GUT Course 22/23

Lecture II

25/10/2022

LMU

Fall 2022

From SU(2) to SM

SU(2) gauge theory

= ew theory ?

$\Downarrow$

1. charge quantization $\rightarrow$ WRONG
2. $g = e$

$\Downarrow$ weak (g_w)

em coupling

---

3. NO way of making em = P,

weak = $\cancel{P}$

$\downarrow$

Glashow '61

$$\boxed{\underset{g}{SU(2)_L} \times \underset{g'}{U_Y(1)}}$$

$$\Downarrow \qquad \boxed{T_a = \frac{\sigma_a}{2}}$$

1. NO charge quantization

2. lose unif. $g \neq e$

---

but

3. prediction of NC (Z boson)

$\longrightarrow$ exp. with 1 parameter:

$$\boxed{\tan\theta_W = g'/g}$$

4. Higgs mechanism (Weinberg '67)

→ origin of all(?) masses

(observed elementary particles)

Matter (fermions)

$$\left.\begin{array}{l} \begin{pmatrix} u \\ d \end{pmatrix}_L \quad T^- (W^-) \\ \begin{pmatrix} \nu \\ e \end{pmatrix}_L \end{array}\right\}$$ weak int. (charged)

f_R — have no charged weak int.

$\Downarrow$

f_R = singlets under $SU(2)_L$

$\Downarrow$

$\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv q_L$	u_R, d_R
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv l_L$	e_R

$\Downarrow$

$$Q_{em} = T_3 + c\,\frac{Y'}{2}$$

$$\boxed{Q_{em} = T_3 + \frac{Y}{2}}$$

$$\Downarrow$$

$$\Rightarrow \boxed{Y = 2\,(Q_{em} - T_3)} \Leftarrow$$

$$Y\,u_R = 2 \cdot 2/3 = 4/3 \qquad (t_3 = 0)$$

$$Y\,d_R = 2\,(-1/3) = -2/3$$

$$Y\,u_L = 2\left(2/3 - 1/2\right) = 1/3$$

$$Y\,d_L = 2\left(-1/3 - (-1/2)\right) = 1/3$$

$$G_{SM} = SU(2)_L \times U(1)$$

$$T_a \qquad\qquad Y$$

$$a = 1,2,3$$

$$[T_a, Y] = 0$$

$$[T_a, T_b] = i\,\varepsilon_{abc} T_c$$

$$\boxed{Y u_L = Y d_L \;, \quad Y \nu_L = Y e_L}$$

$$\boxed{\partial_\mu \to D_\mu = \partial_\mu - i g\, T_a A_\mu^a - i g' \frac{Y}{2} B_\mu}$$

$$a = 1,2,3$$

$$\Downarrow$$

$$\mathcal{L}_D^{kin} \longrightarrow i\, \bar{q}_L \gamma^\mu D_\mu q_L =$$

$$= i\bar{q}_L \gamma^\mu \partial_\mu q_L + g\, \bar{q}_L \gamma^\mu T_a A_\mu^a q_L .$$

$$+ g'/2\, \bar{q}_L \gamma^\mu Y B_\mu q_L .$$

$$\boxed{T_a = \frac{\sigma_a}{2}}$$

- c. c. (charged current) (T_1, T_2)

(c. c.)

$$\longrightarrow g/2 (\bar{u}\,\bar{d})_L \gamma^\mu \begin{pmatrix} 0 & A_1 - i A_2 \\ A_1 + i A_2 & 0 \end{pmatrix}_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$= g/2\, \bar{u}_L \gamma^\mu d_L \; (A_1 - i A_2)_\mu + h.c.$$

$$= g/\sqrt{2} \quad \bar{u}_L \gamma^\mu d_L \left(\frac{A_1 - i A_2}{\sqrt{2}}\right)_\mu + h.c.$$

$$W^\pm_\mu = \left(\frac{A_1 \mp i A_2}{\sqrt{2}}\right)_\mu$$

• n.c. (T_3, Y) <u>neutral current</u> (N.C.)

$$\rightarrow \bar{f}\gamma^\mu \left(g T_3 A_3 + g'/_2 \, Y B_\mu\right) f$$

$Y = 2(Q - T_3)$

distinguishes L from R

$$\boxed{T_3 f_L = \frac{\sigma_3}{2} f_L} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$T_3 f_R = 0$$

$$\Updownarrow$$

$$U f_L = e^{i \sigma_a/2\, \theta_a} f_L$$

$$U f_R = f_R = e^0 f_R$$

$\Rightarrow$ N. C. $\longrightarrow$

$$\bar{f}\left(g T_3 A_3 + g'(Q - T_3) B\right)_\mu \gamma^\mu f$$

$$= \bar{f}\left[\underbrace{(g A_3 - g' B)_\mu}_{Z \quad \textcircled{\text{¥}}} T_3 + g' Q B_\mu\right]\gamma^\mu f$$

$Q \equiv Q_{em}$;

$\exists$ photon A_μ that couples to Q_{em} (Q_A)

