GUT Course
2022/2023

Lecture IV

4/11/2022

LMU
Fall 2022

SM – more

Φ = doublet $\quad \Phi \to U\Phi$

$Y(\Phi) = 1$

$\Rightarrow \quad \langle \Phi \rangle = \Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$

$\Downarrow$

$M_W = \frac{g}{2} v \qquad M_Z = \frac{M_W}{\cos\theta_W}$

$$\boxed{M_A = 0}$$

Φ_0 : $\quad T_a \Phi_0 \neq 0$

$Y \Phi_0 \neq 0$

$$T_3\, \Phi_0 = -\frac{1}{2}\, \Phi_0$$

$$Y\, \Phi_0 = \Phi_0$$

$$\Downarrow$$

$$Q_{em} = T_3 + Y/2$$

$$\boxed{Q_{em}\, \Phi_0 = 0}$$

$$\boxed{\Leftrightarrow \quad M_A = 0}$$

$$A = \sin\theta_W \underset{SU(2)}{A_3} + \cos\theta_W \underset{U(1)}{B}$$

$$\sin\theta_W = \frac{e}{g}\,, \quad \cos\theta_W = \frac{e}{g'}$$

$$\left(\tan\theta_W \equiv g'/g\right)$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \frac{1}{e} A &= \frac{1}{g} A_3 + \frac{1}{g'} B \\ Q &= T_3 + \frac{Y}{2} \end{aligned}}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$e \qquad g \qquad g'$$

$\boxed{\text{to be generalized}}$

- $\Phi_0{}^{G} = \begin{pmatrix} 0 \\ v \end{pmatrix}$

- $\Phi_0{}^{M} = \begin{pmatrix} v \\ 0 \end{pmatrix}$ what then?

$$\left.\begin{aligned} T_3\, \Phi_0{}^{M} &= \frac{1}{2}\, \Phi_0{}^{M} \\ \frac{Y}{2}\, \Phi_0{}^{M} &= \frac{1}{2}\, \Phi_0{}^{M} \end{aligned}\right\} \Rightarrow Q_{em}\, \Phi_0{}^{M} \neq 0$$

?!?!?!?!

$$\Downarrow$$

$$Q_{em}^{M} = T_3 - \frac{Y}{2}$$

$$\boxed{Q_{em}{}^{M}\, \Phi_0{}^{M} = 0}$$

$$\Updownarrow$$

$$\boxed{Q_{em}^{G}\,\Phi_0^{G} = 0}$$

$$Q_{em}^{G} = T_3 + Y/2$$

- $\Phi_0^{J} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \Rightarrow \nexists$

Q_{em}^{J} $\therefore$ $Q_{em}^{J}\,\Phi_0^{J} = 0$

$$\boxed{Q_{em}^{J} = 0\ ??}$$

$\boxed{SU(N)}$ $[T_a, T_b] = i f_{abc} T_c$

$$T_a' = U T_a U^{\dagger}$$

$$[T_a', T_b'] = [U T_a U^\dagger, U T_b U^\dagger]$$

$$= U [T_a, T_b] U^\dagger = U i f_{abc} T_c U^\dagger$$

$$= i f_{abc} T_c'$$

$$\boxed{\begin{aligned} SM \Rightarrow \quad & Q_{em} \Phi_0 \neq 0 \\ & \mu_A = 0 \end{aligned}}$$

$$\Updownarrow$$

$$\boxed{\mu_A^{exp} \leq 10^{-16}\, eV}$$

$$\mathcal{L}_Y = y_e \bar{l}_L \bar{\Phi} e_R + y_d \bar{q}_L \bar{\Phi} d_R +$$

$$+ y_u \bar{q}_L i \sigma_2 \Phi^* u_R + h.c.$$

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow \quad Y_f \bar{f}_L f_R v + h.c. =$$

$$= Y_f v \bar{f} f$$

$$\Downarrow$$

$$\boxed{m_f = Y_f v} \longleftarrow$$

$$\boxed{M_W = g/2 \, v} \longleftarrow$$

the scale of weak int.

$$\Rightarrow \boxed{Y_f = \frac{m_f}{v} = \frac{g}{2} \frac{m_f}{M_W}}$$

• $\Phi \longrightarrow U(x)\Phi$ $\qquad [T_a, T_b] = i\varepsilon_{abc} T_c$

$$U(x) = e^{i\,\theta_a(x)\,T_a}$$

$$\Phi \xrightarrow[U]{} \underbrace{\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}}_{\text{Higgs boson}} = \Phi_{un}$$

Φ (4 real d.o.f.) $\to$ Φ_{un}
(1 d.o.f.)

