GUT and Neutrino

Lecture III

28/10/2022

LMU

Fall 2022

SM : The theory

$U(1) \longrightarrow SU(2)_L^{g} \times U(1)_Y^{g'}$

QED

ew gauge

1961

$i = 1,2,3$

$$D_\mu = \partial_\mu - ig\, T_i A_\mu^i - ig' \frac{Y}{2} B_\mu$$

$$T_{iL} = \frac{\sigma_i}{2} \qquad T_{iR} = 0$$

$$Q = T_3 + \frac{Y}{2}$$

$$Q \in C = \{ [T_\alpha, T_\beta] = 0 \}$$

$\Longleftrightarrow$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv q_L \qquad\qquad u_R, \; d_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv l_L \qquad\qquad e_R$$

$\Rightarrow$ $\mathcal{L}_{\text{weak}} (W) =$

$$= \frac{g}{\sqrt{2}} \left(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L \right) W^+_\mu$$

$$W^\pm_\mu = \frac{(A_1 \mp i A_2)_\mu}{\sqrt{2}}$$

$\Rightarrow$ If $\exists$ A_μ $\therefore$

$$\mathcal{L}_A = e \bar{f} \gamma^\mu f \, Q \, A_\mu$$

$$\Downarrow$$

$$\mathcal{L}_Z = \frac{g}{\cos\theta_W} \bar{f} \gamma^\mu \left[T_3 - Q \sin^2\theta_W \right] f Z_\mu$$

$$\tan\theta_W = g'/g$$

$$e = g \sin\theta_W$$

$$\longleftrightarrow \quad \theta_W^{exp} \simeq 30^\circ$$

$$\sin^2\theta_W^{exp} = 0.23$$

$$e^2/4\pi \equiv \alpha = \alpha_{em} \simeq 1/100$$

$$g^2/4\pi \equiv \alpha_2 = \alpha_W \simeq 1/25$$

renormalizable = good theory

$\Rightarrow$ finite amplitudes

$\Downarrow$

gauge symmetry

$\Downarrow$ Weinberg '67

Higgs mechanism in SM

('t Hooft '71-'73)

$\Rightarrow$ add a scalar multiplet ϕ to the theory :

$$\langle \phi \rangle \neq 0$$

$\Rightarrow \quad M_{W,Z} \propto \langle \phi \rangle$

Q. Why scalar $(s=0)$?

Why not $\langle \psi \rangle \neq 0$?

$\langle A_\mu \rangle \neq 0$?

$\langle e \rangle \neq 0$?

$\langle \nu \rangle \neq 0$?

break Lorentz!

but Lorentz = sacred

$\Downarrow$

scalar: <u>Lorentz</u>!

Task of Weinberg:

$$M_W \neq 0, \quad M_Z \neq 0$$

$$M_A = 0$$

$$m_e \neq 0, \quad m_\nu \neq 0$$

e: $$m_e \bar{e} e = m_e e^{+} \gamma_0 e$$

$$e = e_L + e_R \equiv L e + R e$$

$$L\,(R) = \frac{1 \pm \gamma_5}{2} \quad (\gamma_5^2 = 1)$$

$$\{\gamma_5, \gamma_\mu\} = 0$$

$$\Rightarrow [\gamma_5, \Sigma_{\mu\nu}] = 0$$

$$\Rightarrow L, R = \text{Lorentz invariant}$$

$$m_e \bar{e} e = m_e (e_L^+ + e_R^+) \gamma_0 (e_L + e_R)$$

$$= m_e \, e^+ (L + R) \gamma_0 (L + R) \, e$$

$$= m_e \, e^+ \gamma_0 R \, e + h.c.$$

$$= m_e (\bar{e}_L e_R + h.c.)$$

$$\Downarrow$$

mass term : $L \leftrightarrow R$

$$\boxed{\begin{array}{c}\bar{e}_L e_R \leftarrow \text{breaks } SU(2) \\ \text{FORBIDDEN}\end{array}}$$

