Lecture XIV

13/12/2022

SU(5) GUT (3)

- SU(5) = minimal GUT
- matter (fermions = q, ℓ)

$$\bar{5}_F \; (5_F{}^c) = \begin{pmatrix} d^c \\ \hdashline \ell \end{pmatrix}_L \begin{matrix} \}\, SU(3)_c \\ \}\, SU(2)_L \end{matrix}$$

$$\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$10_F = \begin{array}{c} \overbrace{}^{SU(3)_c} \\ \left(\begin{array}{c:cc} u^c & u & d \\ \hdashline & 0 & e^c \\ & -e^c & 0 \end{array} \right)_L \end{array} \begin{matrix} \}\, SU(3)_c \\ \}\, SU(2)_L \end{matrix}$$

(AS)

$$\underbrace{}_{SU(2)_L}$$

$$Tr\, Q(10_F) = \sum Q(10_F) = 0$$

$$\Downarrow$$

$$\boxed{3\, Q(d) + Q(e^c) = 0}$$

$$\Downarrow$$

$$\boxed{Q(e) = 3\, Q(d)} \quad (1)$$

- $\psi_L \Rightarrow \psi^c = (\psi^c)_R$

$$\bar{5}_F = LH \Rightarrow 5_F = RH$$

$$\underbrace{10_F = 5_F \times 5_F}$$

means SU(5) quantum numbers

but

chirality = ?

not determined

chirality vs gauge structure

SM

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$? \quad l_R = \begin{pmatrix} u \\ d \end{pmatrix}_R \quad ?????$$

$$\frac{g}{\sqrt{2}} W_\mu^+ j_W^\mu = \mathcal{L}_{int}(W)$$

$$j_W^\mu \stackrel{?}{=} \bar{\nu}_L \gamma^\mu e_L + \bar{u}_R \gamma^\mu d_R$$

Is this OK?

However

$$10 = (5 \times 5)_{AS}$$

$$5_F = \begin{pmatrix} d \\ \hline \begin{pmatrix} e^c \\ -\nu^c \end{pmatrix} \end{pmatrix}_R$$

$$\Rightarrow \quad Q(u^c) = 2\,Q(d)$$

$$Q(u) = -2\,Q\,d \qquad (2)$$

$$\boxed{\text{but} \quad Q(u) = Q(d) + 1} \qquad (3)$$

$$\Rightarrow \quad Q(d) = -1/3$$

$$Q(u) = 2/3$$

$Q(e) = -1$

$\boxed{Q(\nu) = 0}$

Neutrino = neutral

Conclusion

chirality of 5_F $(\bar{5}_F)$

$\uparrow$

weak int.

$\uparrow$

chirality $\bar{5}_F$ = chirality of l

QED

$$e = e_L + e_R$$

$$e: \quad e_L\,, \quad (e^c)_L = C\bar{e}_R^{\,T}$$

$$\Downarrow$$

$$(10)_L \longleftrightarrow (e^c)_L$$

$$\parallel$$

LH is a prediction

$$\Downarrow$$

$$\boxed{q = \begin{pmatrix} u \\ d \end{pmatrix} \longleftrightarrow LH}$$

- Higgs sector $\Leftrightarrow$ SSB

$$
\begin{array}{ccc}
SU(5) & \xrightarrow[\langle \Sigma \rangle \ (a)]{} & SU(3)_c \times SU(2)_L \times U(1)_Y \\
& & \Big\downarrow \langle \phi \rangle \ (b) \\
& & U(1)_{Qem} \times SU(3)_c
\end{array}
$$

(a) $\Sigma = 24_H =$ adjoint

$$\therefore \quad \Sigma \to U \Sigma U^{+}, \quad \Sigma = \Sigma^{+}$$

$$\mathrm{Tr}\, \Sigma = 0$$

$$\langle \Sigma \rangle \equiv V_{GUT} \equiv V_X$$

(GUT) $v_{GUT} \gg v_W$ (SM)

(b) $\phi \subseteq 5_H \Longleftarrow \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

SM doublet

$(Y = 1)$

same

from $5_F = \begin{pmatrix} d \\ e^+ \\ -\nu^c \end{pmatrix}_R$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longleftrightarrow \tilde{\phi} = i\sigma_2 \phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

$\langle 24_H \rangle$ leaves G_{SM}

$\Downarrow$

Q.

$$\langle 5_H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ v_W \end{pmatrix} \text{ by } SU(5)?$$

A.

