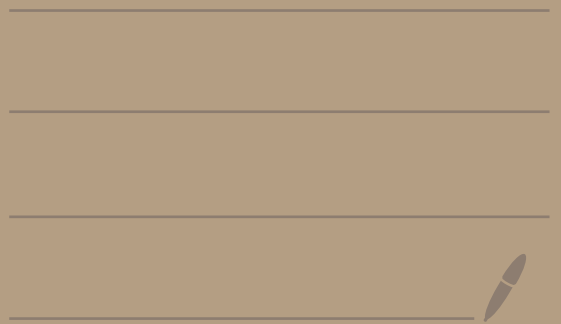

Lecture XIV

13 / 12 / 20 22



SU(5) GUT (3)

- SU(5) = minimal GUT
- matter (fermions = q, l)

$$\bar{5}_F (5_F^c) = \begin{pmatrix} d^c \\ \vdots \\ l \end{pmatrix} \left. \begin{array}{l} \} SU(3)_c \\ \} \end{array} \right\} SU(2)_L$$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix} \leftarrow$$

$$10_F (A_5) = \begin{pmatrix} \overbrace{u^c}^{SU(3)_c} & & & & \\ & u & d & & \\ \vdots & & & & \\ & & & e^c & \\ \vdots & & & & \\ & & & e^c & 0 \end{pmatrix} \left. \begin{array}{l} \} SU(3)_c \\ \} \end{array} \right\} SU(2)_L \leftarrow$$

$SU(2)_L$

$$T_r Q(10_F) = \sum Q(10_F) = 0$$

\Downarrow

$$\boxed{3Q(d) + Q(e^c) = 0}$$

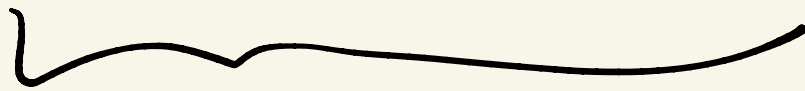
\Downarrow

$$\boxed{Q(e) = 3Q(d)} \quad (1)$$

• $\psi_L \Rightarrow \psi^c = (\psi^c)_R$

$$\bar{5}_F = LH \Rightarrow 5_F = RH$$

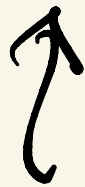
$$10_F = 5_F \times 5_F$$



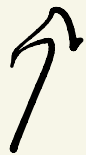
means $SU(5)$ quantum
numbers

but

chirality = ?



not determined



chirality vs gauge structure

SM

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$? \quad l_R = \begin{pmatrix} u \\ d \end{pmatrix}_R \quad ? \cdot ? \cdot ? \cdot ? \cdot ?$$

$$\frac{g}{\sqrt{2}} W_\mu^+ j_\mu^+ = \mathcal{L}_{\text{int}}(W)$$

$$j_\mu^+ \stackrel{?}{=} \bar{\nu}_L \gamma^\mu e_L + \bar{u}_R \gamma^\mu d_R$$

Is this OK?

However

$$10 = (5 \times 5)_{As}$$

$$5_F = \begin{pmatrix} d \\ \vdots \\ (u^c) \\ -u^c \end{pmatrix}_R$$

$$\Rightarrow Q(u^c) = 2Q(d)$$

$$Q(u) = -2Qd \quad (2)$$

$$\boxed{\text{but } Q(u) = Q(d) + 1} \quad (3)$$

$$\Rightarrow Q(d) = -1/3$$

$$Q(u) = 2/3$$

$$Q(e) = -1$$

$$Q(\nu) = 0$$

Neutrino = neutral

Conclusion

chirality of ψ_F ($\bar{\psi}_F$)



weak int.

chirality $\bar{\psi}_F$ = chirality of l

QED

$$e = e_L + e_R$$

$$e: \quad e_L, \quad (e^c)_L = C \bar{e}_R^T$$



$$(10)_L \leftrightarrow (e^c)_L$$

\parallel

LH is a prediction



$$\boxed{\mathcal{L} = \begin{pmatrix} u \\ d \end{pmatrix} \leftrightarrow LH}$$

- Higgs sector \Leftrightarrow SSB

$$\begin{array}{ccc}
 SU(5) & \longrightarrow & SU(3)_c \times SU(2)_L \times U(1)_Y \\
 \langle \Sigma \rangle & & \downarrow \langle \Phi \rangle (6) \\
 (a) & & U(1)_{Q_{ew}} \times SU(3)_c
 \end{array}$$

(a) $\Sigma = 24_H = \text{adjoint}$

$$\therefore \Sigma \rightarrow U \Sigma U^\dagger, \quad \Sigma = \Sigma^\dagger$$

$$\text{Tr} \Sigma = 0$$

$$\langle \Sigma \rangle \equiv V_{GUT} \equiv V_X$$

$$(GUT) V_{GUT} \gg V_W (SM)$$

(b) $\phi \subseteq 5_H \leftarrow \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \Bigg\} \begin{matrix} \\ (Y=1) \end{matrix}$

\uparrow
SM doublet

from $5_F = \begin{pmatrix} d \\ e^+ \\ -\nu^c \end{pmatrix}_R$ same

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \leftrightarrow \tilde{\phi} = i\sigma_2 \phi^*$$

$$\parallel$$

$$\begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

$\langle 24_H \rangle$ leaves G_{SM}

Q. $\langle 5_H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \nu_w \end{pmatrix}$ by $SU(5)$?

A.