$\Downarrow$ instead

$$\mathcal{L}_Y^{(e)} = y_e \bar{l}_L \bar{\Phi} e_R + h.c.$$

$$\Phi \to U \Phi$$

$$l_L \to U \Phi \qquad !. \; U^+ U = 1$$

$$Y = 2[Q - T_3]$$

$\Downarrow$

$$Y(e_R) = -2 \quad , \quad Y(l_L) = -1$$

$$\Rightarrow \boxed{Y(\Phi) = 1}$$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad Q = T_3 + Y/2$$

$$\Updownarrow$$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$\mathcal{L}_Y^{(q)} = y_d \, \bar{q}_L \, \Phi \, d_R \quad +$$

$$Y: \quad -1/3 \; +1 \; -2/3 = 0$$

$$+ \quad \bar{q}_L \quad \Phi^* u_R \qquad i\sigma_2$$

$$Y: \quad -\frac{1}{3} \; -1 + 4/3 = 0$$

$$q_L \to U q_L \,, \quad \Phi \to U \Phi \,, \quad u_R \to u_R$$

$$\bar{q}_L \, \Phi^* u_R \to \bar{q}_L \underbrace{U^+ U^*}_{\neq 1} \phi^* u_R$$

$$S = 0 : \quad | \uparrow\downarrow - \downarrow\uparrow \rangle$$

$$D = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \Leftrightarrow \Phi$$

$$\Phi_1 , \ \Phi_2 \Leftrightarrow$$

$$\Phi_1^T \epsilon \, \Phi_2 \quad \therefore \quad \epsilon^T = -\epsilon$$

$$\Downarrow$$

$$\boxed{\varepsilon = i\sigma_2} \quad \Leftarrow \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Phi_1^T \epsilon \, \Phi_2 \longrightarrow \Phi_1^T U^T \epsilon \, U \phi_2$$

$$= \Phi_1^T U^T i\sigma_2 \, U \phi_2$$

$$U = e^{i\theta_i \sigma_i/2}$$

$$(\sigma_1, \sigma_3)^T = (\sigma_1, \sigma_3)$$

$$\sigma_2^T = -\sigma_2$$

$$[\sigma_2, \sigma_2] = 0$$

$$\{\sigma_2, \sigma_{1,3}\} = 0$$

$$\Rightarrow \quad U^T i\sigma_2 U = e^{i\theta_i \sigma_i^T} i\sigma_2 e^{i\theta_i \frac{\sigma_i}{2}}$$

$$= i\sigma_2 \underbrace{e^{-i\theta_i \frac{\sigma_i}{2}} e^{i\theta_i \frac{\sigma_i}{2}}}_{1}$$

$$= i\sigma_2$$

$$\boxed{\Phi_1^T i\sigma_2 \Phi_2 = SU(2) \text{ INV}}$$

$$\Leftrightarrow \quad \boxed{\begin{aligned} \Phi &\to U\Phi \\ i\sigma_2 \Phi^* &\to U\, i\sigma_2 \Phi^* \end{aligned}}$$

$\boxed{\text{PROVE!}}$

$$\boxed{\begin{aligned} T_a^\dagger &= T_a \\ \Leftrightarrow \quad T_a^* &= T_a^T \end{aligned}}$$

$$\boxed{\text{SSB}}$$

$$\mathcal{L}_\Phi = \frac{1}{2}(D_\mu \Phi^\dagger)(D^\mu \Phi) - V(\Phi)$$

$$\boxed{V(\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger \Phi - v^2\right)^2}$$

Why only $\Phi^\dagger \Phi$?

why not: $\Phi^T i\sigma_2 \Phi$? $(=0)$

— " — $(\Phi^\dagger \sigma_i \Phi)(\Phi^\dagger \sigma_i \Phi)$?

— " — $(\Phi^T i\sigma_2 \sigma_i \Phi)(\Phi^T i\sigma_2 \sigma_i \Phi^*)$?

