Seesaw mechanism of neutrino mass

- $G_{SM} = SU(2)_L \times U_Y(1)$

- minimal f sector

$$q = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R,\ d_R$$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$$

- minimal Higgs

$$\Phi$$

$$\Downarrow$$

$$\boxed{m_\nu = 0}$$

why ?

(a) $\exists\ \nu_L$ only

$$\Downarrow$$

(b) $\underbrace{\nu_L^T C \nu_L}_{\cancel{SU(2)},\ \cancel{U(1)}}$ not allowed

$$\Downarrow$$ natural

$\exists\ \nu_R$

$Y(\nu_R) = 0$

$T_a(\nu_R) = 0$

$a = 1, 2, 3$

$$\left(\begin{array}{c} \cancel{E = ma^2} \\ \cancel{E = mb^2} \\ E = mc^2 \quad ! \end{array} \right)$$

$$\Downarrow$$

$$\mathcal{L}_Y(e) = \overline{l_L}\ i\sigma_2 \Phi^* \nu_R\, y_D + h.c.$$

$$+ \frac{1}{2} \nu_R^T C \nu_R M_R + h.c.$$

if $M_R = 0$

$$\Rightarrow \quad m_\nu = y_D \langle \Phi \rangle$$

$$y_D = \frac{m_\nu}{\langle \phi \rangle} = \frac{g}{2} \frac{m_\nu}{M_W}$$

$$\boxed{m_\nu \lesssim 1\,\mathrm{eV}}$$

$$\Rightarrow \quad y_D < 10^{-11}$$

$$\Gamma(h \to \nu\bar{\nu}) \propto y_D^2$$

$$\Rightarrow \quad B(h \to \nu\bar{\nu}) \leq 10^{-22} \; (=0)$$

true only when $M_R = 0$

$$\bullet \quad N_L \equiv C\,\bar{\nu}_R^{\,T} = C\gamma_0 \nu_R^* = i\gamma_2 \nu_R^*$$

$$M_R\, \nu_R^T C \nu_R + \nu_R^\dagger C^\dagger \nu_R^* M_R^*$$

$$\Rightarrow N_L^T = \bar{\nu}_R C^T = \nu_R^\dagger \gamma_0 C^T$$

$$\Rightarrow N_L^T C N_L = \nu_R^\dagger \gamma_0 \overbrace{C^T C}^{=1} C \gamma_0 \nu_R^*$$

$$= \nu_R^\dagger \gamma_0 C \gamma_0 \nu_R^*$$

$$= \nu_R^\dagger (-C) \nu_R^* = \nu_R^\dagger C^\dagger \nu_R^*$$

$$\Downarrow$$

$$M_R\, \nu_R^T C \nu_R + M_R^* \nu_R^\dagger C^\dagger \nu_R^* =$$

$$= M_N N_L^T C N_L + h.c.$$

$$\boxed{\mathcal{M}_N = M_R^*}$$

$$\langle \mathcal{L}_Y \rangle = \big(\overline{\nu_L}\, y_D\, \nu_R + \overbrace{\overline{\nu_R}\, y_D^*}^{\mathcal{M}_D} \nu_L \big) \langle \phi \rangle$$

$$+ \frac{1}{2} N_L^T C\, M_N\, N_L + h.c.$$

but: $\overline{\nu_R}\, \nu_L = \, ?$

$$N_L = C\, \overline{\nu_R}^T$$

$$\Rightarrow N_L^T = \overline{\nu_R}\, C^T$$

$$\Rightarrow N_L^T C = \overline{\nu_R}\, C^T C = \overline{\nu_R}$$

$$\Downarrow$$

$$\overline{\nu_R}\,\nu_L = N_L^T C\, \nu_L$$

$$\Downarrow$$

$$\mathcal{M}_{(\nu)} = N_L^T C\, M_D\, \nu_L + h.c.$$

$$+ \frac{1}{2} N_L^T C\, M_N\, N_L + h.c.$$

$$\Downarrow$$

$$\boxed{M_N^T = M_N}$$

$$= \frac{1}{2} N_L^T C\, M_D\, \nu_L + \frac{1}{2} N_L^T C\, M_D\, \nu_L$$

$$+ \frac{1}{2} N_L^T C\, M_N\, N_L + h.c.$$

$$= \frac{1}{2} N_L^T C\, M_D\, \nu_L + \frac{1}{2}(-)\,\nu_L^T \overset{(-C)}{\overset{\shortparallel}{C^T}} M_D^T N_L$$