Recall : $\langle 24 \rangle_H = \text{diag}(\dots)$

by $SU(5)$

NO

Instead:

$$\langle 5_H \rangle = \left(\begin{array}{c} v_c \\ 0 \\ 0 \\ \hline 0 \\ v_w \end{array} \right) \left. \begin{array}{l} \} SU(3) \\ \} SU(2) \end{array} \right\}$$

$$V = V(24_H) + \underbrace{24_H \rightarrow -24_H}_{\text{ease pain}}$$

$$\left. \begin{array}{l} -\frac{\mu_5^2}{2} 5_H^\dagger 5_H + \frac{\lambda}{4} (5_H^\dagger 5_H)^2 \\ + \frac{\alpha}{4} 5_H^\dagger 5_H \text{Tr}(24_H^2) \end{array} \right)$$

$$+ \frac{\beta}{4} \phi_H^\dagger (2\phi_H^2) \phi_H \quad \downarrow$$

~~NO~~ $\phi_H^\dagger 2\phi_H \phi_H$

$$\phi_H^\dagger \underbrace{U^\dagger U}_{1} 2\phi_H^2 \underbrace{U^\dagger U}_{1} \phi_H = \text{inv.}!$$

\Downarrow mass term (eff)

$$V(\phi_H) = -\frac{\mu_s^2(\text{eff})}{2} \phi_H^\dagger \phi_H + \frac{\lambda}{4} (\phi_H^\dagger \phi_H)^2$$

$$+ \frac{\beta}{4} 5_H^\dagger \langle 24_H \rangle 5_H \quad \leftarrow$$

$$\mu_5^2(\text{eff}) = \mu_5^2 - \frac{\alpha}{2} \text{Tr} \langle 24_H \rangle^2$$

$$\langle 24_H \rangle = V_{\text{GUT}} \text{diag} (1, 1, 1, -\frac{3}{2}, -\frac{3}{2})$$



$$\beta: \frac{1}{4} (v_e^2 + \frac{9}{4} v_w^2) V_{\text{GUT}}^2$$



$(-\mu^2, \lambda \dots)$

$$V(\text{vac}) = f(v_c^2 + v_w^2) + \frac{v_{\text{GUT}}^2}{4} \beta (v_c^2 + \frac{9}{4} v_w^2)$$

$$= f(v_c^2 + v_w^2) +$$

$$\frac{v_{\text{GUT}}^2}{4} \beta (v_c^2 + v_w^2 + \frac{5}{4} v_w^2)$$

$$= f'(v_c^2 + v_w^2) +$$

$$\boxed{\frac{5 v_{\text{GUT}}^2}{16} \beta v_w^2}$$

$f' =$ has no idea who
is zero or not

but

from exp $v_w \neq 0$

$$\Downarrow$$

$\beta < 0$

$$5_4 = \left(\begin{array}{c} T^a \\ \phi \end{array} \right) \left. \begin{array}{l} \} SU(3) \\ \} SU(2)_L \end{array} \right\} (\alpha = r, Y, b)$$

$$\begin{aligned}
V(S_H) &= -\frac{\mu_5^2}{2} (T^\dagger T + \phi^\dagger \phi) \\
&\quad + \frac{\lambda}{4} (T^\dagger T + \phi^\dagger \phi)^2 \\
&\quad + \frac{f^2}{4} v_{GUT}^2 \left(T^\dagger T + \frac{9}{4} \phi^\dagger \phi \right) \\
&= \left(-\frac{\mu_5^2}{2} + \frac{f^2}{4} v_{GUT}^2 \right) T^\dagger T \\
&\quad + \left(-\frac{\mu_5^2}{2} + \frac{9}{4} f^2 v_{GUT}^2 \right) \phi^\dagger \phi \\
&\quad \underline{\hspace{10em}} \\
&\quad + \frac{\lambda}{4} (\dots)^2
\end{aligned}$$

$$= -\frac{\mu\phi^2}{2} \phi^\dagger \phi + \left(\cancel{\frac{-\mu\phi^2}{2}} - \frac{9\beta V_{GUT}^2}{2} \right) T^\dagger T$$

\uparrow \swarrow *small*
 $(\mu\phi^2 \equiv \mu_5^2 - \frac{9}{2}\beta V_{GUT}^2)$

• FACT: T mediates
 p decay

$$M_T > 10^{12} \text{ GeV}$$

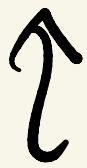
• FACT: $\mu\phi \approx 100 \text{ GeV}$

$$\beta < 0 \Leftrightarrow m_T^2 > 0$$

$$\Leftrightarrow \langle T \rangle = 0$$

guarantees $v_c = \langle T \rangle = 0$
= no color breaking

$$\beta \neq 0 \therefore m_T = \text{large}$$



β is not negligible

$$\mu_{\phi}^2 = \mu_5^2 - \frac{g}{2} (\beta V_{\text{GUT}} \approx M_T^2)$$

$(100 \text{ GeV})^2 \approx (10^{12} \text{ GeV})^2$



$$\mu_5 \approx 10^{12} \text{ GeV}$$



FINE-TUNING (FT)
ISSUE of GUT

⇓

$$V(\phi) = -\frac{\mu\phi^2}{2} \phi^\dagger\phi + \frac{\lambda}{4} (\phi^\dagger\phi)^2$$

⇓

SM SSB

$$+ \exists T^a \quad \therefore \quad m_T \gtrsim 10^{12} \text{ GeV}$$