# GUT and Neutrino

## Lecture I

18/10/2022

LHU
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# Grand Unification:

## Why? How?

Standard Model:

$$\underbrace{\underbrace{SU(3)_c}_{\text{strong}} \otimes \underbrace{SU(2)_L \otimes U(1)}_{\text{ew}}}_{\text{gauge interactions}}$$

unified framework

$$G_{min} = SU(5)$$

$$\alpha = \frac{g^2}{4\pi}$$

gauge: gauge fields

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - i g T_a A_\mu^a$$

$$[T_a, T_b] = i f_{abc} T_c$$

$$\psi \longrightarrow e^{i T_a \theta_a} \psi \quad \text{global}$$

local: $\theta = \theta_a(x) \qquad x \equiv x_\mu$

$= (\bar{x}, t)$

<u>strong</u> : $\alpha_s \gg 1$ , $E \ll \text{GeV}$

$m_p \simeq m_n \simeq \text{GeV}$

<u>em</u>: $\alpha_{em} \simeq 1/100$

<u>weak</u> $\alpha_w > \alpha_{em}$

$\underbrace{W^+, W^-, Z}$ weak messengers

$M_W \simeq M_Z \simeq 100 \text{ GeV}$

reminder:

$$\mathcal{H}_{eff}^{weak} \simeq \frac{4 G_F}{\sqrt{2}} J_\mu^{w} J_w^{\mu}$$

$$\boxed{J_w^{\mu} = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L}$$

$$G_F \simeq 10^{-5} \, GeV^{-2}$$

$$\mathcal{H}_{eff}^{em} \simeq \frac{e^2}{q^2} J_\mu^{em} J^\mu_{em}$$

$\boxed{q \ll GeV}$

$$J_\mu^{em} = \bar{f} \gamma_\mu Q f$$

$$Q f = q f$$

$q_e = -1$, $\underbrace{q_u = 2/3, \quad q_d = -1/3}$

$q_p = 1 \ (uud)$

$q_n = 0 \ (udd)$

$\boxed{\text{MIRACLE}}$

TRUE THEORY

1) guess : principle

2) minimal framework

3) leave it

4) compute predictions

$\Downarrow$

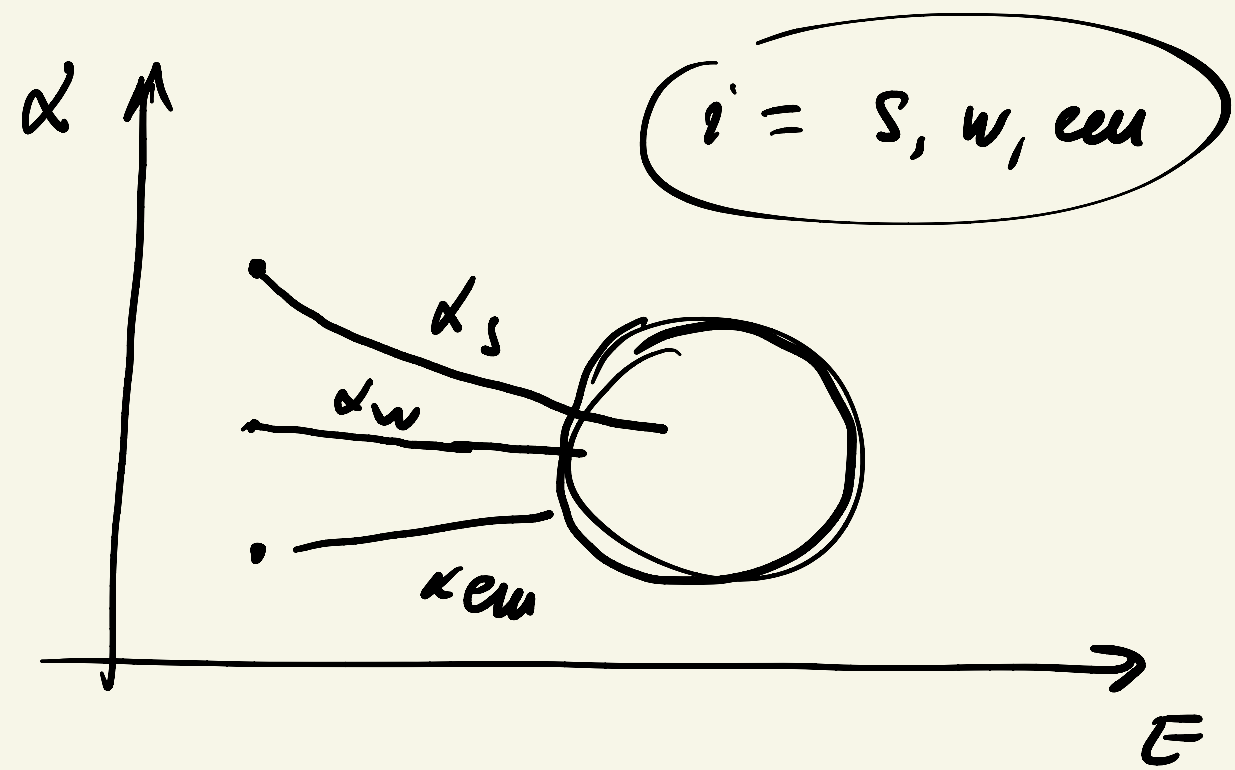
$\boxed{\text{experiment}}$

$\boxed{\text{coupling constant} \neq \text{constant}}$

$$\boxed{\alpha = \alpha(E)}$$

$E \simeq \text{TeV}$ (LHC) :

$\alpha_s \simeq 1/10, \quad \alpha_w \simeq 1/30, \quad \alpha_{em} \simeq 1/100$




$$E_{GUT} = M_{GUT} \;\therefore\; \alpha_i = \alpha_{GUT}$$
$$i = 1,2,3$$

Grand Unified Theory = GUT

(1) UNIF = gauge

$\boxed{U(1) \text{ of em}}$

$$\mathcal{L}_M = \underbrace{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{\text{em field}} + e j_\mu A^\mu + i \bar{\Psi} \gamma^\mu \partial_\mu \Psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{0i} = E_i, \quad F_{ij} = \varepsilon_{ijk} B_k$$

$$j_\mu = \bar{\Psi} \gamma^\mu Q_{em} \Psi$$

$$\psi \to e^{i\theta(x) Q_{em}} \psi$$

