

GUT course 22/23

Lecture 15

25/10/2022

LMU

Fall 2022

From SU(2) to SM

SU(2) gauge theory

= ew theory?

⇓

1. change quantization → WRONG

2. $g = e$

⇓ ↗

weak (g_w)

em coupling

3. NO way of making $em = P,$

$$\text{weak} = \cancel{P}$$



Glashow '61

$$\boxed{SU(2)_L \times U(1)_Y}$$

g g'



$$\boxed{T_a = \frac{\sigma_a}{2}}$$

1. NO charge quantization
2. lose unif. $g \neq e$

but

3. prediction of NC (Z boson)
→ exp. with I pemeeter:

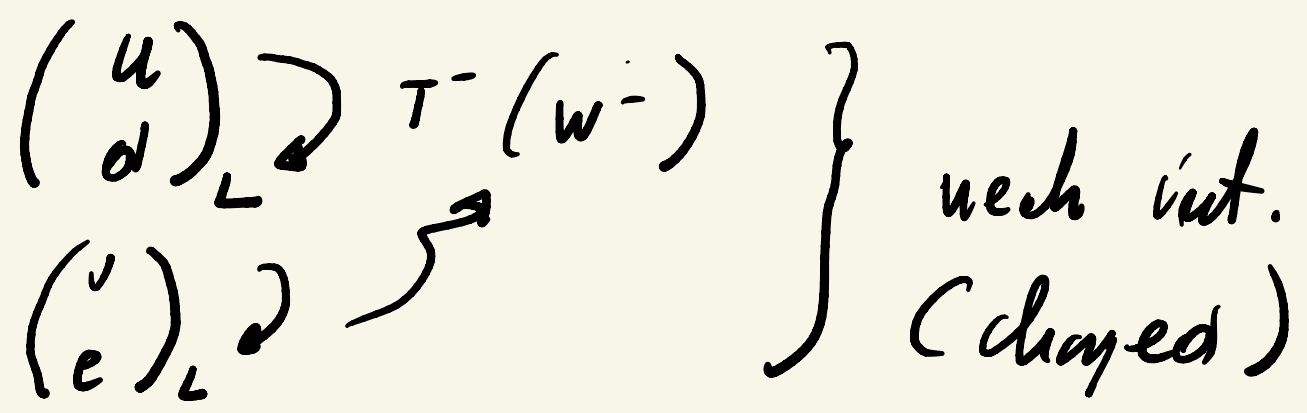
$$\tan \theta_w = g' / g$$

4. Higgs mechanism (Weinberg '67)

→ origin of all(?) masses

(observed elementary particles)

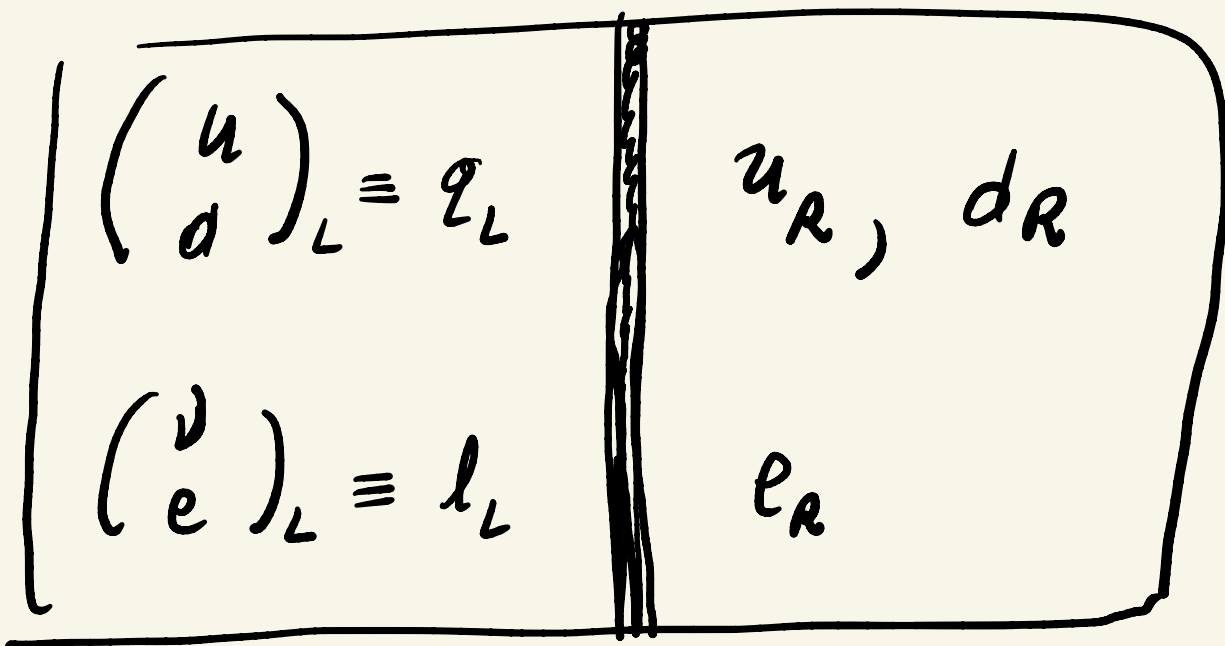
Matter (fermions)



f_R — have no charged
weak int.



