GUT Course 22/23

Lecture XI

29/11/2022

LMU
Fall 2022

$$d_H = \frac{M_p}{T^2}$$

Monopole problem

$$n_M^{in} = \frac{1}{(d_H)^3} = \frac{T_{in}^6}{M_p^3}$$

Preskill '80

$$T_0 = v_{GUT}$$

$$n_M^{in} = \frac{v_{GUT}^6}{M_p^3}$$

$$\boxed{v_{GUT} \simeq 10^{16}\,\text{GeV}}$$

$$n_\gamma^{in} = n_B \simeq n_{\bar{B}} \simeq T_{in}^3 = v_{GUT}^3$$

$$\frac{n_M^{in}}{n_\gamma^{in}} \simeq \left(\frac{v_{GUT}}{M_p}\right)^3 \simeq 10^{-9}$$

$\boxed{T \simeq MeV}$

$$\frac{n_H}{n_H^0} = \left(\frac{T}{T_{in}}\right)^3$$

$\boxed{R\, T = \text{const.}}$

$$\to \quad n_B \equiv n_{B-\bar{B}} = 10^{-10}\, n_\gamma$$

inflation

$d_H \longrightarrow$

$R(t)$

$\boxed{\text{stat}}$

Affleck, Dine '80s

$\boxed{\text{genesis}}$ $\leftarrow$ baryon

to take place later

$\underline{T > T_c}$: $\Delta B \neq 0$

$$\Downarrow$$

$$n_B = n_{\bar{B}} \quad \text{iff}$$

$$\alpha^2 T = \Gamma_{\Delta B \neq 0} > H = T^2/M_p$$

- $\Delta B \neq 0$ Sakharov '64-'67
- $\cancel{C}$, $\cancel{CP}$
- out-of-equilibrium

$$\Longrightarrow \boxed{n_B - n_{\bar{B}} = 10^{-10}\, n_\gamma}$$

Our universe

- $R^0 \simeq d_H^0 \simeq 10^{29}\,\mathrm{cm}$, $T^0 \simeq 10^{-4}\,\mathrm{eV}$

$(k=1)$

- $n_B^0 \simeq 10^{-10}\, n_\gamma^0$

- $n_\nu^0 \simeq 10\, n_B^0 \quad (DM)$

$$\Downarrow$$

$$n_M^0 \leq 10\, n_B^0$$

$$E_B^0 = \underset{\equiv}{\overset{m_B}{}}\ \mathrm{GeV} \quad (\text{single Layer})$$

$$E_\gamma^0 \simeq T^0 \simeq 10^{-13}\,\mathrm{GeV}$$

$$\boxed{\varepsilon_B^0 \simeq 10^3\, \varepsilon_\gamma^0}$$

matter dominated

$$\varepsilon_B = 10^{-10}\, m_B\, T^3$$

$$\varepsilon_\gamma = T^4$$

decoupling: $\varepsilon_B = \varepsilon_\gamma$

$$T_D \simeq 10^{-10}\, m_B \simeq 1/10\ \mathrm{eV}$$

Matter dominated universe

$$H^2 = G_N \rho \qquad H = \frac{1}{t}$$

$$\rho_m = 10^{-10} m_B T^3 \qquad H \equiv \dot{R}/R$$

$$H \simeq 10^{-5} \frac{\sqrt{m_B T^3}}{M_p}$$

$$\boxed{d_H = t = 10^5 \frac{M_p}{\sqrt{m_B T^3}}}$$

$$\Rightarrow d_H^0 = 10^5 \frac{10^{19}}{\sqrt{10^{-39}}} \text{GeV}^{-1} \simeq 10^{44} \text{GeV}^{-1}$$

$T_0 = 10^{-13}\,\mathrm{GeV}$

$\mathrm{GeV}^{-1} = 10^{-14}\,\mathrm{cm}$

$$\boxed{d_H^0 \simeq 10^{30}\,\mathrm{cm}}$$

Mohayee, Sorkin ...

$\downarrow$

<u>NO</u> acceleration

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu} + g_{\mu\nu} \Lambda G_N$$

C. C.

$$\Lambda = \langle V \rangle \simeq v_{SM}^4 \simeq 10^8\,\mathrm{GeV}^4$$

$$\boxed{\Lambda \leq \left(10^{-4}\,\mathrm{eV}\right)^4}$$

DIRAC MONOPOLE

Dirac ?

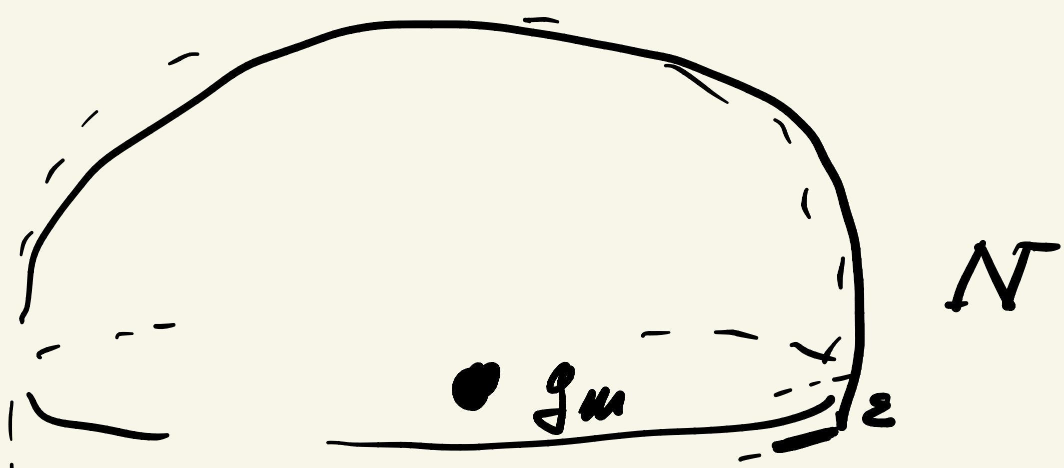
if $\exists$ monopole ?