$$\Downarrow$$

$$\boxed{\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^\mu D_\mu \psi}$$

$$D_\mu = \partial_\mu - ie\, Q\, A_\mu$$

electricity + magnetism

$\Rightarrow$ em

$\boxed{\cdot \ \text{em} + \text{weak} \Rightarrow \text{ew}}$ ??

$$n \longrightarrow p + e + \bar{\nu}_e$$

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} J_\mu^w \, J^\mu_w$$

<u>1957</u>

Marshak, Sudarshan

"V − A"

$$J_\mu^w = \bar{u}_L \gamma_\mu d_L + \bar{\nu}_L \gamma_\mu e_L$$

"V-A was the key"

Weinberg 2009

$$\mathcal{L}_f = \frac{g}{\sqrt{2}} W_\mu^+ J_W^\mu + h.c.$$

gauge theory ?

$$\boxed{W^+, \; W^-, \; A}$$

3 generators

$$\Downarrow$$

$$\boxed{G^{ew}_{min} = SU(2)}$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad l = \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$\downarrow$$

$$SU(2)$$

$SU(2)$ :

$$[T_a, T_b] = i\,\varepsilon_{abc}\,T_c$$

$$a = 1, 2, 3$$

$$D_\mu = \partial_\mu - ig\, T_a\, A_\mu^a$$

$$\Rightarrow T_a = \frac{\sigma_a}{2}$$

$$\psi \to e^{i\theta_a T_a}\psi \equiv U\psi$$

$$\boxed{U^+U = UU^+ = 1 \,, \quad \det U = 1}$$

$$\Rightarrow \quad T_a = T_a^+, \quad \mathrm{Tr}\, T_a = 0$$

$$\Uparrow$$

$$U = e^{iH}, \quad H = H^+, \quad \mathrm{Tr}\, H = 0$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$UU^+ = 1 \qquad \det U = 1$$

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$$Q_{em} = \sum c_a T_a$$

$r(SU(2)) = 1$ rank

$= \#$ of Cartan

$$\text{Cartan} \equiv C = \{T_i, [T_i, T_j] = 0\}$$

$$= \{T_3\}$$

$$\Rightarrow \boxed{Q = T_3}$$

charge is quantized

$\Downarrow$

't Hooft '74

Polyakov '74

$\boxed{\text{Magnetic monopoles}}$

$$i\bar{\psi}\gamma^\mu D_\mu \psi \longrightarrow \mathcal{L}_{int}$$

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$i\bar{\psi}\left(\partial_\mu - ig\,\frac{\sigma_a}{2} A^a_\mu\right)\gamma^\mu \psi$$

$$\to g/2\ \bar{\psi}\gamma^\mu \begin{pmatrix} A_3 & A_1 - iA_2 \\ A_1 + iA_2 & -A_3 \end{pmatrix}_\mu \psi$$

$$= \frac{g}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right) A_{3\mu}$$

$$+ \frac{g}{2}\left[\bar{u}\gamma^\mu (A_1 - iA_2)_\mu d + h.c.\right]$$

$$= g\, \bar{\psi}\, Q_{em} \gamma^{\mu} \psi\, A_{\mu} + \frac{g}{\sqrt{2}}\, \bar{u}\, \gamma^{\mu} \frac{A_1 - iA_2}{\sqrt{2}}\, d + h.c.$$