$$+ \frac{1}{2} N_L^T C M_N N_L + h.c$$

$$\Downarrow$$

$$\mathcal{M}_{(\nu)} = \frac{1}{2} \Big(N_L^T C M_D \nu_L + \nu_L^T C M_D^T N_L$$

$$+ M_N N_L^T C N_L \Big) + h.c.$$

$$\Downarrow$$

$$\underline{M}_{(\nu, N)} = \begin{array}{c c} & \begin{array}{cc} \nu & N \end{array} \\ \begin{array}{c} \nu \\ N \end{array} & \begin{pmatrix} 0 & M_D^T \\ \underline{M}_D & \underline{M}_N \end{pmatrix} \end{array}$$

$\Downarrow$ 1 gen

$$
M_{(\nu, N)} = \begin{array}{c|cc} & \nu & N \\ \hline \nu & 0 & m_D \\ N & m_D & m_N \end{array}
$$

$$
\boxed{m_N \gg m_D}
$$

"seesaw" approximation

$$
m_1 + m_2 = m_N \quad (1)
$$

$$
m_1 \cdot m_2 = - m_D^2 \quad (2)
$$

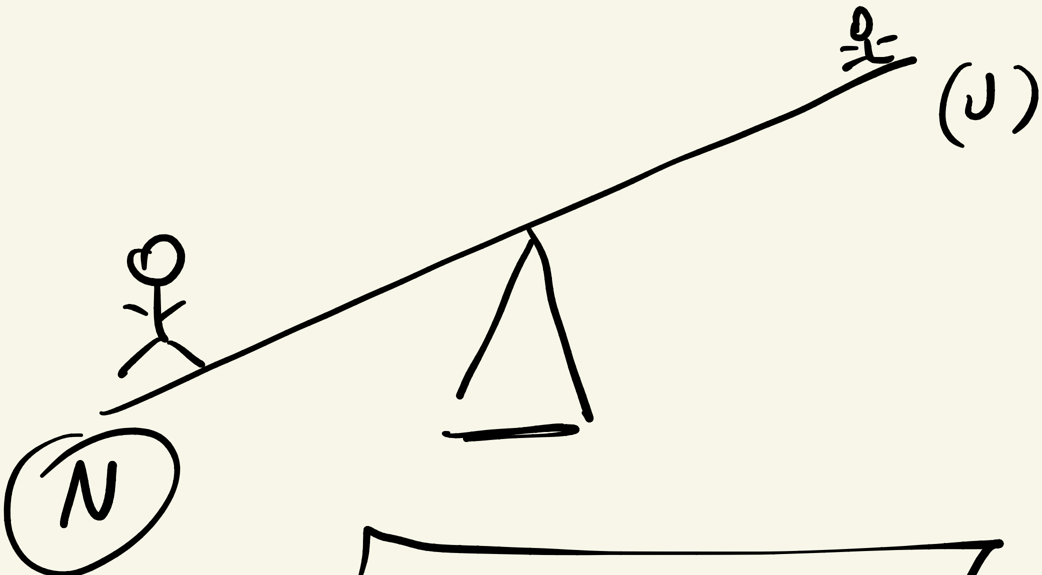
$$
\Downarrow
$$

$$
m_1 \ll m_2
$$

$$
(1) \Longrightarrow \boxed{m_2 \simeq m_N}
$$

$$\Downarrow$$

$$(2) \quad m_1 = -\frac{m_D^2}{m_2} \simeq \frac{m_D^2}{m_N} \simeq m_\nu$$



$\boxed{\text{seesaw}}$

physical

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = U \begin{pmatrix} \nu \\ N \end{pmatrix}$$

unphysical

$$\begin{pmatrix} m_{\nu_1} \equiv m_1 \\ m_{\nu_2} \equiv m_2 \end{pmatrix}$$

1 gen.