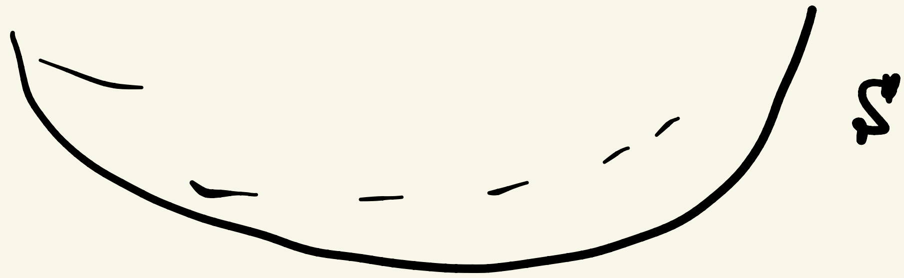
Wu - Yang '50s version

QM:

$$\vec{B} = \nabla \times \vec{A}$$

$$\Rightarrow \quad \nabla \cdot \vec{B} = 0 \quad (\underline{\text{no}} \text{ pole})$$





$$\vec{A}_N = \frac{g_m}{4\pi r}\,\frac{1-\cos\theta}{\sin\theta}\,\hat{\varphi}$$

$$\theta = 0 \Rightarrow \vec{A}_N \to 0$$

$$\theta = \pi \Rightarrow \vec{A}_N \to \infty$$

$$\vec{A}_S = -\frac{g_m}{4\pi r}\,\frac{1+\cos\theta}{\sin\theta}\,\hat{\varphi}$$

$$\theta = 0 \Rightarrow \vec{A}_S \to \infty$$

$$\theta = \pi \Rightarrow \vec{A}_S \to 0$$

$$\vec{B} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, A\right)\hat{r} + 0\cdot\hat{\theta} + 0\cdot\hat{r}$$

$$\Downarrow$$

$$\vec{B}_N = \frac{g_m}{4\pi r^2\sin\theta}\left(\sin\theta\right)\hat{r}$$

$$\Downarrow$$

$$\boxed{\vec{B}_N = \vec{B}_S = \frac{g}{4\pi r^2}\,\hat{r}}$$

$$\vec{A}_N - \vec{A}_S \,(\theta = \pi/2) = \frac{g_m}{2\pi r \sin\theta}\,(\theta = \pi/2)\,\hat{\varphi}$$

overlap

$$\Downarrow$$

$$\vec{A}_N - \vec{A}_S = \frac{g_m}{2\pi r}\hat{\varphi} = \nabla \chi$$

$$\Rightarrow \boxed{\chi = \frac{g_m}{2\pi}\varphi}$$

$$\nabla \chi = \frac{1}{r}\frac{\partial}{\partial \varphi}(\chi)\,\hat{\varphi} + \underbrace{\cdots}_{0}$$

$$\underbrace{\chi(2\pi) \neq \chi(0)}_{\text{\underline{not} single-valued}}$$

$$\psi_q \longrightarrow e^{iq\chi}\,\psi_q$$

gauge transformation

$$\boxed{\psi \overset{!}{=} \text{single-valued}}$$

$$\Rightarrow \boxed{q\,\chi(2\pi) = 2\pi n}$$

but $\chi(2\pi) = gn$

$$\Downarrow$$

$$\boxed{e g_m = 2\pi n}$$

Dirac: if $\exists$ monopole

$\Rightarrow$ charge is quantized

$$\Updownarrow$$

'tHooft - Polyakov: if charge is quantized $\Rightarrow$ $\exists$ monopole

$$\overset{g}{SO(3)} \xrightarrow[\langle \phi \rangle]{} \overset{e}{SO(2) = U(1)}$$

triplet

$(g = e)$

$$g\, g_m = 4\pi \quad (n = 1)$$

basic charge : spinor of $SO(3)$

$$q = \frac{1}{2}\,(g)$$

$$\Rightarrow \; e\, g_m = 2\pi n$$

general

$$\Updownarrow$$

$$\boxed{\vec{B}(x) = \frac{n}{g}\frac{\hat{r}}{r^2}}$$

<u>charge quantization of SM</u>

SM particles (q, e)

$\overset{\downarrow}{=}$ charges quantized

in order to get rid of axial gauge anomalies

Noether:

$$\psi \to e^{i\alpha}\psi$$

$$\Rightarrow \quad \partial_\mu j^\mu = 0$$

$\Uparrow$

classical physics

QFT

$$\psi \to e^{i\beta\gamma_5}\psi$$

$\Uparrow$

when $m_\psi = 0$

$$j_\mu^5 = \bar{\psi}\gamma_\mu\gamma_5\psi \quad \therefore$$

$$\partial_\mu j^\mu_5 = 0 \quad (\text{tree})$$

$\Downarrow$ anomaly

$$\partial_\mu J_5^\mu = \frac{g^2}{32\pi^2} F \tilde{F}$$

$$\tilde{F}_{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$