GUT Course 22/23

15/11/2022

LECTURE VII

LMU

Fall 2022

Domain Walls and Gravity

HW: weak field, small velocity

$$\nabla^2 V_{gr} = 4\pi G_N \left(T_{00} - \frac{1}{2} T \right)$$

matter: $T_{00} = \rho, \quad T_{ii} = 0$

$T_{ij} \; (i \neq j) = 0$

dw: $T_{00} = T^0_0 = \rho$

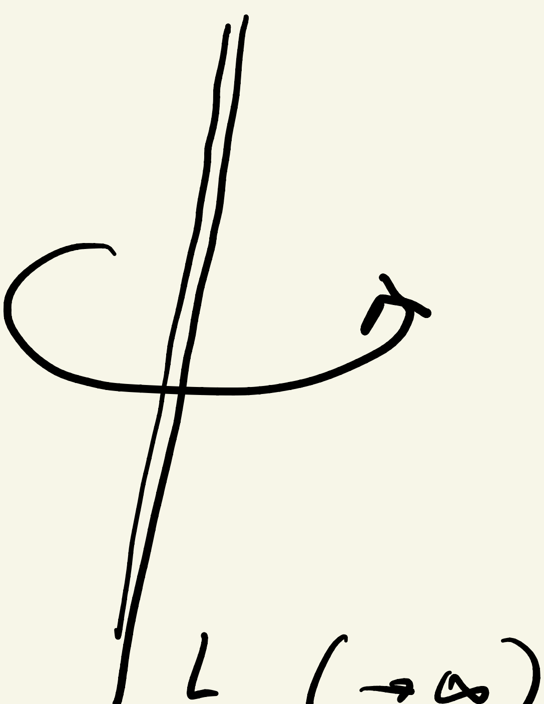
$T_i^{\,i} = -T_{ii} \neq 0$

$\Downarrow$

$\boxed{\text{anti - gravity}}$

(Cosmic) Strings

static classical solutions



S_1 = circle symmetry

$\Downarrow$ suggestive

U(1) gauge theory

$\phi \in \mathbb{C}$ Nielsen, Olesen, 1971

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi)^*(D^\mu \phi) - V(|\phi|)$$

$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu \phi = (\partial_\mu - i g A_\mu)\phi$$

$U(1)$: $\phi \to e^{i\alpha(x)} \phi \quad (Q = 1)$

$$V(|\phi|) = \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2$$

Brout, Englert '64

Higgs '64

$$\mathcal{M}_0 = \{ \phi_0 : V = V_{min} = 0 \}$$

$$= \{ \phi_0 : |\phi_0|^2 = v^2 \}$$

$$= S_1$$

- $\phi = \phi_{un} = v + h$

Higgs

A : $M_A = g v$

$$\Uparrow$$

$$\frac{1}{2} |D_\mu \phi_0|^2 = \frac{1}{2} g^2 v^2 A_\mu A^\mu$$

$$V(h) = \frac{\lambda}{4}\left((v+h)^2 - v^2\right)^2$$

$$= \frac{\lambda}{4}\left(2vh + h^2\right)^2$$

$$\boxed{V_{Higgs} = \frac{1}{2} m_h^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 \qquad m_h^2 = 2\lambda v^2}$$

$$\Downarrow$$

Static solution of finite (per unit length) energy

(ρ, θ, z)

$$\mathcal{E} \equiv E/L = \int ds \left[\frac{1}{2} |D_i \phi|^2 + V(|\phi|) + \frac{1}{2}\left(\bar{E}^2 + \bar{B}^2\right)\right]$$

$$ds = \rho\, d\rho\, d\theta$$

$\Downarrow$

finite E/L $\Rightarrow$ $\boxed{\begin{array}{l} D_i\phi \xrightarrow[\infty]{} 0 \\ V(\phi) \xrightarrow[\infty]{} 0 \\ \vec{E}, \vec{B} \xrightarrow[\infty]{} 0 \end{array}}$

$\infty \Rightarrow \rho = R \to \infty$

$M_\infty = S_1$

$V(\phi_\infty) = 0$

$\Downarrow$

$|\phi_\infty|^2 = v^2 \Rightarrow \underbrace{\phi_\infty \in M_0}_{\text{Map}:\, M_\infty \to M_0}$

$S_1 \to S_1$

(a) trivial $\phi_\infty = \phi_0 = v$

$\Rightarrow \phi_{static} = v \leftarrow$ vacuum

$S_\infty \longrightarrow 1$ (of S_0)

