GUT Course 22/23

Lecture VI

11/11/2022

LMU
Fall 2022

SSB: Domain Walls

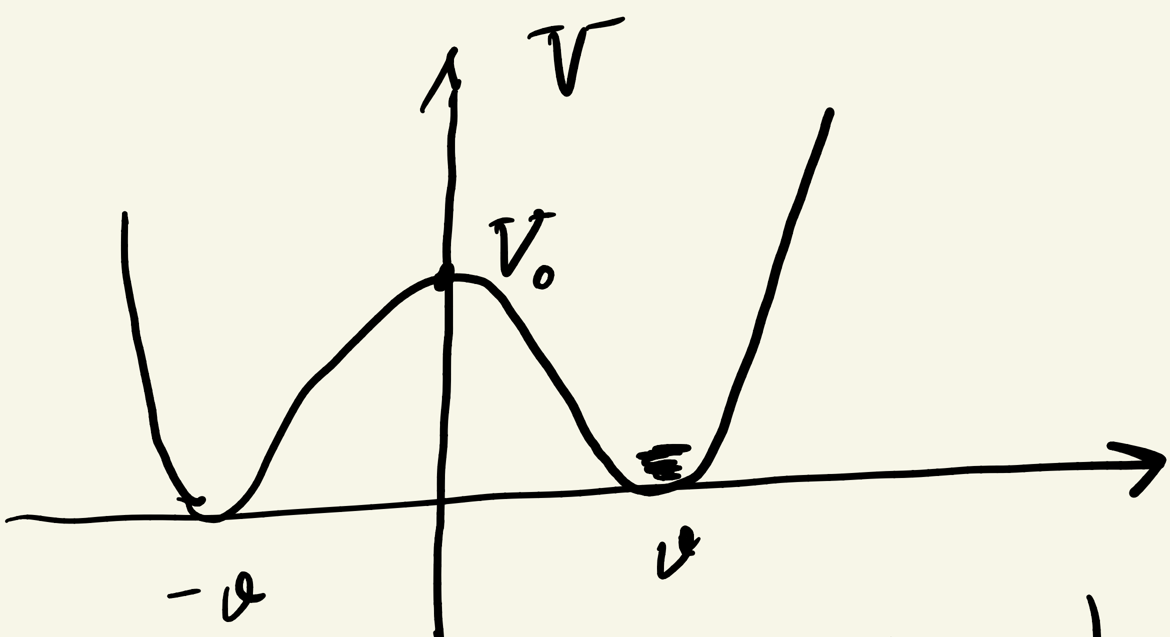
D: $\phi \in \mathbb{R}$, $\boxed{\phi \to -\phi \quad (\mathbb{Z}_2)}$

