right)_L$$

new

new

$$\text{(new)} = \begin{cases} SU(2)_L & \text{doublet} \\ SU(3)_c & \text{triplet} \end{cases}$$

$$\|$$

$$\rightarrow \begin{pmatrix} X \\ Y \end{pmatrix}^{\alpha = r, y, b} (6) \quad + \quad \begin{pmatrix} \bar{Y} \\ \bar{X} \end{pmatrix}^{\alpha} (6)$$

$$6 + 6 = 12$$

$$24 = 12 + 12$$

$$= \underbrace{8_c + 3_w + 1_s}_{SM} + 6 + 6$$

$$\boxed{(f^c)_L = C \bar{f}_R}$$

$$\begin{pmatrix} X \\ Y \end{pmatrix} : \quad l \to d^c$$

$$u^c \to q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\Downarrow$$

$$\mathcal{L}_{(X,Y)} = \bar{X}_\mu \Big[ \underbrace{\bar{u}^c_L \gamma^\mu u_L}_{4/3} + \bar{e}_L \gamma^\mu d^c_L + \bar{d}_L \gamma^\mu e^c_L \Big]$$

Q: (-4/3)

Q: 4/3

$SU(2)_L$ ↓ ↓ $Q: 4/3$

$$+ \bar{Y}_\mu \left[ \bar{u}^c_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu d^c_L \right.$$

$Q: (-1/3)$

$$\left. + \bar{u}_L \gamma^\mu e^c_L \right]$$

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SM: $(\bar{u} d + \bar{\nu} e) W$

$(\bar{u} u, \bar{d} d, \bar{e} e, \bar{\nu} \nu) Z$

$( \quad -\|- \quad , \quad \times \quad ) A$

$\bar{f} f h$

$\Rightarrow$ $\boxed{\Delta B = \Delta L = 0}$

$$B(W) = L(W) = 0$$

$$B(Z) = L(Z) = 0$$

$$B(A) = L(A) = 0$$

$$B(h) = L(h) = 0$$

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new int.

$$\bar{X} : \underbrace{\overline{u^c}\, u}_{B = 2/3} + \underbrace{\bar{e}\, d^c}_{B = -1/3}$$

$B = 2/3 \;\Downarrow\; B(X) = 2/3$

$B = -1/3 \;\Downarrow\; B(X) = -1/3$

$\Downarrow$ $\Delta B = 0$ | $\Downarrow$ $\Delta B = 0$

but both int.