$$\underbrace{A_i,\ B}_{\text{4 massless}} \longrightarrow \underbrace{W^+, W^-, Z}_{\text{3 massive}}\ ;\ \underbrace{A}_{M_A = 0}$$

$\Downarrow$

3 extra d.o.f.

$$\boxed{3+1=4}$$

it matches

$$\Downarrow$$

$$\mathcal{L}_{kin} = \frac{1}{2}(D_\mu \Phi_{un})^\dagger (D^\mu \Phi_{un})$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2} M_W^2 W^+ W^- \left(1+\frac{h}{v}\right)^2$$

$$+ \frac{1}{2} M_Z^2 Z Z \left(1+\frac{h}{v}\right)^2$$

$$- y_f v \bar{f} f \left(1+\frac{h}{v}\right) +$$

$$\Phi_{un} = \begin{pmatrix} 0 \\ v+h \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix}\left(1+\frac{h}{v}\right)$$

$$\boxed{\begin{aligned} M_W &= \tfrac{g}{2} v \\ m_f &= y_f v \end{aligned}}$$

$\Downarrow$ $\qquad v = \frac{2 M_W}{g}$

$\Downarrow$

h : couples to your mass

(if it gives you a mass)

$\Uparrow$

$$\frac{1}{2} M_W^2 W^+ W^- \left(1 + \frac{h}{v}\right)^2$$

$$= \frac{1}{2} M_W^2 W^+ W^- + M_W^2 W^+ W^- \frac{h}{v}$$

$\Downarrow$

$$2 \frac{g}{M_W} M_W^2 W^+ W^- h$$

$\Downarrow$

$$\boxed{2\, h\, m_W g\, W^+ W^-}$$ ✓

- fermions

$$\boxed{h\, y_f\, \bar{f} f = \frac{g}{2} \frac{m_f}{m_W} h\, \bar{f} f}$$

mass = dynamical

$$\Gamma(h \to f\bar{f}) \propto y_f^2\, m_h$$

$\Downarrow$

$$\boxed{\Gamma(h \to f\bar{f}) \propto g^2 \frac{m_f^2}{M_W^2} m_h}$$

today $\boxed{(m_h \simeq 125\ \text{GeV})}$

$$\boxed{\underbrace{W,\ Z,\ t,\ b,\ \tau}_{\text{Higgs origin of mass}}}$$

- $\left.\begin{array}{l} W^- \to e\bar{\nu} \\ Z \to e\bar{e}\ (\bar{f}f) \end{array}\right\}$ g

$$\frac{g}{\sqrt{2}} W^-_\mu \, \bar{e} \gamma^\mu L \nu$$

$$\frac{g}{\cos\theta_W} Z_\mu \, \bar{f} \left[T_3 L - Q \sin^2\theta_W \right] f$$

$$\Gamma(W \to e + \bar{\nu}) \propto g^2 f(m_e)$$

kinematics

$$\underbrace{\Phi_1, \quad \Phi_2}$$

$$\Phi_1^0 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2^0 = v_2 \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\boxed{\text{Q}_{\text{em}}\, \Phi_0 = 0 \quad \text{NOT a prediction}}$$

$$V = \dots + \underline{(\Phi_1^{+} \Phi_2)(\Phi_2^{+} \Phi_1)\,\lambda}$$

$$\therefore \quad \Phi_2^{0} = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1 + h_1 \end{pmatrix}$$

$$\Phi_2 = \begin{pmatrix} H^{+} \\ v_2 + h_2 + i\,G \end{pmatrix}$$

$$h = f(h_1, h_2)$$

$$H_0 \neq h$$

limit on its mass

<u>Higgs sector</u>

$$V = \frac{\lambda}{4}\left(\Phi^{+}\Phi - v^2\right)^2$$

$$\Phi_{u.} = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\Downarrow$$

$$V = \frac{\lambda}{4}\left(\cancel{v^2} + h^2 + 2vh - \cancel{v^2}\right)^2$$

$$= \frac{1}{2} \underbrace{2\lambda v^2}\, h^2 + \frac{\lambda}{4} h^4 + \lambda v h^3$$

$$\boxed{m_h^2 = 2\lambda v^2}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} m_h &= \sqrt{2\lambda}\, v \\ M_W &= \frac{g}{2} v \\ m_f &= y_f v \end{aligned}}$$

Higgs mass on same footing

$$\alpha_w = \frac{g^2}{4\pi}, \qquad \alpha_f = \frac{y_f^2}{4\pi},$$

$$\alpha_h = \frac{2\lambda}{4\pi}$$

$\therefore$

$$\boxed{\alpha_i \ll 1}$$

$\boxed{\text{perturbativity}}$

new fermions:

$$m_f \ll \text{TeV}$$

$$(m_h \ll \text{TeV})$$

cut-off $\Lambda \simeq M_{Pl}$
$(\simeq 10^{19}\ \text{GeV})$

$$\boxed{G_N \simeq \frac{1}{M_{pl}^2}}$$

$$\alpha_N = \alpha_G \simeq \frac{E^2}{M_{pl}^2} \ll 1$$

$$E \ll M_{pl}$$

- Higgs boson mass <u>not</u> predicted !?

