GUT Course 22/23

Lecture V

8/11/2022

LMU

Fall 2022

Hierarchy Problem?

$$\mathcal{L}_D = i\bar{f}\gamma^\mu\partial_\mu f - m_f\bar{f}f$$

$$= i\left(\bar{f}_L\gamma^\mu\partial_\mu f_L + L \leftrightarrow R\right) -$$

$$- m_f^0\left(\bar{f}_L f_R + \bar{f}_R f_L\right)$$

- $U_V(1)$ $\quad f_L \to e^{i\alpha} f_L,\ f_R \to e^{i\alpha} f_R$

$\boxed{\text{vector-like}}$: $L \Leftrightarrow R$

- $m_f^0 = 0$ $\quad f_L \to e^{i\beta} f_L,\ f_R \to f_R$

$\quad f_L \to f_L,\ f_R \to e^{i\gamma} f_R$

$\Uparrow$

Chiral symmetry

$\Updownarrow$

$$f \to e^{i\beta\gamma_5} f \quad \therefore$$

$$f_L \to e^{i\beta} f_L, \quad f_R \to e^{-i\beta} f_R$$

$$m_f = m_f^0 \left[1 + \frac{\alpha}{\pi} \ln \frac{\Lambda}{m} + \cdots \right]$$

Scalar

$$m_s^2 \phi^* \phi$$

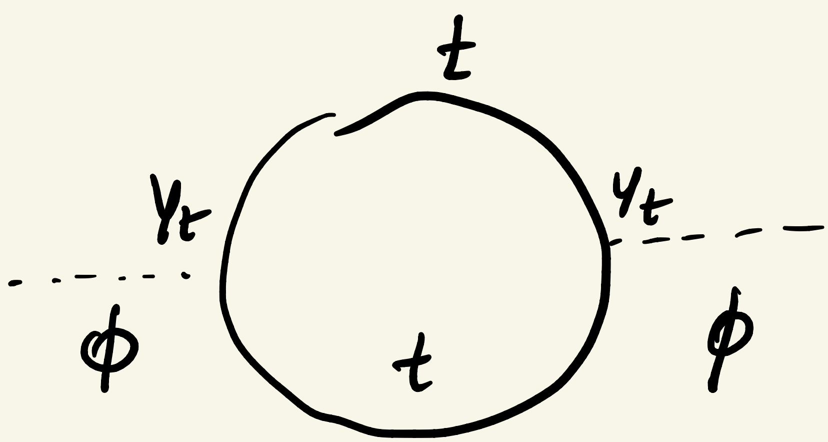
$\Rightarrow$ no extra sym.

$\boxed{\text{when } m_s \to 0}$

Higgs

$$\mu_0^2 \Phi^+ \Phi + \ldots$$

$$\mu_0^2 = \lambda v^2$$



$$\mu^2 = \mu_0^2 + \frac{y_t^2}{16\pi^2}\left(\Lambda^2 + m_t^2 \ln\frac{\Lambda}{\mu}\right)$$

HIERARCHY PROBLEM?

NO

$$\mu_0^2 = \mu^2 - \ldots$$

Λ dependence

<u>but</u>

$$\boxed{\text{physical amplitudes} = \text{independent of } \Lambda \; (\Lambda \to \infty)}$$

$$\Leftrightarrow A = A_{SM} + O\left(\frac{M_W}{\Lambda}\right)^n$$

$$\Downarrow$$

Supersymmetry

1971 – 1974

Gelfand, Likhtman

Wess, Zumino

$(s=\frac{1}{2})\ f \longleftrightarrow \tilde{f}$ (sfermion)

$s=0$

$W \longleftrightarrow \tilde{W}$ (gauginos)