$\Downarrow$

$$\boxed{\Delta B \neq 0}$$
$$\boxed{\Delta L \neq 0}$$

$\Downarrow$

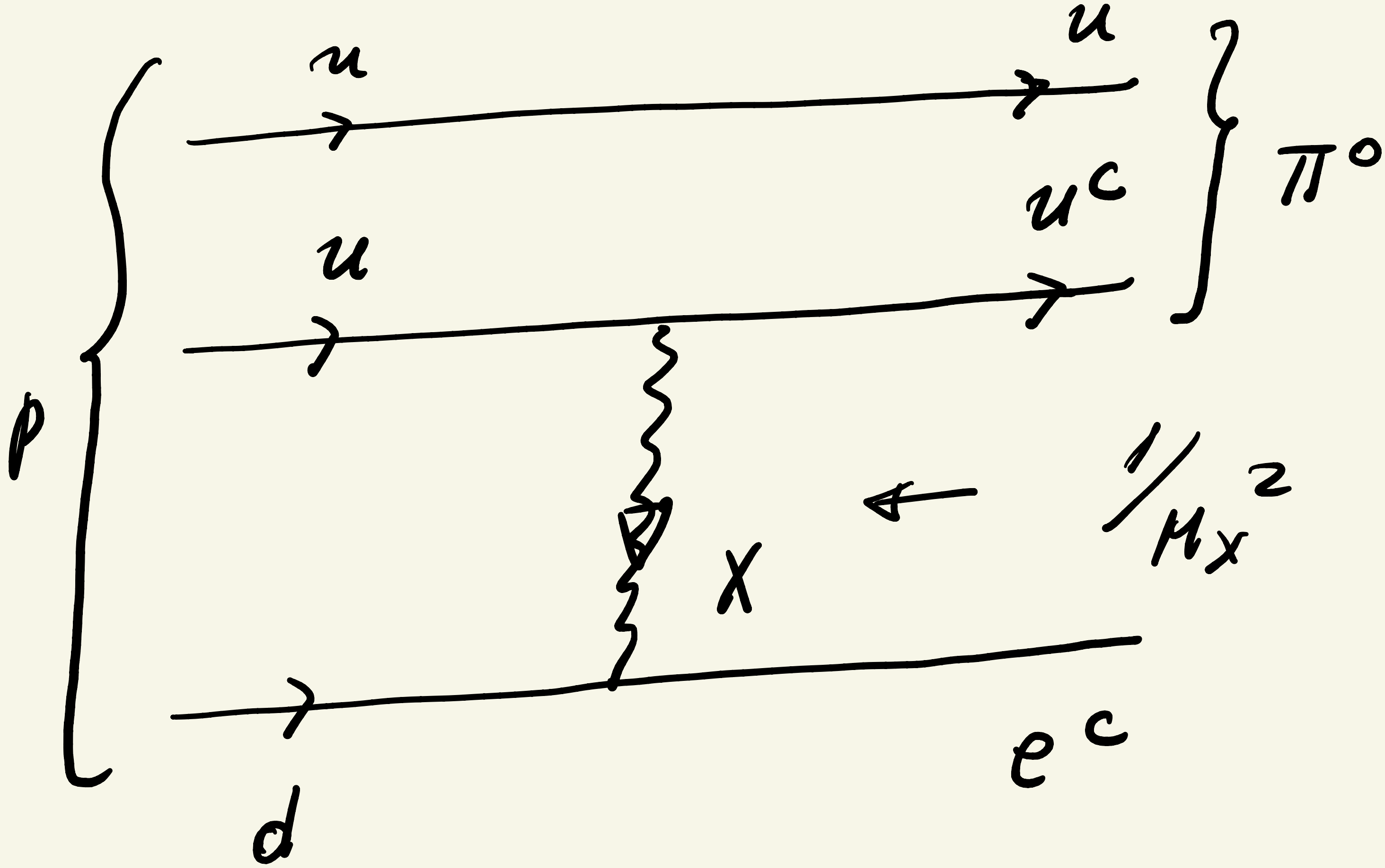
proton decay

(nucleon)

$$\bar{X}\left[\,\bar{u}^c u + \bar{e}\, d^c + \bar{d}\, e^c\,\right]$$

$$X\left[\,\bar{u}\, u^c + \bar{d}^c e + \bar{e}^c d\,\right]$$

$$\Downarrow$$




$$\boxed{p \to \pi^0 + e^c \ (e^+, \bar{e})}$$

$$\boxed{\tau_p(\pi^0 e^+) \geq 10^{34}\, yr}$$

↑

super Kamiokande ← Kamioka mine (Japan)

$$\Gamma_p \propto \frac{1}{M_X^4} m_p^5 \quad (p \to \pi^0 e^+)$$

$$\Gamma_\mu \propto \frac{1}{M_W^4} m_\mu^5 \quad (\mu \to e + \nu + \bar{\nu})$$

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$$\Downarrow \quad \tau = 1/\Gamma$$

$$\Rightarrow \boxed{\tau_p/\tau_\mu = \left(\frac{M_X}{M_W}\right)^4 \left(\frac{m_\mu}{m_p}\right)^5}$$

$$\Downarrow$$

$$\tau_p = 10^{-5} \left( \frac{M_X}{M_W} \right)^4 \times 10^{-6} \sec$$

$$\tau_p > 10^{34}\ yr \simeq 10^{41} \sec$$

$$\left( \frac{M_X}{M_W} \right)^4 \geq 10^{11} \cdot 10^{41} = 10^{52}$$

$$\Downarrow$$

$$M_X / M_W > 10^{13}$$

$$\Downarrow$$

$$\boxed{M_X > 10^{15}\ GeV}$$

Georgi, Glashow '74

• Georgi, Quinn, Weinberg '74

unification:

$\Rightarrow M_X \simeq 10^{15}$ GeV

Q. $M_X = M_0$ (measure)

$M_0 \geq 10^{15}$ GeV

$M_Y = ?$

A. $\begin{pmatrix} X \\ Y \end{pmatrix} \Rightarrow \Delta M \lesssim M_W$ (!)

$\exists$ $M_Y \simeq M^0$

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**Equation (1) : discussion**

$$G_{SM} = SU(2)_L \times U(1)$$

scale of breaking = $v_W$

$$M_W = g/2 \; v_W$$

scale of breaking =

= difference of masses

in a multiplet

Examples:

- $q = \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad t_R,\ b_R$

$$\left.\begin{aligned} m_t &= y_t v_W \\ m_b &= y_b v_W \end{aligned}\right\} :$$

$$\Downarrow$$

$$\Delta m = (y_t - y_b)\, v_W$$

$$\simeq O(v_W)$$

because: $y_t \leq O(1)$

↑

perturbative theory
of weak int.

$$\Rightarrow \boxed{m_t \leq \text{few } 100 \text{ GeV}}$$

. Higgs

$$m_h^2 \simeq \lambda v^2$$

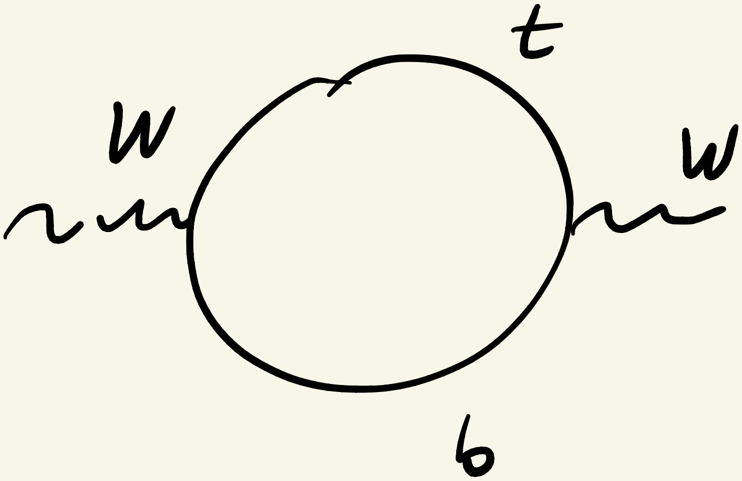
$$\Rightarrow \boxed{m_h \leq \text{few } 100 \text{ GeV}}$$

