GUT Course 22/23

Lecture IX

22/11/2022

LHU

Fall 2022

Magnetic Monopoles (2)

$$SO(3) \text{ gauge } \xrightarrow{\langle\phi\rangle} U(1) = SO(2)$$

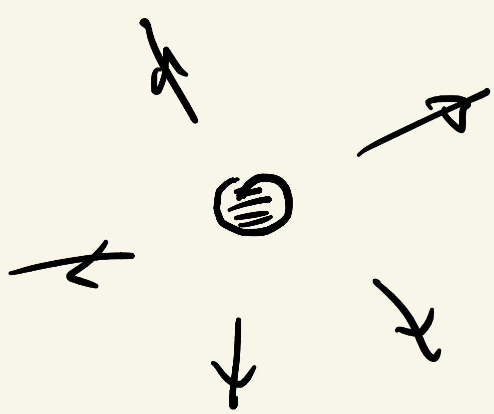
$\hookrightarrow$ em

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \qquad \boxed{g = e}$$

$$\Downarrow$$

$\exists$ static classical solution

$$\boxed{\vec{B} \xrightarrow[r\to\infty]{} \frac{1}{g}\,\frac{\hat{r}}{r^2}} \qquad g_m = \frac{4\pi}{g}$$



't Hooft
Polyakov '74?
(hedgehog)

$$\Uparrow$$

$$\vec{B}_m = \frac{g_m}{4\pi}\frac{\hat{r}}{r^2}$$

Monopole: $\quad \phi_a(\infty) = v\,\frac{x_a}{r}$

$$\downarrow$$

$$S_2 = \mathcal{M}_\infty \longrightarrow \mathcal{M}_0 = S_2$$

<u>n map</u> $\quad \phi_1^\infty = v \sin\theta \cos n\phi$

$$\phi_2^\infty = v \sin\theta \sin n\phi$$

$$\phi_3^\infty = v \cos\theta$$

$$\Rightarrow \vec{B}(\infty) = \frac{n}{g}\frac{\hat{r}}{r^2} \Rightarrow g_m = \frac{4\pi n}{g}$$

$$\Downarrow$$

$$g_m g = 4\pi n$$

$$SO(3) \qquad O = e^{i\Theta L}$$

$$[L_i, L_j] = i\,\varepsilon_{ijk} L_k$$

$$SU(2) \longleftarrow \text{spinor} \quad f = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$L_i \to T_i \qquad T_i = \tau_i/2$$

$$\boxed{Q_{em} = T_3}$$

$$q_f = \pm 1/2 \; (g)$$

$$g = e \iff \boxed{\begin{aligned} A_\mu &= A_\mu^3 \\ M_A &= 0 \end{aligned}} \qquad \phi_0 = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

$$\boxed{M_{A_1} = M_{A_2} = vg}$$

$$g_m\, g = 4\pi n$$

$$\Rightarrow \boxed{g_m\, q_e = 2\pi n}$$

<u>Dirac</u> 1948 ?

$\exists$ monopole

QM electron +

ψ_e = single-valued

$$\Rightarrow \boxed{g_m\, q_e = 2\pi n}$$

QED

$$\partial_\mu F^{\mu\nu} = j^\nu \qquad (k^\nu = 0)$$

$$\Rightarrow \boxed{\partial_\nu j^\nu = 0}$$

QED + monopole

$$\partial_\mu \tilde{F}^{\mu\nu} = k^\nu \quad \Rightarrow \boxed{\partial_\nu k^\nu = 0}$$

$$\tilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$$

$$\Rightarrow \quad k^0 = \partial_\mu \epsilon^{\mu 0 \alpha\beta} F_{\alpha\beta}$$

$$= \epsilon_{ijk} \partial_i F_{jk} = \partial_i B_i$$

$$Q_m = g_m = \int dV\, k^0 = \oint d\vec{S}\cdot\vec{B}$$

$$\Uparrow$$

<u>From the mapping point of view</u>

$$k^\mu = \frac{1\,(?)}{8\pi v^3}\,\epsilon^{\mu\nu\alpha\beta}\,\partial_\nu\phi_a\,\partial_\alpha\phi_b\,\partial_\beta\phi_c\,\varepsilon_{abc}$$