f_R = Singlets under $SU(2)_L$



$$Q_{em} = T_3 + \rho \frac{y'}{2}$$

$$Q_{em} = T_3 + \frac{y}{2}$$



$$\Rightarrow y = 2(Q_{em} - T_3) \Leftarrow$$

$$y u_R = 2 \cdot \frac{2}{3} = \frac{4}{3} \quad (t_3 = 0)$$

$$y d_R = 2 \left(-\frac{1}{3}\right) = -\frac{2}{3}$$

$$y u_L = 2 \left(\frac{2}{3} - \frac{1}{2}\right) = \frac{1}{3}$$

$$y d_L = 2 \left(-\frac{1}{3} - \left(-\frac{1}{2}\right)\right) = \frac{1}{3}$$

$$G_{SM} = SU(2)_L \times U(1)_Y$$

$$T_a \quad Y$$
$$a = 1, 2, 3$$

$$[T_a, Y] = 0$$

$$[T_a, T_b] = i \epsilon_{abc} T_c$$



$$Y u_L = Y d_L, \quad Y \nu_L = Y e_L$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ig T_a A_\mu^a - ig' \frac{Y}{2} B_\mu$$



$$a = 1, 2, 3$$

$$\begin{aligned}
\mathcal{L}_D^{\text{kin}} &\rightarrow i \bar{\psi}_L \partial^\mu D_\mu \psi_L = \\
&= i \bar{\psi}_L \partial^\mu \partial_\mu \psi_L + g \bar{\psi}_L \partial^\mu T_a A_\mu^a \psi_L \cdot \\
&\quad + g' \frac{1}{2} \bar{\psi}_L \partial^\mu Y B_\mu \psi_L \cdot
\end{aligned}$$

$$T_a = \frac{\sigma_a}{2}$$

- c. c. (charged current) (T_1, T_2)
(c. c.)

$$\rightarrow g' \frac{1}{2} (\bar{u} \bar{d})_L \gamma^\mu \begin{pmatrix} 0 & A_1 - i A_2 \\ A_1 + i A_2 & 0 \end{pmatrix}_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$= g' \frac{1}{2} \bar{u}_L \gamma^\mu d_L (A_1 - i A_2)_\mu + \text{h.c.}$$

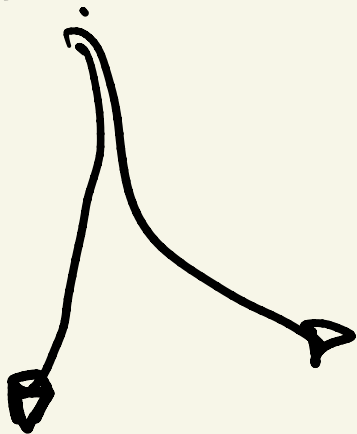
$$= g/\sqrt{2} \bar{u}_L \gamma^\mu d_L \left(\frac{A_1 - i A_2}{\sqrt{2}} \right)_\mu + h.c.$$

$$W_\mu^\pm = \frac{(A_1 \mp i A_2)_\mu}{\sqrt{2}}$$

• n.c. (T_3, Y) neutral current
(N.C.)

$$\rightarrow \bar{f} \gamma^\mu (g T_3 A_3 + g' \frac{1}{2} Y B_\mu) f$$

$$\uparrow Y = 2(Q - T_3)$$



distinguishes L from R

$$\left| T_3 f_L = \frac{\sigma_3}{2} f_L \right.$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\boxed{T_3 f_R = 0}$$



$$U f_L = e^{i \sigma_{ab} \theta_a} f_L$$

$$U f_R = f_R = e^0 f_R$$

⇒ N.C. →

$$\bar{f} (g T_3 A_3 + g' (Q - T_3) B)_\mu \delta^\mu f$$

$$= \bar{f} \left[\underbrace{(g A_3 - g' B)_\mu}_{\text{Z}} T_3 + g' Q B_\mu \right] \delta^\mu f$$

Z ⊗

Q ≡ Q_{em} ?