$$\left( A_{\mu} = A_{3\mu}, \quad Q = T_3 \right)$$

$$W_{\mu}^{+} = \frac{(A_1 - iA_2)_{\mu}}{\sqrt{2}}$$

weak

$$W_{\mu}^{-} = \frac{(A_1 + iA_2)_{\mu}}{\sqrt{2}}$$

$$= e\, \bar{\psi}\, Q_{em} \gamma^{\mu} \psi\, A_{\mu} + \frac{g}{\sqrt{2}} W_{\mu}^{+}\, \bar{u}\, \gamma^{\mu} d + h.c.$$

$$\boxed{e = g}$$

Imagine SU(2) watched

$\Downarrow$

$$\text{"SM"} = \underbrace{\overset{r=2}{SU(3)} \times \overset{r=1}{SU(2)}}_{\text{unif.}}$$

$\Downarrow$

$$\boxed{\begin{aligned} G_{\text{unif}} &= SU(5) \\ r &= 4 \end{aligned}}$$

$r(SU(2)) = 1$

$r(SU(3)) = 2$

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$\Uparrow$

<u>SU(2)</u> $\quad \text{Tr}\, T_a = 0 \quad$ Cartan

$\Downarrow$

$$T_3 = \frac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

<u>SU(3)</u>

$$\left\{ \begin{aligned} T_3 &= \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \\ T_8 &= \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \end{aligned} \right\}$$

Cartan

$$\boxed{\mathrm{Tr}\, T_a T_b = \frac{1}{2}\delta_{ab}}$$

$$\Rightarrow \boxed{\mathrm{Tr}\, T_3 T_8 = 0}$$

$$r(SU(5)) = 4$$

$$\Uparrow$$

$$\left\{\begin{array}{l} T_3 = \frac{1}{2}\,\mathrm{diag}\,(1,-1,0,0,0) \\ T_8 = \frac{1}{2\sqrt{3}}\,\mathrm{diag}\,(1,1,-2,0,0) \\ T_{15} = \cdots\,\mathrm{diag}\,(1,1,1,-3,0) \\ T_{24} = \cdots\,\mathrm{diag}\,(1,1,1,1,-4) \end{array}\right\}$$

Cartan

$$G_{max} \subseteq SU(5)$$

$$= SU(3) \times \underbrace{SU(2) \times U(1)}$$

ew

$\Rightarrow$ SU(5) = minimal GUT that unifies SU(2) + SU(3)

$\Downarrow$

SU(5) predicts the extra U(1)

<u>but</u>

SU(2) $\neq$ ew theory

$\hookrightarrow$ $Q_{em} = \pm 1/2$

$$Q_{em} = \begin{pmatrix} 2/3 & 0 \\ -1/3 & -1 \end{pmatrix}$$

(a) failure: charge quantization =

= wrong

(b) chirality of em = —||— of weak

||

wrong

Nature:

$$\frac{g}{\sqrt{2}} W_\mu^+ \left( \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L \right)$$

$$e A_\mu \bar{f} \gamma^\mu Q_{em} f \leftarrow$$

$$f (\equiv \psi) = f_L + f_R$$

$$L(R) \equiv \frac{1 \pm \gamma_5}{2}$$

Conventions

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma_i & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\{ \gamma^\mu, \gamma^\nu \} = 2 g^{\mu\nu}$$

$$\{ \gamma_5, \gamma_\mu \} = 0, \quad \boxed{\gamma_5^2 = 1}$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\Sigma^{\mu\nu} = \frac{1}{4i} [ \gamma^\mu, \gamma^\nu ]$$ <u>Lorentz</u>

$$\psi \to \Lambda \psi$$

$$\Lambda = \exp ( i \Theta^{\mu\nu} \Sigma_{\mu\nu} )$$

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)$$

$$[T_\alpha^c, T_a^L] = 0, \quad [T_a^L, Y] = 0$$

$$a = 1, 2, 3$$

$$\alpha = 1, 2, \dots, 8$$

$$[T_\alpha^c, Y] = 0$$

$$G_{min} \supseteq G_{SM}$$

quark

$$\psi_{min} = \left(\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \hline \boxed{\begin{array}{c} \times \\ \times \end{array}} \end{array}\right) \begin{array}{l} \left.\vphantom{\begin{array}{c}\bullet\\\bullet\\\bullet\end{array}}\right\} \text{color} \\ \left.\vphantom{\begin{array}{c}\times\\\times\end{array}}\right\} \text{weak} \end{array}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$4 \text{ of } SU(4) = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ * \end{pmatrix} \begin{matrix} \\ \\ \\ \} \text{ lacks } SU(2) \text{ sibling} \end{matrix}$$

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