$$\partial_\mu k^\mu = \ldots\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu\partial_\nu \cdots + \cdots\right) = 0$$

$$\phi_a = \phi_a^\infty = v\,\frac{x^a}{r}$$

$$k^0 = \frac{1}{8\pi}\,\epsilon_{ijk}\,\partial_i\left(\frac{x_a}{r}\right)\partial_j\left(\frac{x_b}{r}\right)\partial_k\left(\frac{x_c}{r}\right)\varepsilon_{abc}$$

$$\Rightarrow \delta \pi k^0 = \varepsilon_{ijk}\left(\frac{\delta_{ia}}{r} - \frac{x_i x_a}{r^3}\right)\left(\frac{\delta_{jb}}{r} - \frac{x_j x_b}{r^3}\right)$$

$$\times \left(\frac{\delta_{kc}}{r} - \frac{x_k x_c}{r^3}\right) \varepsilon_{abc}$$

$$= \varepsilon_{abc}\, \varepsilon_{abc} \frac{1}{r^3} \quad +$$

$$+ \varepsilon_{ajc} \frac{x_j x_b}{r^5} \varepsilon_{abc} \quad (\,?\,)$$

Instead

$$\delta \pi k_i^0 = \varepsilon_{ijk}\, \partial_i \left(\frac{x_a}{r}\, \partial_j \frac{x_b}{r}\, \partial_k \frac{x_c}{r} \right) \varepsilon_{abc}$$

$$- \varepsilon_{ijk} \frac{x_a}{r}\, \partial_i \partial_j \cdots\cdots = 0$$

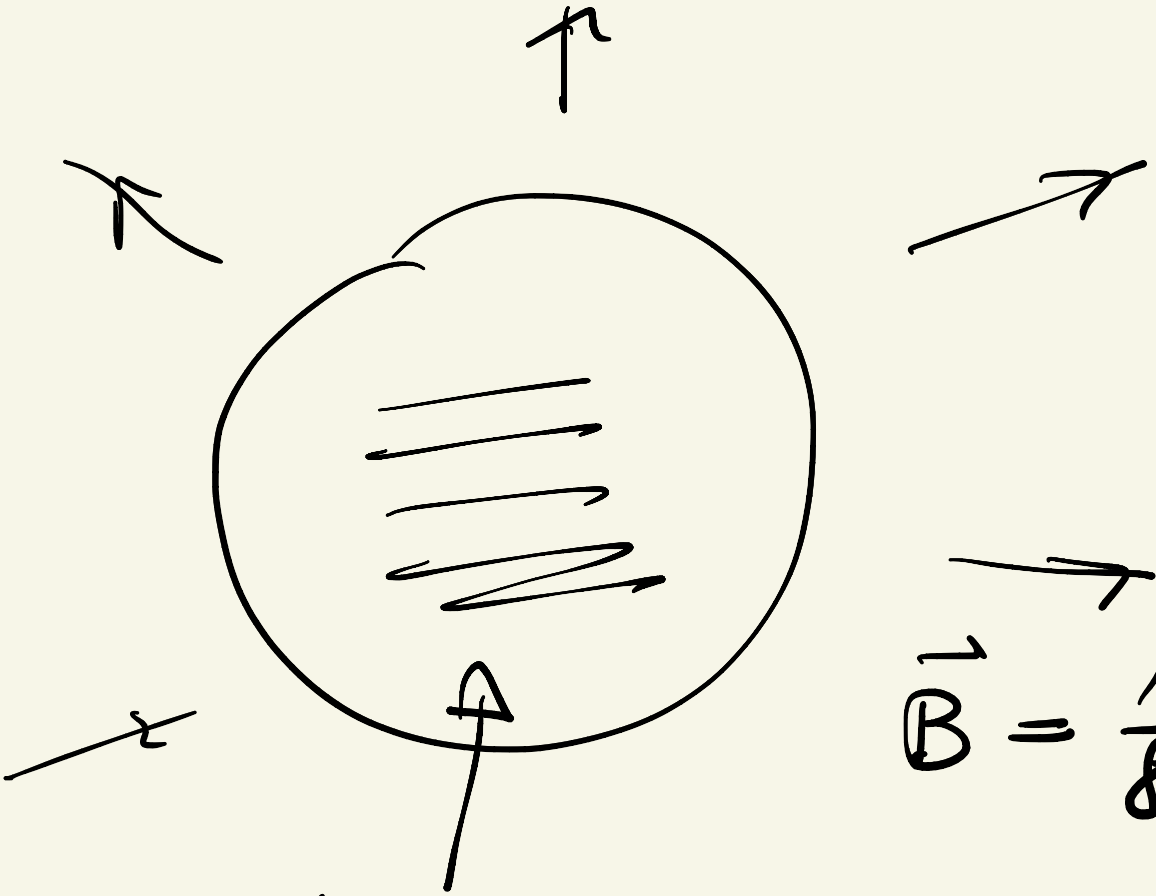
$$8\pi \int h^0 \, dV = 8\pi \int dS_i \, V_i$$

