GUT Course 22/23

Lecture XII

2/12/2022

LHU

2022

$\boxed{SU(5)\ \text{GUT}}$

EW: $\quad SU(2)$ = minimal theory

$\underbrace{\hphantom{SU(2)\ \text{minimal theory}}}$

tragedy ?

$$\overset{r=1}{SU(2)} \otimes \overset{r=2}{SU(3)} \subseteq \overset{r=4}{SU(5)}$$

$T_a \qquad\qquad T_\alpha$

$a = 1,2,3 \qquad \Updownarrow \qquad \alpha = 1,2,\dots,8$

$$[T_a, T_\alpha] = 0$$

$$\underset{F}{5} = \left(\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \hline \times \\ \times \end{array}\right) \begin{array}{l} \left.\vphantom{\begin{array}{c}\cdot\\\cdot\\\cdot\end{array}}\right\} SU(3) \\ \left.\vphantom{\begin{array}{c}\times\\\times\end{array}}\right\} SU(2) \end{array}$$

$$\underset{F}{5} \longrightarrow U_5 \, \underset{F}{5}$$

$$\boxed{\begin{matrix} U_5 U_5^{+} = 1 \\ \det U = 1 \end{matrix}} = SU(5)$$

<u>$SU(N)$</u>

- $U = e^{iH}$

  $UU^{+} = 1 \Rightarrow H = H^{+}$

- $\det U = 1 \Rightarrow Tr\, H = 0$

$\underline{M} \in C \Rightarrow 2n^2$ elements (real)

$U = U^+$ (n^2 conditions) $\Rightarrow n^2$ element

$\det U = 1 \Rightarrow \boxed{n^2 - 1 \text{ elements}}$

$$H = \sum_{i=1}^{N^2-1} c_i T_i \qquad \boxed{\begin{array}{c} T_r T_i = 0 \\ T_i^+ = T_i \end{array}}$$

$\Rightarrow N-1$ diagonal T_{cartan}

$$\text{Cartan} = \{ T_\alpha : [T_\alpha, T_\beta] = 0 \}$$

$\underset{\text{Cartan}}{SU(2)} : \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} 1/2 = T_3$

$$\boxed{\mathrm{Tr}\, T_i T_j = \frac{1}{2}\delta_{ij}}$$

$$[T_i, T_j] = i f_{ijk} T_k$$

$$\boxed{SU(2): \quad f_{ijk} = \varepsilon_{ijk}}$$

<u>$SU(3)$</u>

Cartan

$$T_3 = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

$$T_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$\exists$ $\boxed{N-1 \text{ such } T_{\text{Cartan}}}$ $\boxed{SU(N)}$

$\Downarrow$ charge arbitrary

$$SU(2) \times \boxed{U(1)} \times SU(3)$$

$$\cap\!|$$

$$SU(5)$$

Maximal subgroup of G (H_{max}) :

$$\boxed{\begin{gathered} H_{max} \subseteq G \\ r(H_{max}) = r(G) \end{gathered}}$$

H_{max} in $SU(5)$!

1) $\underset{r=1}{SU(2)} \times \underset{r=1}{U(1)} \times \underset{r=2}{SU(3)}$

$\boxed{r=4}$

2) $\underset{r=3}{SU(4)} \times \underset{r=1}{U(1)}$

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1) $SU(3) \begin{cases} T_3 = \frac{1}{2} \, \mathrm{diag}\,(1, -1, 0, 0, 0) \\ T_8 = \frac{1}{2\sqrt{3}} \, \mathrm{diag}\,(1, 1, -2, 0, 0) \end{cases}$

$\vdots$

$SU(2) \{ \; T_{23} = \frac{1}{2} \, \mathrm{diag}\,(0, 0, 0, 1, -1)$

$U(1) \quad T_{24} = \underset{?}{N} \, \mathrm{diag}\,(1, 1, 1, -3/2, -3/2)$

Cartan in $\boxed{U(n)}$ $\Big\}$ $\mathrm{diag}\,(1,0,0,0,0\ldots)$

$\mathrm{diag}\,(0,1,\,---\,)$

$$\boxed{SU(5)}$$

$$T_3 = \frac{1}{2}\begin{pmatrix} \begin{matrix} 1 & & \\ & -1 & \\ & & 0 \end{matrix} & \\ & \begin{matrix} 0 & \\ & 0 \end{matrix} \end{pmatrix}$$