$$V = \frac{1}{4}\left(\phi^2 - v^2\right)^2$$

mass scale

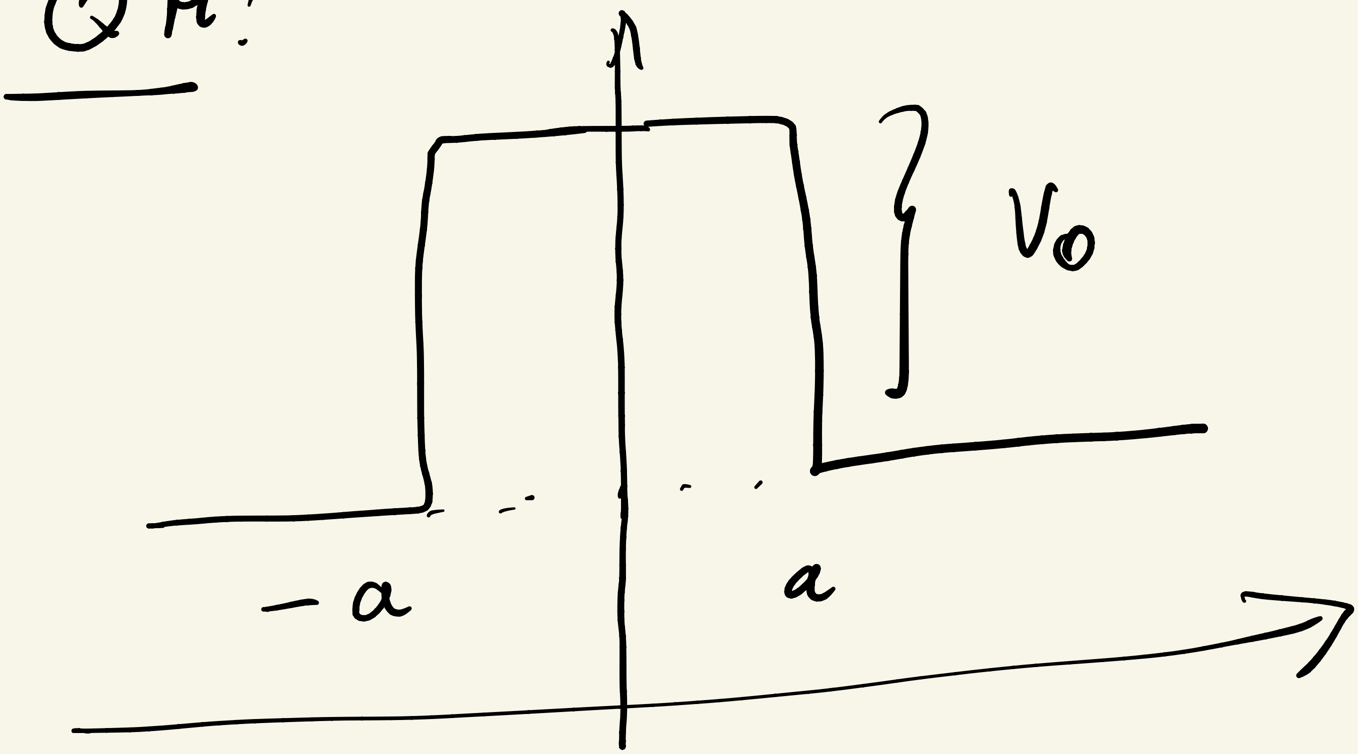
$$\mathcal{M}_0 = \{\phi_0 : V = V_{min} = 0\}$$

$$= \{\phi_0^2 = v^2 : \phi_0 = \pm v\}$$



$V_0 = \frac{1}{4} v^4$

QM:



$$\Gamma_{tun} \propto e^{-\int_{-a}^{a} \sqrt{V_0}\, dx}$$

$$\psi_0 = \text{symmetric}$$

NO SSB

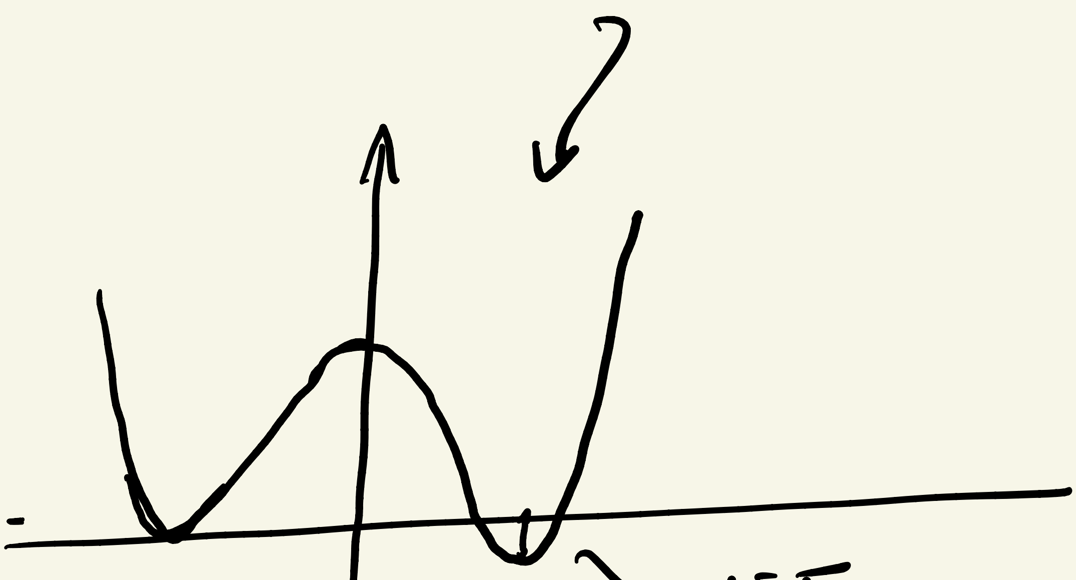
(tunneling)