$\Downarrow$

$\exists$ Z_μ that couples to Q_Z

$$\boxed{Q_Z = T_3 - \sin^2\theta_W \, Q_{em}}$$

Task:

$$\begin{aligned} A &= f(A_3, B) \\ Z &= f'(A_3, B) \end{aligned}$$

$$\boxed{A \perp Z}$$

$$\Downarrow$$

$$Z = \frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}} \qquad (*)$$

$$\Downarrow$$

$$A = \frac{g' A_3 + g B}{\sqrt{g^2 + g'^2}}$$

$$\tan \theta_W = g'/g$$

$$\Downarrow$$

$$A = \sin\theta_W\, A_3 + \cos\theta_W\, B$$

$$Z = \cos\theta_W\, A_3 - \sin\theta_W\, B$$

$$\Downarrow$$

$c_W \equiv \cos\theta_W$

$s_W \equiv \sin\theta_W$

$$\begin{pmatrix} A \\ Z \end{pmatrix} = O \begin{pmatrix} A_3 \\ B \end{pmatrix}$$

$$O = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} A_3 \\ B \end{pmatrix} = O^T \begin{pmatrix} A \\ Z \end{pmatrix}$$

$$\Downarrow$$

$$A_3 = \sin\theta_W \, A + \cos\theta_W \, Z$$

$$B = \cos\theta_W \, A - \sin\theta_W \, Z$$

$$\Downarrow$$

$$N.C. = \bar{f}\left[\left(g A_3 - g' B\right)_\mu T_3 + Q B_\mu\right]\partial^\mu f$$

$$= \bar{f}\left(Z_\mu \sqrt{g^2+g'^2}\, T_3 g + Q g'\left(c_W A - s_W Z\right)_\mu\right)\partial^\mu f$$

$$= \bar{f}\left[g' c_W Q_{em} A_\mu + \left(\frac{g}{\cos\theta_W} T_3 - g' s_W Q\right) Z_\mu\right]\partial^\mu f$$

$$\boxed{g' c_W \equiv e = g \sin\theta_W}$$

$$= \bar{f}\gamma^\mu \left[\underbrace{e\, Q_{em} A_\mu}_{em} + \frac{g}{\cos\theta_W} \underbrace{\left(T_3 - \sin^2\theta_W\, Q\right) Z_\mu}_{Z} \right] f$$

A_μ : e, $Q_{em} \equiv Q_A$

Z_μ : $\dfrac{g}{\cos\theta_W}$, $Q_Z = T_3 - \sin^2\theta_W\, Q_{em}$

$$\boxed{\begin{aligned} \theta_W &\simeq 30^\circ \\ \sin^2\theta_W &\simeq 0.23 \end{aligned}}$$

$$\boxed{\begin{aligned} Q_Z &= T_3 - Q \sin^2\theta_W \\ && \parallel \\ Q_Z' &= T_3 L - \quad -\text{"}- \end{aligned}}$$

$$L = \frac{1+\gamma_5}{2} \qquad R = \frac{1-\gamma_5}{2}$$

$$f_L \equiv L f = \frac{1+\gamma_5}{2} f$$

$$f_R = R f = \frac{1-\gamma_5}{2} f$$

$$\boxed{\begin{aligned} L^2 &= L \\ R^2 &= R \\ LR &= 0 \end{aligned}}$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\cdot\ f = e = \begin{pmatrix} u \\ v \end{pmatrix} \Rightarrow$$

$$f_L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} f = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$f_R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} f = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$T_3 f_L = \frac{\sigma_3}{2} f_L \quad , \quad T_3 f_R = 0$$

$$T_3 L f_L = T_3 f_L = \frac{\sigma_3}{2} f_L \quad , \quad T_3 L f_R = 0$$

$$\Downarrow$$

$$\boxed{T_3 \Longleftrightarrow T_3 L}$$

$$\Downarrow$$

$$\boxed{Q_z = Q'_z}$$

$$f \equiv \psi \equiv \psi_f \quad \therefore$$

$$\psi \to \Lambda \psi$$

$$\Lambda = e^{i\theta_{\mu\nu} \Sigma^{\mu\nu}} \quad (\theta_{\mu\nu} = -\theta_{\nu\mu})$$

$$\Sigma^{\mu\nu} = \frac{1}{4i} [\gamma^\mu, \gamma^\nu] \quad (\text{Lorentz})$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$f_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \Rightarrow$$

$$u_L \to e^{i\,\vec{\sigma}/2\cdot(\underbrace{\vec{\theta}}_{\text{ROT}} + i\underbrace{\vec{\chi}}_{\text{BOOST}})}\, u_L$$

$$\chi_i = \theta_{0i}$$

$$\theta_{ij} = \varepsilon_{ijk}\,\theta_k$$

$$u_R \to e^{i\,\vec{\sigma}/2\cdot(\vec{\theta} - i\vec{\chi})}\, u_R$$

$$\psi_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$\bar{\psi} = \psi^\dagger \gamma^0$$

$$\psi \to \Lambda\,\psi$$

$$\psi^c \to \Lambda\,\psi^c = C\,\bar{\psi}^T$$

$$\boxed{\psi^c = C\gamma_0\psi^*}$$

$$C = i\gamma_2\gamma_0$$

$$\psi^c = i\gamma_2\psi^*$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\Rightarrow \quad \psi^c = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}\psi^*$$

$$\Downarrow$$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \rightarrow \psi^c = \begin{pmatrix} 0 \\ -i\sigma_2 u_L^* \end{pmatrix}$$

$$\Rightarrow \boxed{(\psi_L)^c = (\psi^c)_R}$$

$$\Downarrow$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R,\ d_R$$

OR

$$\boxed{\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad (u^c)_L,\ (d^c)_L}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\propto u_R^*, \qquad \propto d_R^*$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \longleftarrow \quad \text{OK}$$

$$\cancel{\begin{pmatrix} u_L \\ d_R \end{pmatrix} \longleftarrow \quad \text{Ok ?}}$$

$$\boxed{\text{break Lorentz}}$$