(b) non-trivial

$$\phi_\infty = v \, e^{i\theta n}$$

$$\Uparrow$$

$$\phi_\infty(2\pi) = \phi_\infty(0)$$

$$\Downarrow$$

$\phi(0) = 0$ (expect)

$$\Downarrow$$

$V(0) = V_{max}$ (local) $= \frac{1}{4} v^4$

$\Downarrow$

String

Proof: $(D_i \phi)_\infty \to 0$

$$\partial_i \phi_\infty = ig A_i^\infty \phi_\infty$$

$$(\phi_\infty = v e^{in\theta})$$

$$\Rightarrow \boxed{A_i^\infty = \frac{n}{g} \partial_i \theta}$$

$\Downarrow$

Magnetic flux

$$\text{Flux} = \int \vec{B} \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l} \qquad \vec{B} = \nabla \times \vec{A}$$

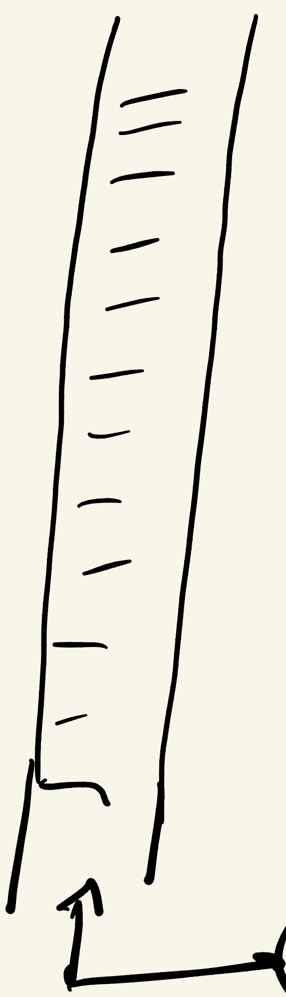
$\Downarrow$

$$\text{Flux} = \oint dx_i \, \frac{1}{g} \, u \, \partial_i \theta$$

$$= \frac{u}{g} \oint d\theta = \frac{2\pi u}{g}$$

$$\Downarrow$$

$\boxed{\text{string of a "magnetic" field}}$



α (outside)

$\vec{E} = 0 = \vec{B} = V$

$(\vec{B}, V(0))$

$(\vec{E}=0)$

$$\mathcal{E} \equiv E/L = \text{energy density}$$

$$E = \int_{-\infty}^{+\infty} \mathcal{E}\, dz = \mathcal{E} L$$

$(L \gg \delta)$

"know": $\delta \sim \frac{1}{v}$, $v > \text{TeV}$

$(M_A = g v)$

$$\Rightarrow \boxed{\delta < 10^{-17} \text{cm}} \quad \boxed{\text{GeV}^{-1} \simeq 10^{-14} \text{cm}}$$

- $\text{Flux} = \oint \vec{B}\, d\vec{S} = B \cdot S = B \cdot \pi \delta^2$

(radius of

string)

$$B \cdot \pi \delta^2 = \frac{2\pi n}{g}$$

$$\boxed{B = \frac{2n}{g\,\delta^2}} \rightarrow \text{(inside)}$$

$$\boxed{V = V(0) = \frac{1}{4} v^4} \rightarrow \text{(inside)}$$

$$\boxed{U(1) \xrightarrow[\text{SSB}]{} \mathbb{1} \Rightarrow \text{string}}$$

· Is there a SM string?

NO : $SU(2) \times U(1) \rightarrow U(1)$

$SU(2) \rightarrow \mathbb{1}$

$$\left.\begin{array}{l}\phi(\rho) = e^{in\theta} f(\rho)\, v \\ f(\rho) \therefore \begin{cases} f(0) = 0 \\ f(\infty) = 1 \end{cases}\end{array}\right\} \boxed{\text{string}}$$

$$E \doteq L \int \left[\frac{1}{2} \bar{B}^2_{(0)} + V(0) \right] dS$$

$$= L \cdot \pi \delta^2 \left[\frac{1}{2} \frac{4}{g^2 \delta^4} + \frac{\lambda}{4} v^4 \right]$$

$$= \pi L \left[\frac{2}{g^2 \delta^2} + \frac{\lambda}{4} v^4 \delta^2 \right]$$

$$\frac{\partial E}{\partial \delta} = 0 \Rightarrow \frac{\lambda}{4} v^4 \delta = \frac{4}{g^2 \delta^3}$$