SSB in QFT

$\Longleftrightarrow$ NO tunneling between degenerate vacua

Okun, Kobzarev, Zeldovich '70s

Coleman '76



$\Delta V \ll V_0$

$$\Gamma_{\text{tun}} \propto e^{-\left(\frac{V_0}{\Delta V}\right)^n} \qquad (n = ?)$$

$$\to 0, \quad \Delta V \to 0$$

$$\Downarrow$$

$$\boxed{\begin{array}{c} \text{SSB}: \Gamma_{\text{tun}} = 0 \\ \text{for } \Delta V = 0 \end{array}}$$

$\Downarrow$ <u>imprint</u>

$$\boxed{\text{Domain wall}}$$

energy density

STATIC solution.

$$E/S = \text{finite}$$

$$E/S = \underbrace{\int_{-\infty}^{+\infty} \left[\frac{1}{2}\left(\frac{d\phi}{dz}\right)^2 + V(\phi) \right] dz}_{\text{finite}} \qquad (1)$$

$$\left(\frac{d\phi}{dz}\right)^2 \downarrow 0 \quad \text{at } \infty \quad V(\phi) \downarrow 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \qquad (2)$$

$$\Downarrow$$

$$\Box \phi = -\frac{\partial V}{\partial \phi} \qquad (3)$$

$$\Downarrow$$

$$\frac{d^2\phi}{dz^2} = \frac{\partial V}{\partial \phi} \Big/ \frac{d\phi}{dz} \qquad (4)$$

$$\frac{d}{dz}\left[\frac{1}{2}\left(\frac{d\phi}{dz}\right)^2\right] = \frac{dV}{dz}$$

$$\Downarrow$$

$$\frac{d}{dz}\left[\frac{1}{2}\left(\frac{d\phi}{dz}\right)^2 - \bar{V}\right] = 0$$

$$\Downarrow$$

$$\frac{1}{2}\left(\frac{d\phi}{dz}\right)^2 - \bar{V}(\phi) = \underset{0 \text{ at } \infty}{\overset{}{\underset{\shortparallel}{\text{const}}}}$$