$$V_i = \varepsilon_{ijk} \, \varepsilon_{abc} \, \frac{x_a}{r} \, \partial_j\left(\frac{x_b}{r}\right) \partial_k\left(\frac{x_c}{r}\right)$$

$$= \varepsilon_{ijk} \, \varepsilon_{abc} \, \frac{x_a}{r} \left(\frac{\delta_{jb}}{r} - \frac{x_j x_b}{r^3} \right) \left(\frac{\delta_{kc}}{r} - \frac{x_k x_c}{r^3} \right)$$

$$= \underbrace{\varepsilon_{ibc} \, \varepsilon_{abc} \, \frac{x_a}{r^3}}_{} - \underbrace{\cancel{\varepsilon_{ibk} \, \varepsilon_{abc} \, \frac{x_a}{r} \, \frac{x_k x_c}{r^3}}}_{0}$$

$$= \frac{x_i}{r^3} \propto B_i$$



$$\vec{B} = \frac{1}{g}\,\frac{\hat{r}}{r^2}$$

$$\underbrace{\phi = 0}_{\text{local maximum}}$$

$$E = \int dV \left[\underbrace{\cancel{\tfrac{1}{2}|\nabla\phi|^2}}_{=\,0} + V(\phi) + \frac{1}{2}\vec{B}^2 \right]$$

$$\propto \int_0^\delta r^2 dr\, V(0) + \int_\delta^\infty r^2 dr\, \frac{1}{g^2}\frac{1}{r^4}$$

$$\propto \lambda v^4 \delta^3 + \frac{1}{g^2}\frac{1}{\delta}$$

$$\Downarrow$$

$$\frac{\partial E}{\partial \delta} \propto \lambda v^4 \delta^2 - \frac{1}{g^2}\frac{1}{\delta^2} = 0$$

$$\Downarrow \quad \lambda \simeq g^2$$

$$\boxed{\delta \simeq \frac{1}{g} v}$$

width of the monopole

Mass

$$M_m = E \propto \lambda \vartheta^4 \delta^3 \simeq$$

$$\propto g^2 \frac{1}{g^3} v$$

$$\Rightarrow \boxed{M_m \doteq \frac{1}{g} v}$$

$$\boxed{M_A = g v}$$

$$\Rightarrow \boxed{M_m = \frac{1}{g^2} M_A}$$

$$M_m \longrightarrow r_c(m) \simeq \frac{1}{M_m}$$

$$\Downarrow$$

$$\boxed{\begin{array}{l} r_c(m) \simeq g\, v^{-1} \\[2ex] \delta^m \simeq \dfrac{1}{g}\, v^{-1} \gg r_c^m \end{array}}$$