$$U = O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \equiv \begin{pmatrix} \nu' \\ N' \end{pmatrix} \leftarrow \text{physical}$$

$$\exists \quad \underbrace{\frac{1}{2}\tan 2\theta}_{\simeq\, \theta \ (\theta \leq 1)} \longrightarrow 0 \qquad m_D \to 0$$

$$\frac{1}{2}\tan 2\theta \longrightarrow 0$$

$$m_N \to \infty$$

$$\Downarrow$$

$$\theta \simeq \frac{1}{2}\tan 2\theta = \frac{m_D}{m_N} \ll 1$$

$$\Uparrow \text{ from}$$

$$S = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

$$\Rightarrow \boxed{\frac{1}{2}\tan 2\theta = \frac{c}{b-a}}$$

bottom line:

$$\nu' \simeq \nu + \theta N$$

$$N' \simeq N - \theta \nu$$

$$\boxed{\text{from now } \nu', N' = \text{called} \quad \nu, N}$$

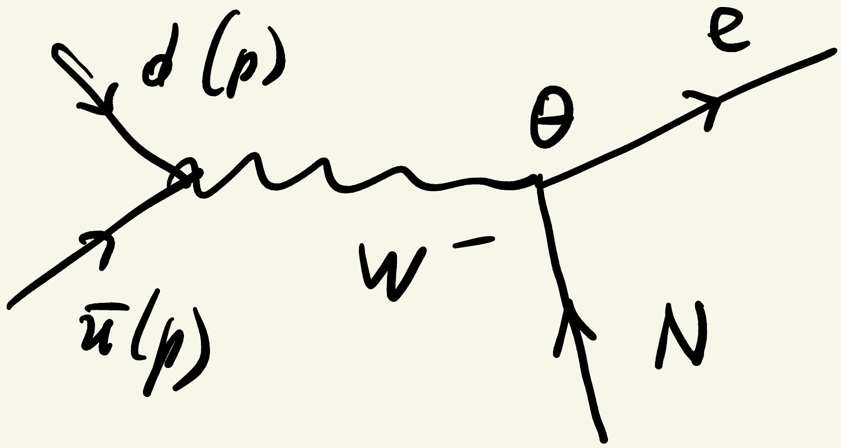
· seesaw $(m_N \gg m_D)$

used as a physical case!

$$m_N > M_W$$

$$\mathcal{L}_{cu} = \frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L W_\mu^+ + h.c.$$

$$\longrightarrow \frac{g}{\sqrt{2}} \bar{N}_L \gamma^\mu e_L W_\mu^+ \theta + h.c.$$



$$\Rightarrow \quad \sigma(N) \propto \theta^2 \simeq \frac{m_D^2}{m_N^2}$$

$$\left|\frac{m_\nu}{m_N}\right| = \frac{m_D^2}{m_N\, m_N} \simeq \frac{m_D^2}{m_N}$$

$$\Rightarrow \boxed{\sigma(N) \propto \frac{m_\nu}{m_N} \longrightarrow 0}$$

N cannot be produced in seesaw picture

• > 1 generation

$\Downarrow$

$$\underline{\underline{M}}_{(\nu, N)} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

$$\begin{pmatrix} \nu \\ N \end{pmatrix} \to U \begin{pmatrix} \nu \\ N \end{pmatrix}$$

$$UU^+ = 1 \quad (\text{up to } \theta^2 \text{ order})$$

$$U = \begin{pmatrix} 1 & \sim\theta^+ \\ \sim\theta & 1 \end{pmatrix}$$

$$U^T M_{(\nu N)} U = \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix}$$

$$(1) \quad \Rightarrow \boxed{\begin{array}{l} M_\nu = -M_D^T \dfrac{1}{M_N} M_D \\ (M_N \gg M_D) \end{array}}$$

$\|$

symmetric !