$$\Downarrow$$

$$\boxed{\frac{1}{2}\left(\frac{d\phi}{dz}\right)^2 = \bar{V} \qquad (5)}$$

$$\Downarrow$$

$$V(\infty) = 0$$

$$\Rightarrow \boxed{\phi(\pm\infty) = \pm v}$$

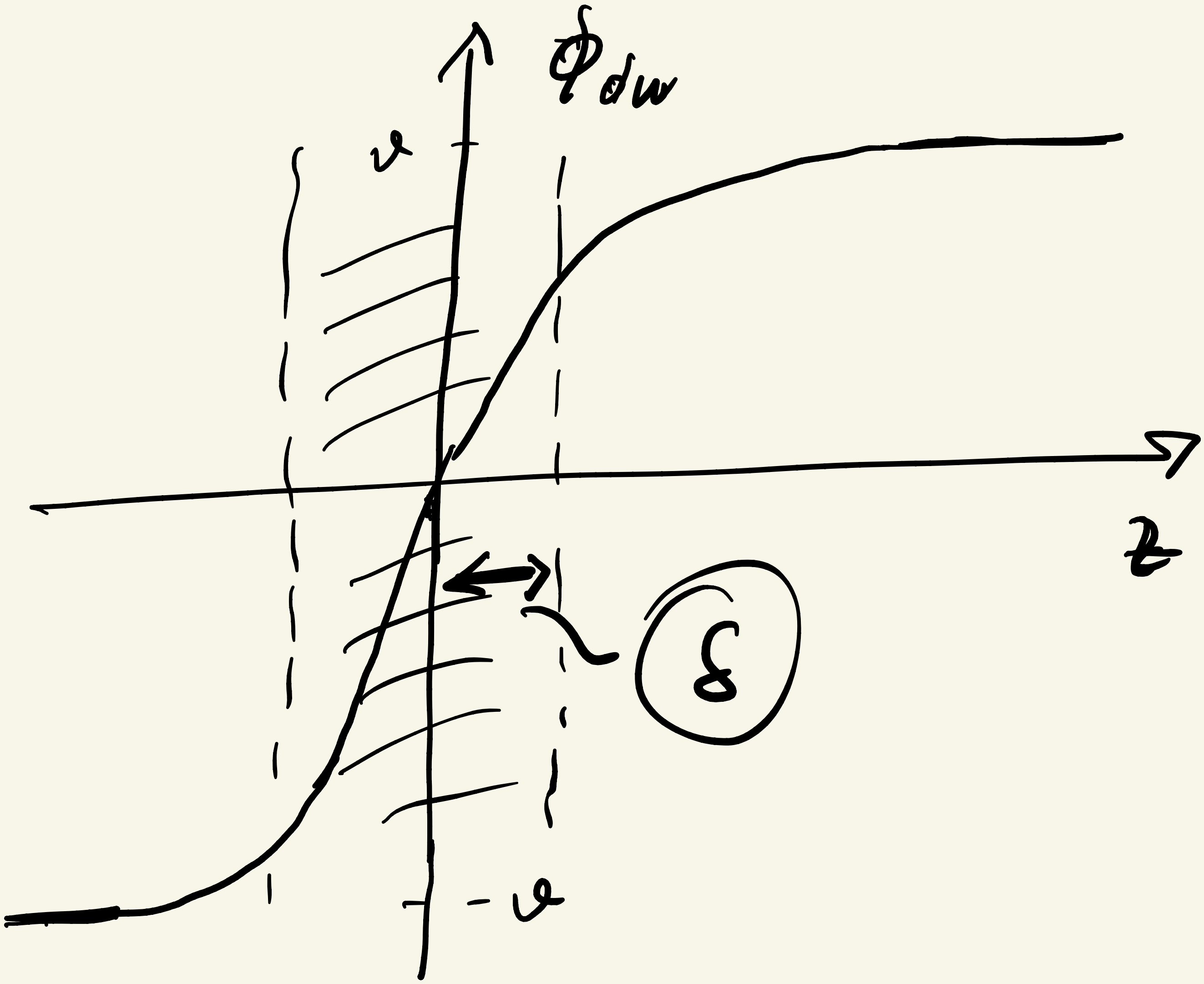
a. <u>trivial solution</u> = <u>vacuum</u>

$$\phi = \phi_0 = v$$

b. <u>non-trivial solution</u>

$$\left.\begin{aligned} \phi_{dw}(+\infty) &= v \\ \phi_{dw}(-\infty) &= -v \end{aligned}\right\} dw$$

$$\Updownarrow$$



Map: $M_\infty = \{+\infty, -\infty\}$

$M_0 = \{+v, -v\}$

$\Phi_{dw} = \text{map}$

$\Downarrow$

let us find ϕ_{dw} :

(eq. (5))

$$\frac{d\phi_{dw}}{dz} = \sqrt{2\,\bar{V}}$$

$$= \sqrt{\frac{\lambda}{2}\left(\phi_{dw}^2 - v^2\right)^2}$$

$$\Downarrow$$

$$\boxed{\frac{d\phi_{dw}}{dz} = \pm\sqrt{\frac{\lambda}{2}}\left(\phi_{dw}^2 - v^2\right)} \tag{6}$$

$$\Downarrow$$

$$\boxed{\phi_{dw} = v \tanh \sqrt{\tfrac{\lambda}{2}}\, v z} \quad (7)$$

$$\boxed{\phi_{dw}^{anti} = - \phi_{dw}}$$

$$\tanh \delta z$$

$$\boxed{\delta = \frac{1}{v}\sqrt{2/\lambda} \quad (8)}$$

- SM : $\phi \to \Phi \to v\Phi$

$$V = \frac{1}{4}\left(\Phi^{+}\Phi - v^{2}\right)^{2}$$

Z_2 : $\Phi \to -1\Phi$?????