$SU(3)$ $\quad$ $SU(2)$ $\quad$ $SU(3)$ $\quad$ $SU(2)$

$$T_1 = \frac{1}{2}\begin{pmatrix} \begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & 0 \\ 0 & 0 \end{pmatrix}$$

$\downarrow$ $\sigma_1 \to \sigma_2$

$$T_2 = \frac{1}{2} \left(\begin{array}{ccc|c} 0 & -i & 0 & \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ \hline & 0 & & 0 \end{array} \right)$$

$$T_4 = \frac{1}{2} \left(\begin{array}{ccc|c} 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & \\ \hline & 0 & & 0 \end{array} \right)$$

$\downarrow$

5 $(\sigma_1 \to \sigma_2)$

$$T_6 = \frac{1}{2} \left(\begin{array}{ccc|c} 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & \\ \hline & 0 & & 0 \end{array} \right)$$

$\downarrow$ $\sigma_1 \to \sigma_2$

$$T_7 = \;- \;-\; -$$

$$T_8 = \frac{1}{2\sqrt{3}} \left(\begin{array}{ccc|c} 1 & & 0 & \\ 0 & 1 & & 0 \\ & & -2 & \\ \hline & 0 & & 0 \end{array} \right)$$

$$T_9 = \frac{1}{2} \left(\begin{array}{ccc|cc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$\vdots$ complete

$\sigma_{4,2}$ = all over $= 20$

$+$ Cartan $= 4$

Building SU(5)

'1974

Georgi, Glashow

$$\bar{5}_F = \begin{pmatrix} d^c_r \\ d^c_y \\ d^c_b \\ \hline l \end{pmatrix}_L \quad \begin{array}{l} SU(3)_{\text{color}} \\ \\ SU(2)_L \end{array}$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad 6^*$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad 2$$

$$u_R, \ d_R \to (u^c)_L, \ (d^c)_L \qquad 3^*, \ 3$$

$$e_R \to (e^c)_L = C \bar{e}_R^T \qquad 1^*$$

$$\psi \longrightarrow \psi^c = C \bar{\psi}^T$$

$$\bar{\psi} = \psi^\dagger \gamma^0 \qquad C = i\gamma_2\gamma_0$$

$$\psi^c = i\gamma_2\gamma_0\gamma_0\psi^*$$

$$= i\gamma_2\psi^* = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}\psi^*$$

$$\psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \Rightarrow$$

$$\psi^c = \begin{pmatrix} i\sigma_2 u_R^* \\ -i\sigma_2 u_L^* \end{pmatrix} \begin{array}{l} \text{L spinor} \\ \text{R} \;\text{—〃—} \end{array}$$

$$SU(5) \Rightarrow Q_{em} \in SU(5)$$

$$\Rightarrow Q_{em} = \sum_{i=1}^{24} c_i T_i$$

$Q_{em} = \text{diag}$ $\Downarrow$

$$\boxed{Q_{em} = \sum_{\text{Cartan}} c\, T}$$

$$\Downarrow$$

$$\boxed{\text{Tr}\, Q_{em} = 0}$$

$\Downarrow$ sum of charges in $(\bar{5}_F) = 0$

$$\boxed{3\, Q(q^c) + (-1) + 0 = 0}$$

SM: $[T_\alpha, T_a] = 0$

$\uparrow$ colour, $\uparrow$ ew

$[T_\alpha, Y] = 0$

$$Q_{em} = T_3 + Y/2$$

$$\Rightarrow [T_\alpha, Q_{em}] = 0$$

$$\Rightarrow \boxed{q^{r,y,b} = \text{same charge}}$$

$$\Rightarrow 3\,Q\,(q^c) = 1$$

$$\Rightarrow \boxed{q^c = d^c}$$

correct relation

$$\boxed{3\,Q(d^c) + Q(e) + Q(\nu) = 0 \qquad Q(\nu) = Q(e) + 1} \quad \boxed{(1)}$$

$$Q = T_3 + Y/2$$

$$[T_3, Y] = 0$$

$$D = \begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow \boxed{Y_u = Y_d}$$

$$\Rightarrow \boxed{Q(u) - Q(d) = T_3(u) - T_3(d)}$$

$$\boxed{Q(x) = Q(d) + 1}$$

I am left with :