$Z \qquad \tilde{Z}$

$A \qquad \tilde{A}$

$s=1$ gauge bosons

$(s=0)\ \phi \longleftrightarrow \tilde{\phi}$ Higgsino

Higgs

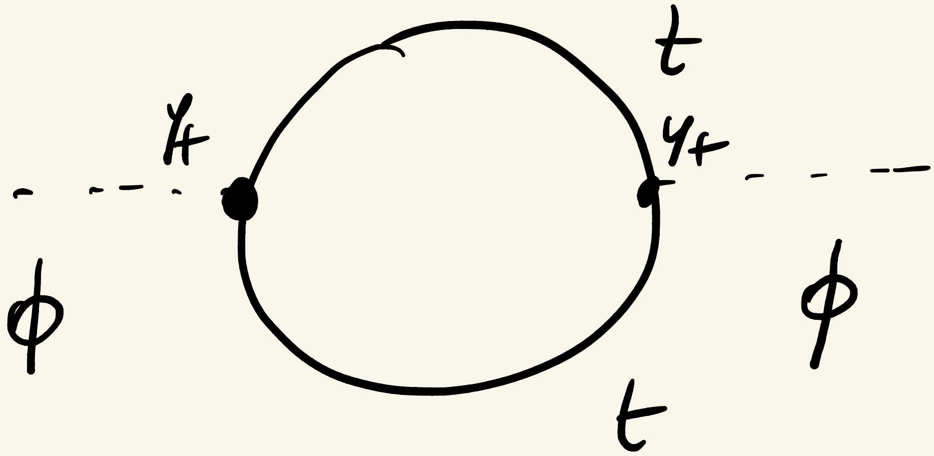
$p \longleftarrow \tilde{p}$

particle $\Downarrow$ sparticle

$\boxed{m_{\tilde{p}} \neq m_p}$

$\boxed{m_{\tilde{p}} > 100 \text{ GeV}}$

$\uparrow$ $\boxed{\text{SUSY}}$



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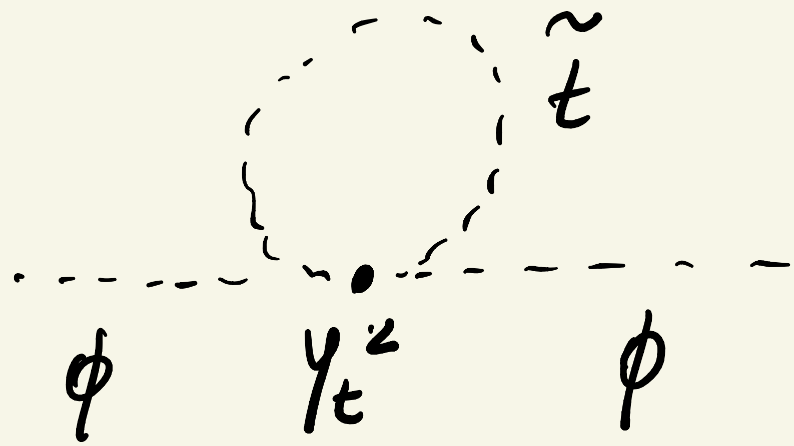

$$\mu^2 = \mu_0^2 + \frac{y_t^2}{16\pi^2}\left(\cancel{\Lambda^2} + m_t^2 \ln \frac{\Lambda}{m_t}\right)$$

2

$(100\ \text{GeV})^2$

$$- \frac{y_t^2}{16\pi^2}\left(\cancel{\Lambda^2} + m_{\tilde{t}}^2 \ln \frac{\Lambda}{m_{\tilde{t}}}\right)$$

$||$

generic of all SUSY loops to all orders

$$\Downarrow \qquad\qquad - \downarrow$$

$$\mu^2 \simeq \mu_0^2 - \frac{y_t^2}{16\pi^2}\left(m_{\tilde{t}}^2 - m_t^2\right)$$

$$m_{\tilde{t}} \leq \mathrm{TeV} \Rightarrow \mu \simeq 100\,\mathrm{GeV}$$

$$(\mu_0 \simeq 100\,\mathrm{GeV})$$

SUSY breaking?

$$\uparrow$$

$m_{\tilde t} \gg m_t$

Fine-Tuning (FT) in SM:

$$\mu^2 = \mu_0^2 + \frac{y_t^2}{16\pi^2}\Lambda^2 + \dots$$

100 GeV

$\Lambda \simeq M_{pl}$

$\mu \simeq m_h$

$\mu_0 \simeq 10^{18-19}$ GeV

μ_0 vs Λ

Cancellation = FT

- $m_{\tilde t} \longrightarrow$ makes ϕ tachyonic

$$+\mu^2 = \mu_0^2 - \frac{y_t^2}{16\pi^2}\left(m_{\tilde t}^2 - \cancel{m_t^2}\right)$$

<u>SM</u>

$$V = \frac{\lambda}{4}\left(\phi^\dagger\phi - v^2\right)^2$$

$$= \frac{\lambda}{4}\left(\phi^\dagger\phi\right)^2 + \frac{\mu^2}{2}\phi^\dagger\phi$$

$$\mu^2 = -\lambda v^2$$

$$\phi = \begin{pmatrix} 0 \\ h+v \end{pmatrix}$$

$$\boxed{m_h^2 = 2\lambda v^2}$$

low energy SUSY

$\Longleftrightarrow$ $m_{\tilde{t}}, \ldots < \mathrm{TeV}$

$\Downarrow$

$\tilde{S}\tilde{S}M$ (SUSY SM)

$\therefore$ 100 parameters =

masses and mixings

SM

$$m_W = g v$$

$$m_f = y_f v$$

$$m_h = \sqrt{\lambda} v$$

$v \ll M_{pe}$

Issue, problem

NO Problem!

but

FT = problem?

$\Downarrow$

SUSY ?

mass = ?

$$\boxed{E^2 = \bar{p}^2 + m^2}$$

$\boxed{\text{impact of } m \propto \frac{m}{E}}$

soft breaking of symmetry

$\Leftrightarrow$ mass breaks symmetry,

but interactions not

SUSY = softly broken

bottom line:

$$\mu^2 \cong \mu_0^2 - \frac{y_t^2}{16\pi^2} m_{\tilde{t}}^2$$

- $m_{\tilde{t}} = 10^{19}\,\mathrm{GeV} = M_{pl}$

  $\Rightarrow$ awful FT

- $m_{\tilde{t}} \simeq 10^{10}\,\mathrm{GeV}$ — " —

- $m_{\tilde{t}} = 10^4$ GeV $\longrightarrow$ (1-10%) FT