$$-\mathbb{1}_2 = U = e^{i\pi\sigma_3}$$

$$= \cos\pi + i(\sin\pi)\sigma_3$$

$\Uparrow$

discrete

$\Rightarrow$ NO DW in SM

$\Downarrow$

$$\boxed{v > 100\ \text{GeV}}$$

$$\boxed{\text{GeV}^{-1} \cong 10^{-14}\ \text{cm}}$$

$$\Rightarrow \boxed{\delta \simeq \frac{1}{v} \leq 10^{-16}\ \text{cm}}$$

$$\cdot \left(\frac{E}{S}\right)_{dw} = \int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\left(\frac{d\phi_{dw}}{dz}\right)^2 + V(\phi) \right]$$

$$= \int_{-\infty}^{+\infty} dz \; 2V = \int_{-v}^{v} d\phi \, \frac{dz}{d\phi} \, 2V$$

$$\frac{dz}{d\phi} = \frac{1}{d\phi/dz} = \frac{1}{\sqrt{2V}}$$

$$= \int_{-v}^{v} d\phi \sqrt{2V} = \int_{-v}^{v} d\phi \sqrt{\frac{\lambda}{2}}\,(\phi_{dw}^2 - v^2)$$

$$\phi_{dw} = v \, \mathrm{th}\, \delta z$$

$$\Downarrow$$

$$\phi_{dw}^2 - v^2 = \pm v^2 \cosh^{-2} \delta z$$

$$\boxed{\left(E/s\right)_{dw} \cancel{\propto} v^3}$$

#

$\neq$

vs universe

Stability

$\mu, \nu = t; z$

$$j_\mu = \epsilon_{\mu\nu} \partial^\nu \phi$$

$\boxed{\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}}$

$$\partial^\mu j_\mu = \epsilon_{\mu\nu} \partial^\mu \partial^\nu \phi = 0$$

$$\Downarrow$$

$$j_0 = \frac{d\phi}{dz}$$

$$\Downarrow$$

$$Q = \int_{-\infty}^{+\infty} j_0 \, dz = \phi(+\infty) - \phi(-\infty)$$

$$\frac{dQ}{dt} = 0$$

- $\phi_{\text{vacum}} = \phi_0 = +v \; (-v)$

$$Q_{\text{vacum}} = 0$$

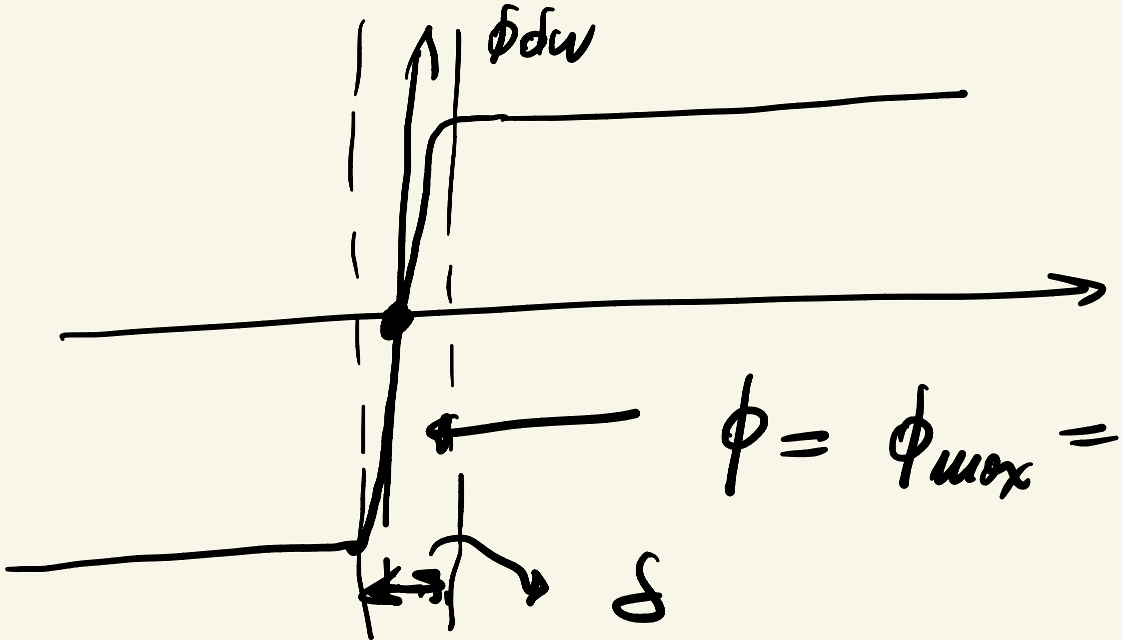
- ϕ_{dw} :

