

Kürzlich willkommen zur Zentralübung der Rechenmethoden

Bspausg. 9 Partielle Integration

$$\begin{aligned}
 (a) \quad I(z) &= \int_0^z dx \underbrace{x}_{u(x)} \underbrace{e^{2x}}_{v'(x)} & v'(x) &= e^{2x} & v(x) &= \frac{e^{2x}}{2} \\
 &= \left[\underbrace{x}_{u(x)} \cdot \underbrace{\frac{e^{2x}}{2}}_{v(x)} \right]_0^z - \int_0^z \underbrace{1}_{u'(x)} \cdot \frac{e^{2x}}{2} dx = \frac{1}{2} z e^{2z} - \left[\frac{e^{2x}}{4} \right]_0^z = \\
 &= \frac{1}{2} z e^{2z} - \left(\frac{1}{4} e^{2z} - \frac{1}{4} \right) =
 \end{aligned}$$

$$= \frac{1}{2} z e^{2z} - \frac{1}{4} e^{2z} + \frac{1}{4} z$$

$$(b) \quad I(z) = \int_0^z dx \underbrace{x^2}_{u(x)} \underbrace{e^{2x}}_{v'(x)} = \left[x^2 \frac{1}{2} e^{2x} \right]_0^z - \underbrace{\int_0^z dx \underbrace{2x}_{u'(x)} \underbrace{\frac{1}{2} e^{2x}}_{v(x)}}_{\text{Integral aus (a)}}$$

$$(c) \quad I(z) = \int_0^z dx \underbrace{\ln x}_{u(x)} \cdot \underbrace{\frac{1}{x}}_{v'(x)} =$$

$$= \left[\underbrace{(\ln x)}_{u(x)} \cdot \underbrace{x}_{v(x)} \right]_0^z - \int_0^z \underbrace{\frac{1}{x}}_{u'(x)} \cdot x \, dx =$$

$$= (\ln z) \cdot z - 0 - \int_0^z 1 \, dx = (\ln z) \cdot z - z$$

$\lim_{x \rightarrow 0} [(\ln x) \cdot x] \quad \text{L'Hôpital}$

$$\begin{aligned}
 (d) \quad I(z) &= \int_0^z dx \underbrace{\ln x}_{u(x)} \cdot \underbrace{\frac{1}{\sqrt{x}}}_{v'(x)} = \left[(\ln x) \underbrace{2\sqrt{x}}_{v(x)} \right]_0^z - \int_0^z dx \underbrace{\frac{1}{x}}_{u'(x)} \cdot 2\sqrt{x} = \\
 &= (\ln z) 2\sqrt{z} - 4\sqrt{z} \quad \text{L'Hôpital}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad I(z) &= \int_0^z \underbrace{\sin x}_{u(x)} \cdot \underbrace{\sin x}_{v'(x)} dx = \left[\underbrace{\sin x}_{u(x)} \underbrace{(-\cos x)}_{v(x)} \right]_0^z - \int_0^z dx \underbrace{\cos x (-\cos x)}_{= -(\cos x)^2} = \\
 &= \left[\sin x (-\cos x) \right]_0^z - \int_0^z \underbrace{-(\cos x)^2}_{= (\sin x)^2 - 1} dx =
 \end{aligned}$$

$$\int_0^z \sin^2 x dx = -\sin z \cos z - \int_0^z \sin^2 x dx + \int_0^z 1 dx$$

$$2I(z) = 2 \int_0^z \sin^2 x dx = -\sin z \cos z + z$$

$$I(z) = \frac{-\sin z \cos z + z}{2}$$

Bsp. 10: Integration durch Substitution

$$(a) \quad I(z) = \int_0^z dx \cdot \underbrace{x}_{f} \cdot \underbrace{\cos(x^2 + \pi)}_{g(x)}$$

$$y(x) = x^2 + \pi$$

$$\frac{dy}{dx} = 2x \quad dy = 2x \cdot dx$$
$$\left(= \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \right) \quad dx = \frac{dy}{2x}$$

$$= \int_{y(0)}^{y(z)} \frac{dy}{2x} \cdot \check{x} \cdot \underbrace{\cos(x^2 + \pi)}_y =$$

$$= \frac{1}{2} \int_{\pi}^{z^2 + \pi} \cos y \, dy = \frac{1}{2} \left[\sin y \right]_{\pi}^{z^2 + \pi} = \frac{1}{2} \sin(z^2 + \pi) \quad (-0)$$

$$\int dx \cdot x \cos(x^2 + \pi) \quad \text{dann Grenzen einsetzen}$$

Übungsaufgabe 4: ①

$$(a) \quad z + \bar{z} = \underbrace{x+iy}_z + \underbrace{x-iy}_{\bar{z}} = 2x = 2 \operatorname{Re}(z)$$

$$(b) \quad z - \bar{z} = iy - (-iy) = 2iy = 2i \operatorname{Im}(z)$$

$$(c) \quad z \cdot \bar{z} = (x+iy) \cdot (x-iy) = \overset{\substack{\uparrow \\ \text{Binom. Formel}}}{x^2 - (iy)^2} = x^2 - \overset{\substack{\uparrow \\ = -1}}{i^2} \cdot y^2 = x^2 + y^2 \quad (+i0)$$

$$(d) \quad \frac{z}{\bar{z}} = \frac{z \cdot z}{\bar{z} \cdot z} = \frac{(x+iy)(x+iy)}{x^2+y^2} = \frac{(x+iy)^2}{x^2+y^2} = \frac{x^2 + 2xiy + (iy)^2}{x^2+y^2} = \frac{x^2 - y^2 + 2xiy}{x^2+y^2}$$

$$= \underbrace{\frac{x^2 - y^2}{x^2 + y^2}}_{\operatorname{Re}} + i \underbrace{\frac{2xy}{x^2 + y^2}}_{\operatorname{Im}}$$

$$(e) \quad \frac{1}{z} + \frac{1}{\bar{z}} = \frac{\bar{z} + z}{z \cdot \bar{z}} \stackrel{(a)}{=} \stackrel{(c)}{=} \frac{2 \operatorname{Re} z}{x^2 + y^2} = \frac{2x}{x^2 + y^2}$$

$(g), (h)$ also multiplizieren, Sortieren