$$6(e) + 3(x^c) + 1(e^c)$$

$$= 10$$

$$5 \times 5 = \cancel{\underbrace{(5\times 5)_S}_{\frac{5\cdot 6}{2} = 15}} + \boxed{\underbrace{(5\times 5)_A}_{\frac{5\cdot 4}{2} = 10}}$$

$$10_F = (5_F \times 5_F)_{AS}$$

but

$$5_F = \begin{pmatrix} d_r \\ d_y \\ d_b \\ \hline e^c \\ \nu^c \end{pmatrix}_R$$

$$\boxed{Q(u) = Q(d) + 1}$$

$$\boxed{Q_{em}(5) = \mathrm{diag}\left(-\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, 1, 0\right)}$$

$$\Downarrow$$

$(u^c)_L\ (3)$ $\qquad\qquad\qquad$ $q_L\ (6)$

$$10_5 = \begin{pmatrix} 0 & u^c_r & u^c_y & u_r & d_r \\ -u^c_r & 0 & u^c_b & u_y & d_y \\ & & 0 & u_b & d_b \\ & & & 0 & e^c\,(1) \\ & & & & 0 \end{pmatrix}_L$$

12 in 10_F

$$\Downarrow$$

$$Q(12) = Q(1) + Q(2)$$

$$= -1/3 - 1/3 = -2/3$$

$$= Q(13) = Q(23)$$

45 in 10_F

$\Downarrow$

$$Q(45) = Q(4) + Q(5) = 1 + 0$$

$$= 1$$

$\uparrow$

$$(2 \times 2)_{AS} = \text{singlet}$$

$$2 \times 2 = 4 = 3 + 1$$

$$= (2 \times 2)_S + (2 \times 2)_{AS}$$

$$|S = 0\rangle = |\uparrow\downarrow - \downarrow\uparrow\rangle$$

$$= |ud - du\rangle$$

$$D = \begin{pmatrix} u \\ d \end{pmatrix} \qquad //$$

$$\Rightarrow \quad D^T \epsilon D$$

$$\varepsilon_{12} = -\varepsilon_{12} = +1$$

$$\varepsilon_{11} = \varepsilon_{22} = 0$$

$$\Rightarrow \boxed{\epsilon = i\sigma_2}$$

<u>14</u> $\quad Q(14) = Q(1) + Q(4) = 2/3$

$$= Q(24)$$

$$= Q(34)$$

Group theory

$$5 \to U\,5$$

$$\Rightarrow 5_i \to U_{ij}\, 5_j$$

$$10_{ij} = 5_i\, 5_j$$

$$10_{ij} \to U_{ik}\, 5_k\, U_{je}\, 5_e$$

$$= U_{ik}\, 5_k 5_e\, U_{je}$$

$$= U_{ik}\, 10_{ke}\, U_{je}$$

$$= U_{ik}\, 10_{ke}\, U^T_{ej}$$

$$\boxed{(10) \to U(10)U^T}$$

$$U = 1 + i\theta_i T_i + \cdots$$

$$= e^{i\theta_i T_i} \qquad i = 1, \ldots, 24$$

$$(10) \to (1 + i\theta_i T_i)(10) \times (1 + i\theta_i T_i^T)$$

$$= 10 + i\left(\theta_i \, T_i (10) + (10) T_i^T\right)$$

$$\Downarrow$$

$$\boxed{\hat{T}_i (10) = T_i (10) + (10) T_i^T}$$

$$U(10) = e^{i\theta_i T_i^1}(10)$$

$$\Downarrow \quad Q_{em} = \sum c T$$

$$\boxed{\hat{Q}_{em}(10) = Q_{em}(10) + (10)\, Q_{em}}$$

$$\Downarrow$$

$$\left(\hat{Q}_{em}(10)\right)_{14} = (Q_{em})_{11}(10)_{14} +$$

$$+ (10)_{14}(Q_{em})_{44}$$

$$= Q(1)(10)_{14} + (10)_{14}\, Q(4)$$

$$\Downarrow \quad = (Q(1) + Q(4))(10)_{14}$$

$$\boxed{\left(\hat{Q}_{em}(10)\right)_{ij} = \left(Q(i)+Q(j)\right)(10)_{ij}}$$

<u>confirmation</u>

$$T_3(e^c) = T_3(10)_{45}$$

$$= \underset{\substack{\| \\ 1/2}}{T_3(4)} + \underset{\substack{\| \\ -1/2}}{T_3(5)} = 0$$