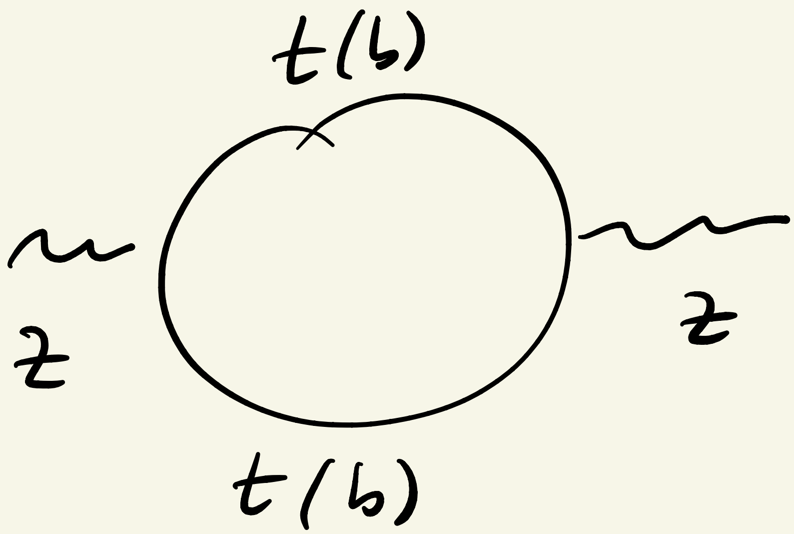
$$\phi = \begin{pmatrix} G^+ \\ h + iG \end{pmatrix} \sim M_W, M_Z$$

a bit more - - -

$$\rho \equiv \frac{M_Z^2 \cos^2\theta_W}{M_W^2} \quad \therefore \rho_{tree} = 1$$

$(t, b)$



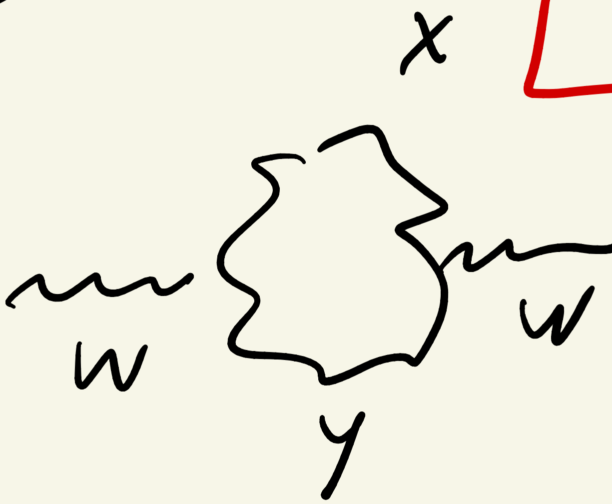



$$\rho = 1 + \frac{\alpha}{\pi}\frac{m_t^2 - m_b^2}{M_W^2}$$

$$+ \frac{\alpha}{\pi}\frac{M_X^2 - M_Y^2}{M_W^2}$$

$(X, Y)$

$$\to \frac{\alpha}{\pi}\frac{M_X^2 - M_Y^2}{M_X^2}$$




$+ \ldots$

$\Downarrow$

$$\frac{\alpha}{\pi}\frac{M_X^2 - M_Y^2}{M_X^2} \ll 1$$

~~⇓~~ (NOT true)

$$M_X^2 - M_Y^2 \leq O(M_W^2)$$

⇓

?? $$M_Y \simeq M_X \simeq M_0$$ ??

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- $M_X \simeq M_Y \simeq \cdots g\, v_{GUT} \; (v_X)$

new gauge bosons can "never" be seen directly

- other new particles

$2\,4_H$ : $8_c,\ 3_w,\ 1_s + \cancel{(12)}$

$m \propto v_{GUT}\,(a, b)$ ← could be small?

eaten by $12\,[(X, Y) + (\bar{Y}, \bar{X})]$
$6 + 6$

$$5_H = \begin{pmatrix} T^\alpha \\ \hline \phi \end{pmatrix}$$ ← mediates p decay

$\Downarrow$

$$\boxed{m_T \gtrsim 10^{12}\,\mathrm{GeV}}\ (?)$$

check

$$M_x = g\, v_{GUT} + a\, g\, v_W$$

$$\parallel$$

$$M_y = g\, v_{GUT} + b\, g\, v_W$$

$$(a, b) \sim O(1)$$

$$\Downarrow$$

$$\boxed{\Delta M \equiv M_x - M_y \leq O(M_W)}$$