<u>NO !</u>

$m_h =$ "predicted"

$M_W = \frac{g}{2} v$

$m_h^2 = 2\lambda v^2$

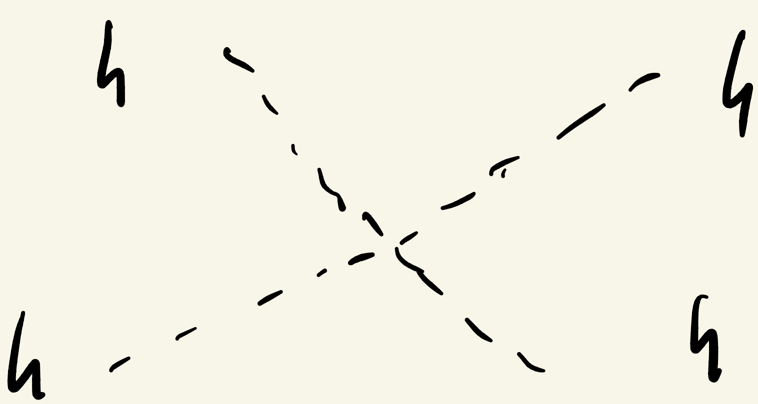
$$\boxed{m_h^2 = 8\lambda/g^2 \; M_W^2}$$

$(g \simeq 0.6)$

$(\lambda \sim 1/10\ ?)$

$\boxed{m_h \longleftrightarrow \lambda \text{ connection}}$

but: λh^4



$h + h \longrightarrow h + h$

$$\sigma(h+h) \propto \lambda^2$$

$$\boxed{\text{if you measure} \Rightarrow \text{predict } m_h!}$$

compare with W mass

$$\boxed{\Delta_{\mu\nu} \propto \frac{g_{\mu\nu}}{\cancel{k^2} - M_W^2} \quad (k \ll M_W)}$$

$$\mathcal{L}_{int} = g/\sqrt{2}\; \bar{u}_L \gamma^\mu d_L W_\mu^+ + h.c.$$

$$\to \frac{g^2}{2} \frac{1}{M_W^2} \bar{u}_L \gamma^\mu d_L \bar{\nu}_L \gamma_\mu e_L$$

$$(d \to u + e + \bar{\nu})$$

$$= \frac{g^2}{8 M_W^2} \bar{u} \gamma_\mu (1+\gamma_5) d \; \bar{\nu} \gamma^\mu (1+\gamma_5) e$$

$$\|$$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}}$$

measure

measure

"predict" M_W

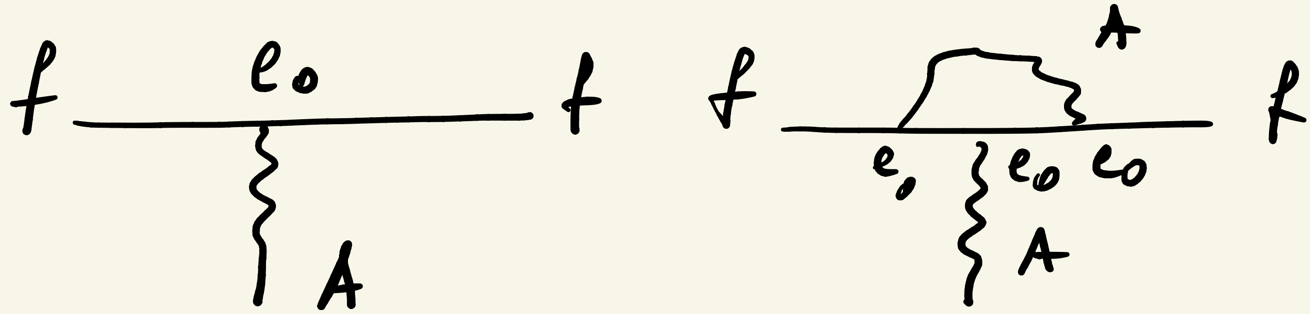
Hierarchy (Higgs mass) issue

$$V = \frac{\lambda}{4}\left(\Phi^\dagger\Phi - v^2\right)^2$$

$$= \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2 \underbrace{- \frac{\lambda}{2} v^2 \Phi^\dagger\Phi}_{-\frac{\mu^2}{2}\Phi^\dagger\Phi} + \ldots$$