- $v > 10^3\ \mathrm{GeV}$

$$\delta \lesssim 10^{-3}\ \mathrm{GeV}^{-1} \simeq 10^{-17}\ \mathrm{cm}$$

- $v_{GUT} \simeq 10^{16}\ \mathrm{GeV}$

$$\Rightarrow \delta_{GUT} \simeq 10^{-30}\ \mathrm{cm}$$

SM: monopoles?

$$\mathcal{M}_0 = \{ \Phi_0 : \Phi_0^\dagger \Phi_0 = v^2 \}$$

$$= S_3$$

map?

$$\mathcal{M}_\infty = S_2$$

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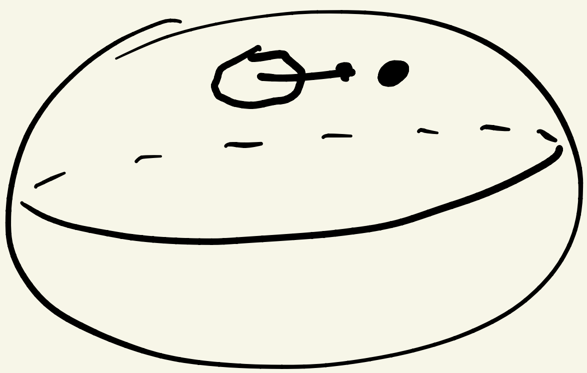
analogy: $S_1 \longrightarrow S_2$



Production of (GUT) monopoles

NO collider!

$\Downarrow$

early universe

$\Rightarrow$ $\boxed{T \gg v}$

- QFT at high T
- cosmology at high T

$T \gg v$

Kirznitch(?) '72

Weinberg '74

$\Downarrow$

$$V_T = V(0) + \underbrace{a\, T^2 \phi^2}_{\text{dim. grounds}} + T^4$$

$$a = \lambda + g^2 > 0$$

$\uparrow$

V = bound

$\Downarrow$

$$\langle \phi \rangle_T = 0 \quad (T \gg v)$$

$T \qquad \langle \phi \rangle = 0$

$\downarrow$

$\langle \phi \rangle \neq 0$

value of ϕ_0 = correlated

Kibble '68 (?)

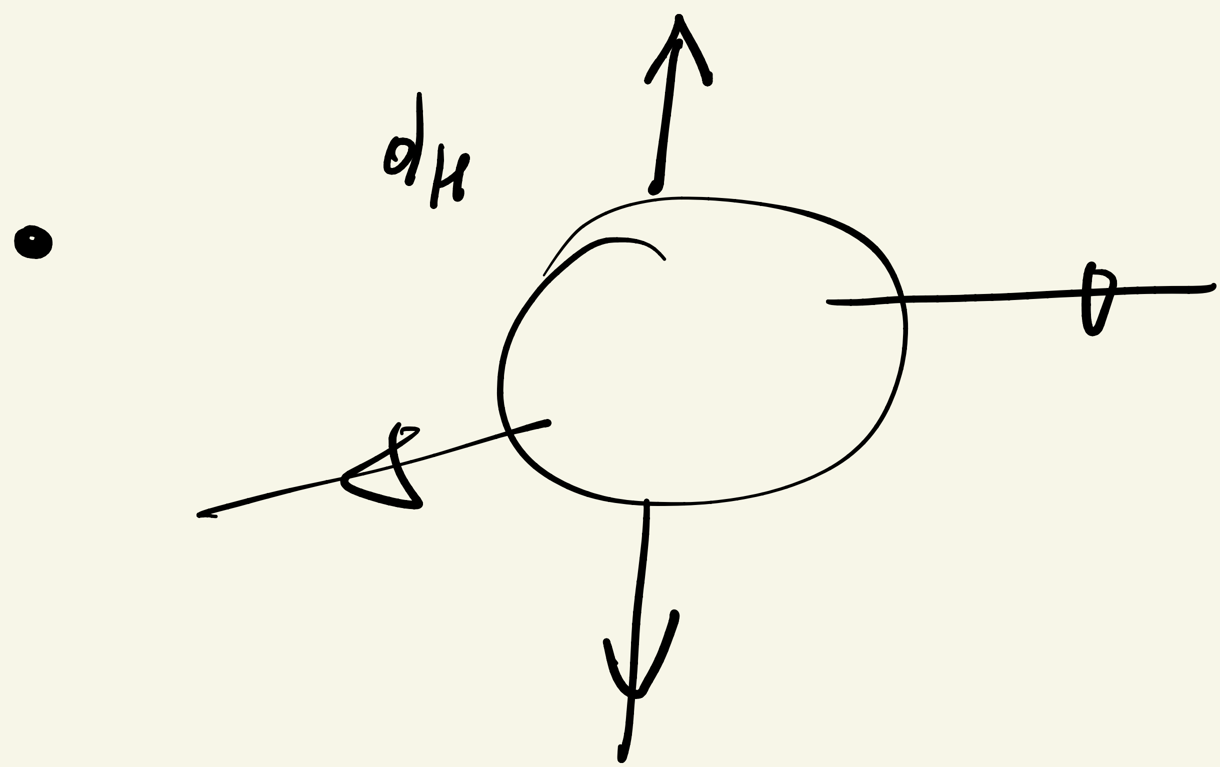
$$\boxed{\phi_0 = \text{fixed} \quad l \leq d_H}$$