Recall : $\langle 24 \rangle_H = \text{diag}(---)$

by SU(5)

$\boxed{\text{NO}}$

Instead:

$$\langle 5_H \rangle = \begin{pmatrix} v_c \\ 0 \\ 0 \\ \hdashline 0 \\ v_W \end{pmatrix} \begin{array}{l} \left.\vphantom{\begin{matrix} v_c \\ 0 \\ 0 \end{matrix}}\right\} SU(3) \\ \left.\vphantom{\begin{matrix} 0 \\ v_W \end{matrix}}\right\} SU(2) \end{array}$$

$$V = V(24_H) + \underbrace{\text{ease pain}}_{24_H \to -24_H}$$

$$- \frac{\mu_5^2}{2} 5_H^+ 5_H + \frac{\lambda}{4} (5_H^+ 5_H)^2$$

$$+ \frac{\alpha}{4} 5_H^+ 5_H \, Tr\langle 24_H^2 \rangle$$

$$+\boxed{\frac{f_3}{4}\, 5_H^+ \langle 24_H^2 \rangle 5_H}$$

NO $\cancel{5_H^+ 24_H 5_H}$

$$5_H^+ \underbrace{U^+ U}_{1} 24_H^2 \underbrace{U^+ U}_{1} 5_H = \text{inv.}!$$

$\Downarrow$ mass term (eff)

$$V(5_H) = -\frac{\mu_5^2(\text{eff})}{2} 5_H^+ 5_H + \frac{\lambda}{4}\left(5_H^+ 5_H\right)^2$$

$$\boxed{+\ \frac{\beta}{4}\ 5_H^{+}\langle 24_H\rangle^2 5_H}$$

$$\mu_5^2(\text{eff}) = \mu_5^2 - \frac{\alpha}{2}\,\text{Tr}\langle 24_H\rangle^2$$

$$\langle 24_H\rangle = V_{GUT}\,\text{diag}\left(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}\right)$$

$$\Downarrow$$

$$\boxed{\beta:\ \frac{1}{4}\left(v_c^2 + \frac{9}{4}v_w^2\right)V_{GUT}^2}$$

$$\Downarrow$$

$$(-\mu^2, \lambda \ldots)$$

$$V(\mathrm{vac}) = f(v_c^2 + v_w^2) +$$

$$+ \frac{v_{GUT}^2}{4} \beta \left(v_c^2 + \frac{9}{4} v_w^2 \right)$$

$$= f(v_c^2 + v_w^2) +$$

$$\frac{v_{GUT}^2}{4} \beta \left(v_c^2 + v_w^2 + \frac{5}{4} v_w^2 \right)$$

$$= f'(v_c^2 + v_w^2) +$$

$$\boxed{\frac{5 v_{GUT}^2}{16} \beta v_w^2}$$

f' = has no idea who is zero or not

but

from exp $v_W \neq 0$

$\Downarrow$

$$\boxed{\beta < 0}$$

$$5_H = \begin{pmatrix} T^\alpha \\ \hdashline \phi \end{pmatrix} \begin{matrix} \} \, SU(3) \\ \} \, SU(2)_L \end{matrix} \quad (\alpha = r, y, b)$$

$$V(5_H) = -\frac{\mu_5^2}{2}\left(T^+T + \phi^+\phi\right)$$

$$+ \frac{\lambda}{4}\left(T^+T + \phi^+\phi\right)^2$$

$$+ \frac{\beta}{4} v_{GUT}^2 \left(T^+T + \frac{9}{4}\phi^+\phi\right)$$

$$= \left(-\frac{\mu_5^2}{2} + \frac{\beta}{4} v_{GUT}^2\right) T^+T$$

$$+ \left(-\frac{\mu_5^2}{2} + \frac{9}{4}\beta v_{GUT}^2\right)\phi^+\phi$$

$$+ \frac{\lambda}{4}\left(--- \right)^2$$

$$= -\frac{\mu_\phi^2}{2}\,\phi^\dagger\phi + \Big(\overbrace{\cancel{-\frac{\mu_\phi^2}{2}}}^{\text{small}} - \frac{4\beta v_{GUT}^2}{2}\Big)T^\dagger T$$

$$\left(\mu_\phi^2 \equiv \mu_5^2 - \frac{9}{2}\beta v_{GUT}^2\right)$$

$$\boxed{\bullet\ \text{FACT: } T \text{ mediates } p \text{ decay}}$$

$$\Downarrow$$

$$\boxed{m_T > 10^{12}\ \text{GeV}}$$

$$\boxed{\bullet\ \text{FACT: } \mu_\phi \simeq 100\ \text{GeV}}$$

$$\boxed{\beta < 0 \iff m_T^2 > 0 \iff \langle T \rangle = 0}$$

guarantees $v_c = \langle T \rangle = 0$

= no color breaking

$\beta \neq 0$ $\therefore$ m_T = large

β is <u>not</u> negligible

$$\underbrace{\mu_\phi^2}_{\sim (100\,\text{GeV})^2} = \mu_5^2 - \frac{9}{2}\underbrace{\left(\beta\, v_{GUT}^2 \simeq m_T^2\right)}_{\gtrsim (10^{12}\,\text{GeV})^2}$$

$$\Downarrow$$

$$\boxed{\mu_5 \gtrsim 10^{12}\,\text{GeV}}$$

$$\Uparrow$$

FINE-TUNING (FT)

ISSUE of GUT

$$\Downarrow$$

$$V(\phi) = -\frac{\mu_\phi^2}{2}\phi^+\phi + \frac{\lambda}{4}(\phi^+\phi)^2$$

$$\Downarrow$$

SM SSB

$+ \ \exists\ T^\alpha \ : \ m_T \gtrsim 10^{12}\,\mathrm{GeV}$