$$M_\nu^T = M_\nu$$

$$M_N^T = M_N$$

$$\nu_i^T (M_\nu)_{ij} \, C \, \nu_j^T =$$

$$= - \nu_j^T C^T (M_\nu)_{ij} \nu_i$$

$$= \nu_j^T C \, (M_\nu^T)_{ji} \nu_i \qquad \text{Q.E.D.}$$

$(C^T = -C)$

(2) $$\boxed{\Theta = \frac{1}{M_N} M_D}$$

$\ll 1, \quad M_N \gg M_D$

- neutrino mass $\Leftarrow$ neutrino oscillations

(a) $\nu_\mu \to \nu_\tau$ (atm. neutrino)

$$\Delta m^2_{ATM} \simeq 10^{-3}\, eV^2$$

(b) $\nu_e \to \nu_\mu$ (solar neutrino)

$$\Delta m^2_\odot \simeq 10^{-5}\,\mathrm{eV}^2$$

$$\Downarrow$$

Q. how many massive neutrinos?

A. <u>at least two!</u>

$$\Downarrow$$

$\boxed{\text{at least two } N}$

Other sources of neutrino mass

$$\underbrace{\bar{l}_L \, \phi^*}_{\bar{D} \times \bar{D} \,=\, S \,+\, T} \left(\nu_R\right) \quad , \quad \left(T_F\right)_R$$

Type I   Type III

$$\bar{D} \times \bar{D} = S + T$$

$$(2) \times (2) \qquad (1) \qquad (3)$$

$$Y(\nu_R) = 0 \, , \quad Y(T_F) = 0$$

$$\nu_R \longrightarrow N_L$$

$$(T_F)_R \longrightarrow (T_F^c)_L$$

$$\therefore \quad \frac{g}{\sqrt{2}} \, \overline{T_{0F}} \, \gamma^\mu \, T_{-F} \, W_\mu^+$$

$$(T_F)_L = \begin{pmatrix} T^+ \\ T^0 \\ T^- \end{pmatrix}_L$$

Q. If no new fermion

$\Longrightarrow$ what?

$\Downarrow$

$$\underbrace{\nu_L^T \, C \, \nu_L}$$

repr. under $SU(2)$?

$$T_3 : \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) = -1$$

scalar triplet (Δ)!

- $l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad , \quad e_R$

$$\Downarrow$$

$$\Delta \longrightarrow U \Delta U^{+}$$

$$\boxed{Y(\Delta)=2} \Rightarrow$$

$$\mathcal{L}_Y(\text{new}) = \ell_L^T C\, i\sigma_2 \Delta\, \ell_L\, Y_\Delta + \text{h.c.}$$

$$\to \ell_L^T C\, U^T i\sigma_2\, U \Delta U^\dagger U \ell_L\, Y_\Delta$$

$$\to \ell_L^T C\, i\sigma_2\, \underbrace{U^\dagger U}_{1}\, \Delta\, \underbrace{U^\dagger U}_{1}\, \ell_L\, Y_\Delta$$

$= \text{inv.}$ Q.E.D.

$$V = \underset{(\phi)}{V_{SM}} + \frac{m_\Delta^2}{2}\, \overbrace{\mathrm{Tr}\,\Delta^\dagger \Delta}^{\Delta_0 \Delta_0^* + \cdots} + \cdots$$

$$+ V_{\phi\Delta}$$

$$\parallel$$

$$-\mu\, \phi^T i\sigma_2\, \Delta^+\, \phi$$

$$\frac{\partial V}{\partial \Delta_0^*} = m_\Delta^2\, \Delta_0 - \mu \langle\phi\rangle\langle\phi\rangle + \ldots = 0$$

$$\Rightarrow \boxed{\langle\Delta_0\rangle = +\mu \frac{\langle\phi\rangle^2}{m_\Delta^2}}$$

induced (small) vev

$$\boxed{\langle\Delta_0\rangle \leq \text{GeV}}$$

$$\boxed{M_\nu = Y_\Delta \langle \Delta_0 \rangle}$$

Type II seesaw

$\mu \sim m_\Delta \Rightarrow$

$$M_\nu \simeq Y_\Delta \frac{M_W^2}{m_\Delta} \qquad (\mathrm{II})$$

$$M_\nu \simeq \frac{Y_D^2 M_W^2}{m_N} \qquad (\mathrm{I})$$