$$\boxed{\begin{aligned} \phi_{dw}(+\infty) &= v \\ \phi_{dw}(-\infty) &= -v \end{aligned}}$$

$$\boxed{\begin{aligned} Q_{dw} &= 2v \\ Q_{dw}^{anti} &= -2v \end{aligned}} \neq 0$$

$$\Rightarrow \boxed{dw \ (anti \ dw) = stable}$$

Physics of dw



$$\Longrightarrow \quad \delta \to 0$$

$$E/S = \int_{-\infty}^{+\infty} \left[\frac{1}{2} \left(\frac{d\phi}{dz} \right)^2 + V(z) \right] dz$$

① ②

$$= E_1 + E_2 \simeq v^4 \delta$$

$$(\delta \to 0)$$

$$\left(\frac{\Delta \phi}{\Delta z} \right)^2 \delta = \left(\frac{2v}{2\delta} \right)^2 \delta \simeq \frac{v^2}{\delta}$$

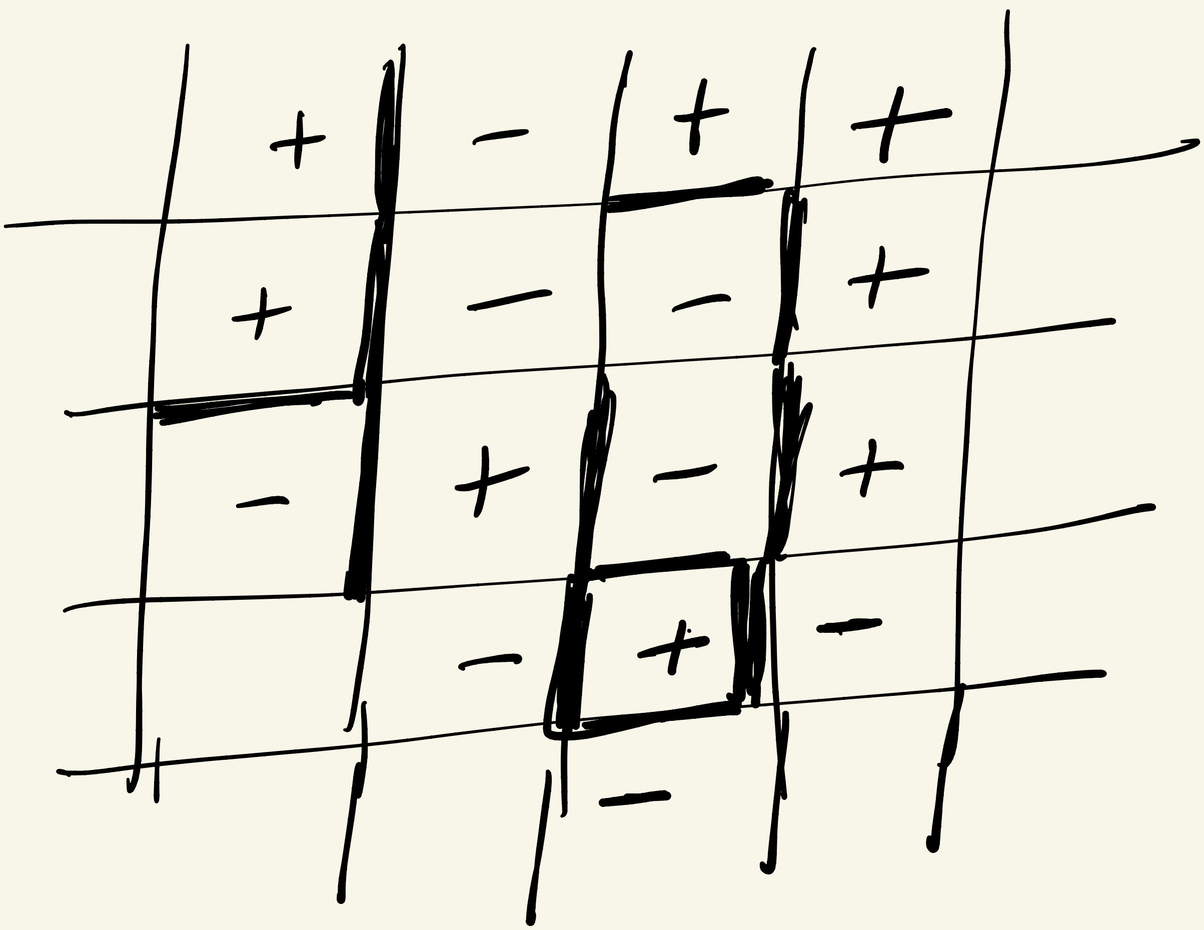
$$\delta \to \infty$$

$$\Downarrow$$

$$(E/S) \simeq \frac{v^2}{\delta} + v^4 \delta \Leftarrow \text{minimal}$$

$$\frac{\partial (E/S)}{\partial \delta} \simeq -\frac{v^2}{\delta^2} + v^4 = 0$$

$$\Rightarrow \boxed{\delta \simeq 1/v}$$



$\Downarrow$

small domains $\longrightarrow 0$

$\Downarrow$

at least 1 large dw

What of dw in universe?

- $R_U \simeq 10^{28}\,cm \simeq 10^{18}\,sec$

size of universe

- $n_\gamma \simeq 400/\text{cm}^3 \simeq n_\nu$ (*)

- $n_B / n_\gamma \simeq 10^{-10}$