$$\Rightarrow \boxed{\delta^4 \cong \frac{16}{g^2 \lambda} \frac{1}{v^4}}$$

$$\Downarrow$$

$$\boxed{\delta \sim \frac{1}{v}} \quad \boxed{\text{as expected}}$$

String stability

$$\phi \in \mathbb{C} \Rightarrow \phi = \phi_1 + i\phi_2$$

$$\boxed{\mu, \nu, \ldots = t, x, y} \qquad a = 1, 2$$

$$j^\mu = \epsilon^{\mu\nu\alpha} \partial_\nu \phi_a \partial_\alpha \phi_b \varepsilon_{ab} N$$

$$\partial_\mu j^\mu = \epsilon^{\mu\nu\alpha} \left[\partial_\mu \partial_\nu \cdots + \partial_\mu \partial_\alpha \cdots \right] = 0$$

$$\boxed{\text{conserved current}}$$

$$Q = \int j_0 \, dS$$

$$j_0 = \varepsilon_{ij} \varepsilon_{ab} \partial_i \phi_a \partial_j \phi_b \; N$$

$$= \varepsilon_{ij} \varepsilon_{ab} \partial_i (\phi_a \partial_j \phi_b) \; N$$

$$\cancel{- \varepsilon_{ij} \varepsilon_{ab} \phi_a \partial_i \partial_j \phi_b \; N}$$

$$\boxed{j_0 = \varepsilon_{ij} \partial_i V_j} = \tilde{j}_0^{\,z}$$

curl

$$\boxed{V_j = \varepsilon_{ab} \phi_a \partial_j \phi_b N}$$

$$Q = \oint dx_i \; V_i$$

$$V_i(\infty) = \varepsilon_{ab} \; \phi_a(\infty) \partial_i \; \phi_b(\infty) N$$

$$\phi_a(\infty) = {}^{x_a}/_{\rho}\, v$$

$$\phi_1(\infty) = v\cos\theta$$

$$\phi_2(\infty) = v\sin\theta$$

$$\Rightarrow \boxed{\bar{V}_i(\infty) = v^2 N \; A_i(\infty)}$$

$$\left(N = \frac{1}{v^2}\right)$$

Used:

$$\boxed{\begin{gathered} \int (\nabla\times\vec{V})\cdot d\vec{S} = \oint \vec{V}\cdot d\vec{l} \\ d\vec{S} = \hat{z}\, dS \\ (\nabla\times\vec{V})_z = j_0 \end{gathered}}$$

$$\boxed{\begin{array}{c} \dfrac{dQ}{dt} = 0 \\[1em] Q \propto \text{Flux} \end{array}}$$

Why?

$$\phi_\infty = e^{in\theta} v$$

$$n = \text{fixed}$$

$$\text{Flux} = \frac{2\pi n}{g} = \text{conserved}$$

Strings in cosmology

$\varepsilon = E/L \cong v^2 \leftarrow$ compute

$$E = L \int ds \left[\frac{1}{2} \bar{B}^2 + V(\phi) \right]$$

$$\cong L \; 2 V(0) \pi \delta^2 \sim v^4 \delta^2 \sim v^2$$

- $E_S = v^2 R_U \longleftarrow$ cosmic string

$$\left(\begin{array}{c} n_B / n_\gamma \simeq 10^{-10} \\ R_U \simeq 10^{29}\,\mathrm{cm} \\ n_\gamma \simeq 400 / \mathrm{cm}^3 \end{array} \right)$$

$$E_U \simeq \underbrace{10 \; E_B}_{DM} \simeq 10^{-9}\,\mathrm{GeV} \frac{10^2}{\mathrm{cm}^3} R_U^{\,3}$$

$$E_U \simeq 10^{-7}\,\mathrm{GeV} \left(\frac{10^{29}\,\mathrm{cm}}{\mathrm{cm}} \right)^3$$

$$E_u \simeq 10^{-7}\,\mathrm{GeV} \times 10^{87} \simeq 10^{80}\,\mathrm{GeV}$$

$$\boxed{\begin{aligned} N_B &\simeq 10^{80} \\ N_\gamma &\simeq 10^{90} \end{aligned}}$$

$$\boxed{E_u \simeq 10^{80}\,\mathrm{GeV}}$$

$$E_s > 10^{6}\,\mathrm{GeV}^{2} \cdot 10^{29}\,\mathrm{cm}$$

$$= 10^{35}\underbrace{(\mathrm{GeV\,cm})}_{10^{14}}\,\mathrm{GeV}$$

$$\boxed{E_s > 10^{49}\,\mathrm{GeV}}$$

- $v = 10^{15}\,\mathrm{GeV} \Rightarrow \quad E_s \simeq 10^{30} \cdot 10^{29} \cdot 10^{14}\,\mathrm{GeV}$
$\simeq 10^{74}\,\mathrm{GeV}$

Vilenkin ?

My notes: Beyond the SM