Higgs mass term μ

$\sim v$



$$e = e_0\left(1 + \frac{\alpha_0}{\pi}\ln\frac{\Lambda}{m_f}\right)$$

$$\simeq e_0\left(1 + \frac{\alpha}{\pi}\ln\frac{\Lambda}{m_f}\right)$$

$$\updownarrow$$

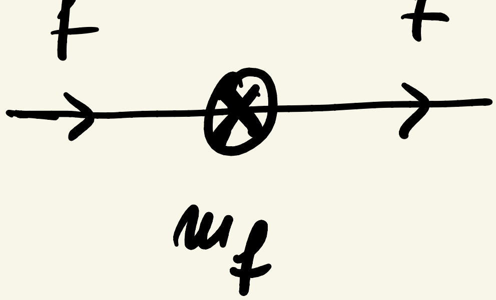
$$e_0 \simeq e\left(1 - \frac{\alpha}{\pi}\ln\frac{\Lambda}{m_f}\right)$$

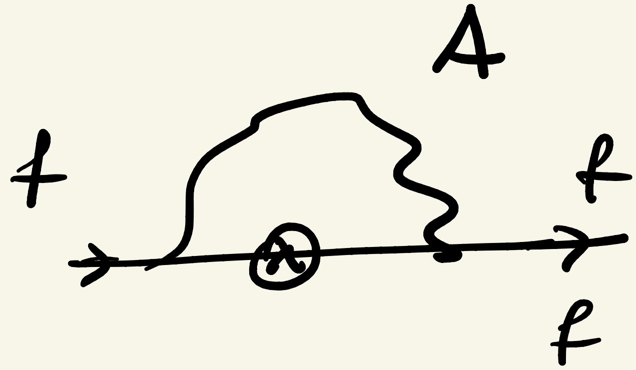
work with e

$$\text{Amplitudes} \propto e^{2u}\left(1+\cancel{\frac{E}{\Lambda}}\right)$$

zero,

$\Lambda \to \infty$



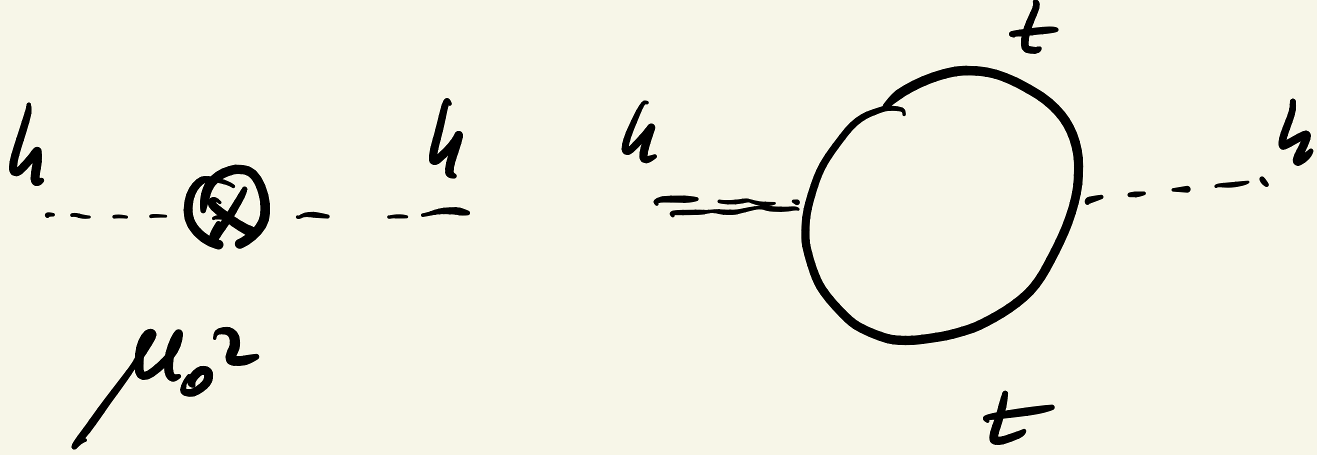


Weisskopf 1950

$$\boxed{m_f = m_f^{0}\Big[1+\underbrace{\frac{\alpha}{\pi}\ln\frac{\Lambda}{m_f}}_{\text{small}\ (<1)}\Big]}$$

Why ?

scalar



$$\mu^2 = \mu_0^2 + \frac{y_t^2}{16\pi^2}\Lambda^2 \simeq \frac{1}{2}\left(125\,\mathrm{GeV}\right)^2$$

hierarchy "problem"

$$\mu^2 = \lambda v^2 = \frac{1}{2} m_h^2$$