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Sheet 00: Differentiation and Integration

Solution Homework Problem 1: Differentiation of polynomials [1]

$$(a) \quad f'(x) = 20x^4 - 3x^2 \qquad f''(x) = 80x^3 - 6x.$$

$$(b) \quad f'(x) = 3x^2 - 4x - 1, \qquad f''(x) = 6x - 4.$$

Solution Homework Problem 2: Derivatives involving powers, sine and cosine: product rule and chain rule [2]

Using $\sin' x = \cos x$, $\cos' x = -\sin x$, and the product and chain rules, we obtain:

$$(a) \quad f'(x) = \sin\left[\pi\left(x + \frac{1}{4}\right)\right] + \pi\left(x + \frac{1}{\pi}\right) \cos\left[\pi\left(x + \frac{1}{4}\right)\right]$$

$$(b) \quad f'(x) = -2x \cos(\pi x) + \pi x^2 \sin(\pi x)$$

$$(c) \quad f'(x) = -\pi \sin[\pi \sin(x)] \cos(x)$$

$$(d) \quad f'(x) = 4 \cos^3\left(\frac{3}{\pi}x^2 - x\right) \sin\left(\frac{3}{\pi}x^2 - x\right) \left(\frac{6}{\pi}x - 1\right)$$

$$(e) \quad f'(x) = -\frac{3x^2 - 4x}{(x^3 - 2x^2)^2}$$

$$(f) \quad f'(x) = \frac{2x}{x^2 + 1} - \frac{(x^2 - 2)2x}{(x^2 + 1)^2} = \frac{6x}{(x^2 + 1)^2}$$

Solution Homework Problem 3: Differentiation of powers, exponentials, logarithms [2]

$$(a) \quad f'(x) = \frac{2}{3\sqrt[3]{x}}$$

$$(b) \quad f'(x) = \frac{1}{(x^2 + 1)^{1/2}} - \frac{1}{2} \frac{x \cdot 2x}{(x^2 + 1)^{3/2}} = \frac{1}{(x^2 + 1)^{3/2}}$$

$$(c) \quad f'(x) = 2xe^{1-x^2}$$

$$(d) \quad f'(x) = \frac{d}{dx} e^{\ln 2 x^2} = \frac{d}{dx} e^{x^2 \ln 2} = e^{x^2 \ln 2} 2x \ln 2 = 2x^2 2x \ln 2$$

$$(e) \quad f'(x) = \frac{1}{\sqrt{\ln x}} \frac{1}{x^2} - 2 \frac{\sqrt{\ln x}}{x^2}$$

$$(f) \quad f'(x) = \frac{1}{\sqrt{x^2 + 1}} \frac{1}{2} \frac{2x}{\sqrt{x^2 + 1}} = \frac{x}{x^2 + 1}$$

Solution Homework Problem 4: Elementary integrals [1]

$$(a) \quad I(x) = \int_0^x dy \frac{1}{2y + 4} = \left[\frac{1}{2} \ln(2y + 4) \right]_0^x = \frac{1}{2} \left[\ln(2x + 4) - \ln(4) \right] = \boxed{\frac{1}{2} \ln\left(\frac{1}{2}x + 1\right)}.$$

(b) $I(x) = \int_0^x dy \sinh(\tfrac{1}{2}y) = \left[2 \cosh(\tfrac{1}{2}y) \right]_0^x = \boxed{2[\cosh(\tfrac{1}{2}x) - 1]}.$

[Total Points for Homework Problems: 6]