if $R \gg d_H$

$+ \quad (d_H)$

• dw

$- \quad (d_H)$



$$\text{Kibble} \lesssim \frac{1 \text{ monopole}}{\text{horizon}} \left(\tfrac{1}{10}\right)$$

$$\boxed{\begin{array}{c}\text{UNIVERSE} \\ (\text{at high } T)\end{array}}$$

$$d_H = t = \frac{M_P}{T^2}$$

$$G_N = 1/M_p^2$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

$(\sim \rho)$

<u>at high T:</u> $\rho,\ p = \rho/3$

$$\rho \sim T^4$$

$$ds^2 = dt^2 - R^2(t)\, d\vec{x}^{\,2}$$

dim. of L

$$\Downarrow$$

$$\dot{R}^2/R^2 \cong G_N\, T^4 (\rho)$$

$$\dot{R}^2 = G_N R^2 \rho$$

$$\cancel{m}\frac{1}{2}\dot{R}^2 = G_N (V\rho)\frac{\cancel{m}}{R}$$

$$\underbrace{\cancel{m}\frac{1}{2}\dot{R}^2}_{KE} \qquad \underbrace{G_N (V\rho)\frac{\cancel{m}}{R}}_{PE}$$

$$\frac{1}{2}\dot{R}^2 = \frac{4\pi}{3}\rho\, G_N \, R^2/R$$

$$\left(\dot{R}/R\right)^2 = \frac{1}{M_p^2} T^4$$

$$H \equiv \dot{R}/R = T^2/M_p$$

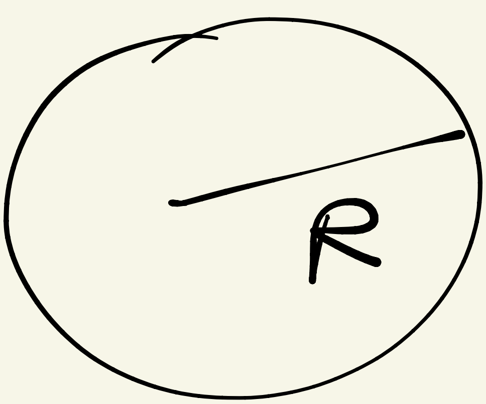
$$\boxed{t = 1/H = \frac{M_p}{T^2}}$$ big-bang

- $d_H(T) = \dfrac{M_p}{T^2}$ = horizon

- $\boxed{R(T) = \dfrac{C}{T} \quad (C = 10^{30})}$

$\Updownarrow$ adiabatic expansion

$$N = V \cdot n = R^3 \cdot T^3$$
$$\simeq 10^{90}$$
$$\Uparrow$$



S = conserved