$$E_\gamma \simeq T_\gamma \simeq 10^{-4}\,\text{eV} \quad (T_\gamma = T_0)$$

$$\simeq 10^{-13}\,\text{GeV}$$

$$\varepsilon_\nu = n_\gamma T_0 + n_B m_B$$

$$\simeq n_B \left(\text{GeV} + \cancel{10^{10}\,10^{-13}\,\text{GeV}}\right)$$

$$+ \,\varepsilon_{dm}$$

$$\varepsilon_\nu = 10\,\varepsilon_B \simeq n_B\,\text{GeV}$$

$\simeq 10^{-10} n_\gamma$ GeV

$$\boxed{\mathcal{E}_m^{max} \simeq 10^{-8}/cm^3 \text{ GeV} \quad \text{(mass)}}$$

$\parallel$

mass / unit volume

$$\boxed{= \left(\frac{\text{mass}}{\text{unit area}}\right)_U \simeq \frac{10^{-8}\,\text{GeV}}{cm^3}\,10^{28}\,cm \simeq 10^{20}\,\frac{\text{GeV}}{cm^2}}$$

universe

• dw

$$\left(E/s \simeq v^3\right)_{dw}$$

$$\left(\frac{dw}{\text{universe}}\right)_{\text{energy}} \simeq \frac{v^3}{10^{20}\ \text{GeV}/\text{cm}^2}$$

$$\simeq \frac{v^3\,\text{cm}^2}{10^{20}\ \text{GeV}} \gtrsim \frac{10^6\,\text{cm}^2\,\text{GeV}^3}{10^{20}\ \text{GeV}}$$

$$\Downarrow$$

$$\left(\frac{\mathcal{E}_{dw}}{\mathcal{E}_{u}}\right) \gtrsim 10^{-14}\underbrace{(\text{cm GeV})^2}_{10^{14}}$$

$$\boxed{\left(\frac{\Sigma dw}{\varepsilon_w}\right) \gtrsim 10^{14}}$$