INV $\Leftrightarrow$ $f(\Phi)$:

$$f(\Phi) = f(U\Phi)$$

$$U\Phi = \begin{pmatrix} 0 \\ \phi \end{pmatrix} = \Phi'$$

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \Rightarrow U : \quad U\Phi$$

$$\Rightarrow \boxed{\text{ONE Inv.}} = f(\phi)$$

$$\boxed{f(\Phi) = \Phi^\dagger \Phi}$$

- $\mathcal{M}_0$ = vacuum manifold

$$= \{ \Phi_0 : V(\Phi^0) = V_{min} = 0 \}$$

$$= \{ \Phi_0^\dagger \Phi_0 = v^2 \}$$

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \phi_i \in C$$

$$= \begin{pmatrix} R_1 + i R_2 \\ R_3 + i R_4 \end{pmatrix}$$

$$\Rightarrow \quad \Phi^{+}\Phi = |\phi_1|^2 + |\phi_2|^2$$

$$= \sum_{i=1}^{4} R_i^{\,2}$$

$$\Rightarrow \quad \underbrace{\Phi_0^{+}\Phi_0^{\,2} = \sum_{i=1}^{4} R_{i0}^{\,2} = v^2}_{S_3}$$

$$\Downarrow$$

$$\Phi_0^{G} = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v \in R$$

$$\Downarrow \qquad \Phi \to \Phi_0$$

$$\bullet\ \mathcal{L}_Y \to y_e \bar{l}_L \begin{pmatrix} 0 \\ v \end{pmatrix} e_R +$$

$$+ y_d \bar{q}_L \begin{pmatrix} 0 \\ v \end{pmatrix} d_R +$$

$$+ y_u \bar{q}_L \begin{pmatrix} v \\ 0 \end{pmatrix} u_R + h.c$$

$$\Rightarrow \boxed{\begin{aligned} m_e &= y_e v \\ m_d &= y_d v \\ m_u &= y_u v \end{aligned}}$$

$$\bullet\ \frac{1}{2}(D_\mu \Phi_0)^\dagger (D^\mu \Phi^0) \qquad Y(\Phi_0) = 1$$

$$D^\mu \phi_0 = \left(-ig\, T_i A_\mu^i - ig' \frac{B_\mu}{2}\right)\Phi_0$$

a) charged gen. (A) (σ_1, σ_2)

$$D_\mu \Phi_0 \to -i \frac{g}{2} \begin{pmatrix} 0 & A_1 - iA_2 \\ A_1 + iA_2 & 0 \end{pmatrix}_\mu \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= -i \frac{g}{2} (A_1 - iA_2)_\mu \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\Downarrow$$

$$(D_\mu \Phi_0)^+ D^\mu \Phi_0 = \frac{g^2}{4} v^2 (A_1^2 + A_2^2)$$

$$W^\pm = \frac{A_1 \mp iA_2}{\sqrt{2}}$$

$$\boxed{M_W = \frac{g}{2} v}$$

b) neutral gen. $(\sigma_3,\ Y=1)$

$$D_\mu \Phi_0 \rightarrow -i/2 \begin{pmatrix} gA_3 + g'B & 0 \\ 0 & -gA_3 + g'B \end{pmatrix} \Phi_0$$

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow |D_\mu \Phi_0|^2 = \frac{1}{4} v^2 \left(-gA_3 + g'B\right)^2$$

$$= \frac{1}{4} v^2 \left(\frac{gA_3 - g'B}{\sqrt{g^2+g'^2}} \right)^2 (g^2 + g'^2)$$

$$\tan\theta_W \equiv g'/g \qquad \underset{Z}{\parallel} \ , \quad \boxed{m_Z^2 = \frac{(g^2+g'^2)}{4} v^2}$$

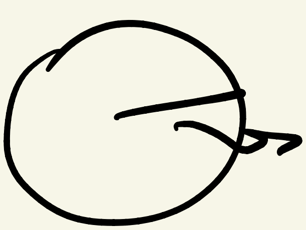
$$\Rightarrow \quad Z = \cos\theta_W A_3 - \sin\theta_W B$$

① $$\boxed{\begin{aligned} A &= \sin\theta_W A_3 + \cos\theta_W B \\ \therefore \ m_A &= 0 \end{aligned}}$$

② $$\boxed{M_Z \cos\theta_W = M_W} \qquad (1)$$

$\boxed{\text{CDF}}$ & (D0)

Tevatron ($E \simeq$ TeV)

 7km

(*) deviation from (1)

Q1. why $m_A = 0$?

Q2. $\Phi_0{}^{\mu} = \begin{pmatrix} v \\ 0 \end{pmatrix} \rightarrow$

what happens?