\exists photon A_μ that couples to
 $Q_{em} (Q_A)$



$\exists Z_\mu$ that couples to Q_2

$$Q_2 = T_3 - \sin^2 \theta_w Q_{em}$$

Task:

$$A = f(A_3, B)$$

$$Z = f'(A_3, B)$$

$$A \perp Z$$

$$Z = \frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}}$$



$$A = \frac{g' A_3 + g B}{\sqrt{g^2 + g'^2}}$$

$$\tan \theta_w = g'/g$$



$$A = \sin \theta_w A_3 + \cos \theta_w B$$

$$z = \cos \theta_w A_3 - \sin \theta_w B$$



$$c_w \equiv \cos \theta_w$$

$$s_w \equiv \sin \theta_w$$

$$\begin{pmatrix} A \\ z \end{pmatrix} = O \begin{pmatrix} A_3 \\ B \end{pmatrix}$$

$$O = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix}$$



$$\begin{pmatrix} A_3 \\ B \end{pmatrix} = O^T \begin{pmatrix} A \\ z \end{pmatrix}$$



$$A_3 = \sin\theta_w A + \cos\theta_w Z$$

$$B = \cos\theta_w A - \sin\theta_w Z$$



$$N.C. = \bar{f} \left[(g A_3 - g' B)_\mu T_3 + Q B_\mu \right] \delta^4 f$$

$$= \bar{f} \left(Z_\mu \sqrt{g^2 + g'^2} T_3 g + Q g' (c_w A - s_w Z)_\mu \right) \delta^4 f$$

$$= \bar{f} \left[g' c_w Q c_w A_\mu + \left(\frac{g}{c_w} T_3 - g' s_w Q \right) Z_\mu \right] \delta^4 f$$

$g' c_w \equiv e$	$= g s_w$
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$$= \bar{P} \partial^\mu \left[\underbrace{e Q_{em} A_\mu}_{em} + \frac{g}{\cos \theta_w} \underbrace{(T_3 - \sin^2 \theta_w Q)}_Z \right]$$

$$A_\mu : e, \quad Q_{em} \equiv Q_A$$

$$Z_\mu : \frac{g}{\cos \theta_w}, \quad Q_Z = T_3 - \sin^2 \theta_w Q_{em}$$

$$\theta_w \approx 30^\circ$$

$$\sin^2 \theta_w \approx 0.23$$

$$Q_2 = T_3 - Q \sin^2 \theta_w$$

$$Q_2' = T_3 L - \text{---} - \text{---}$$

$$L = \frac{1 + \gamma_5}{2}$$

$$R = \frac{1 - \gamma_5}{2}$$

$$f_L \equiv L f = \frac{1 + \gamma_5}{2} f$$

$$f_R = R f = \frac{1 - \gamma_5}{2} f$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$L^2 = L$$

$$R^2 = R$$

$$LR = 0$$

$$f = e = \begin{pmatrix} u \\ \nu \end{pmatrix} \Rightarrow$$

$$f_L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} f = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$f_R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} f = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$T_3 f_L = \frac{\sigma_3}{2} f_L, \quad T_3 f_R = 0$$

$$T_3 L f_L = T_3 f_L = \frac{\sigma_3}{2} f_L, \quad T_3 L f_R = 0$$



$$T_3 \iff T_3 L$$



$$Q_2 = Q'_2$$

$$f \equiv \psi \equiv \psi_f \quad \therefore$$

$$\psi \rightarrow \Lambda \psi$$

$$\Lambda = e^{i\theta_{\mu\nu} \Sigma^{\mu\nu}} \quad (\theta_{\mu\nu} = -\theta_{\nu\mu})$$

$$\Sigma^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu] \quad (\text{Lorentz})$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$P_L = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow$$

$$u_L \rightarrow e^{i \vec{\sigma}/2 \cdot (\vec{\theta} + i \vec{x})} u_L$$

↑
↑
 ROT BOOST

$$x_i = \theta_{0i}$$

$$\theta_{ij} = \epsilon_{iju} \theta_u$$

$$u_R \rightarrow e^{i \vec{\sigma}/2 \cdot (\vec{\theta} - i \vec{x})} u_R$$

$$f_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$\bar{\psi} = \psi^\dagger \gamma^0$$

$$\psi \rightarrow \Lambda \psi$$

$$\psi^c \rightarrow \Lambda \psi^c = C \bar{\psi}^T$$

$$\boxed{\psi^c = C \gamma_0 \psi^*}$$

$$C = i \gamma_2 \gamma_0$$

$$\psi^c = i \gamma_2 \psi^*$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\Rightarrow \psi^c = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \psi^*$$



$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \rightarrow \psi^c = \begin{pmatrix} 0 \\ -i\sigma_2 u_L^* \end{pmatrix}$$

$$\Rightarrow \boxed{(\psi_L)^c = (\psi^c)_R}$$

\Downarrow

$$\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$u_R, d_R$$

OR

$$\boxed{\begin{matrix} \begin{pmatrix} u \\ d \end{pmatrix}_L & & (u^c)_L, (d^c)_L \\ & \downarrow & \downarrow \\ & \propto u_R^* & \propto d_R^* \end{matrix}}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \leftarrow \text{OK}$$

~~$$\begin{pmatrix} u_L \\ d_R \end{pmatrix} \leftarrow \text{OK?}$$~~

break Lorentz