Proton decay and grand unification[§]

Goran Senjanović
International Centre for Theoretical Physics, 34100 Trieste, Italy

Abstract

I review the theoretical and experimental status of proton decay theory and experiment. Regarding theory, I focus mostly, but not only, on grand unification. I discuss only the minimal, well established SU(5) and SO(10) models, both ordinary and supersymmetric. I show how the minimal realistic extensions of the original Georgi - Glashow model can lead to interesting LHC physics, and I demonstrate that the minimal supersymmetric SU(5) theory is in perfect accord with experiment. Since no universally accepted model has of yet emerged, I discuss the effective operator analysis of proton decay and some related predictions from a high scale underlying theory. A strong case is made for the improvement of experimental limits, or better the search of, two body neutron decay modes into charged kaons and charged leptons. Their discovery would necessarily imply a low energy physics since they practically vanish in any theory with a desert in energies between M_W and M_{GUT}.

1 Introduction

As Maurice Goldhaber had put nicely more than half a century ago, we feel it in our bones that proton is long lived, for otherwise the radiation from the decay would kill you. All you need to know is how much radiation is hazardous for you, and you get a lower limit on proton lifetime, on the order of 10^{18} years. It was, as it is today, puzzling that the proton should live so much longer than the neutron, and it was attributed to the conservation of the baryon number, first by Weyl [1] already in 1929, and the again by Stuckelberg [2] and Wigner [3]every ten years afterwards. And then it was becoming a dogma, as usually happens when a cause and a consequence are confused, and the questioning became heretic. It is amusing to see how almost apologetic Reines et al [4] were in justifying their pioneering experimental probe of nucleon decay:

"It has often been surmised that there exists a conservation law of nucleons, i.e., that they neither decay spontaneously nor are destroyed or created singly in nuclear collisions. In view of the fundamental nature of such an assumption, it seemed of interest to investigate the extent to which the stability of nucleons could be experimentally demonstrated."

They established a limit of 10^{21} yr, which they would improve to 10^{26} yr later on. And while the baryon number was becoming sacred, lepton number violation was seriously discussed already in 1937 by Majorana in his classic paper [5] on majorana spinors. In turn Racah [6] and Furry [7] were to discuss at depth neutrino-less double decay still desperately searched for as a probe of lepton number violation. This shows how crucial for experimentalists it is to have a theory behind.

Since there is nothing sacred about global symmetries we will follow a belief that: **the only good global symmetry is a broken global symmetry**. The symmetry point is not a special point in the parameter space of a theory, for an almost exact global symmetry is just as protected and useful as a fully exact one. Furthermore a zero is useless for experimentalists.

And baryon number is not an exact gauge symmetry unless you accept a corresponding gauge coupling to be ridiculously small: $g_B \leq (10^{-20} - 10^{-19})$ (a repulsive competition with gravity).

While Goldhaber et al had to justify their search for proton decay, today the atmosphere has changed completely, as illustrated by the fact that one is asked to give plenary talks on it at major conferences. The point is that there is a theory behind: grand unification of strong, weak and electro-magnetic interactions. It grew out of pioneering ideas of Pati and Salam [8] on the unification of quarks and leptons and is exemplified perfectly on the minimal SU(5) theory of Georgi and Glashow [9]. When Georgi, Quinn and Weinberg [10] computed the unification scale and predicted proton lifetime $\tau_p \simeq 10^{30}$ yr, many experimentalists rushed underground [11], all over the world, from India to Japan to US to Europe. Here is a list of experiments

Calorimeter detector

- Kolar Gold Field - Kolar district (Kamataka, India)
- NUSEX - Mont Blanc (Alps, France)
- FREJUS - Frejus tunnel (Alps, France)
- SOUDAN- Soudan underground mine (Minnesota, US)

Cherenkov detector

- IMB - Morton salt mine (Ohio, US)
- Kamiokande - Mozumi mine (Hida, Japan) $\rightarrow$ Super-Kamiokande $\rightarrow$ atmospheric neutrino oscillations

The search resulted in great improvements on proton decay limits, culminating with SK that recently pushed the pionic channel all the way to about 10^{34} yr. Here are limits on some proton decay channels.

[§]Based on the plenary talks at the SUSY09 and PASCOS09 conferences.

Channel	$\tau_p(10^{33}$ years)
$p \to e^+\pi^0$	8.2
$p \to \mu^+\pi^0$	6.6
$p \to \mu^+K^0$	1.3
$p \to e^+K^0$	1.0
$p \to \nu K^+$	2.3
$n \to e^+K^-$	0.02
$n \to e^-K^+$	0.03

The last two channels are important since they are an indication of low scale as discussed in sections 3 and 5.

In what follows I discuss the minimal SU(5) and SO(10) theories, both ordinary and supersymmetric. Due to the lack of space, many important references are likely to be omitted. For a review and more complete references on the subject, see [12].

2 Minimal realistic SU(5)

It was the minimal SU(5) that caused the underground rush, for it predicted proton lifetime on the order of $\tau_p \simeq 10^{30}$ yr. And on top, it also predicted the nucleon decay branching ratios as shown by Mohapatra [13]; unfortunately these predictions resulted from the wrong mass relations: $m_e = m_d$, wrong for all three generations. These relations can be corrected easily by simply adding higher dimensional operators [31] (at least for the first two generations) , but then the theory stops being predictive.

In any case this is only history now, for the theory is not even consistent:

- gauge couplings do not unify since α_2 and α_3 meet at 10^{16} GeV (as in SM), but α_1 meets α_2 too early at $\approx 10^{13}$ GeV ;
- neutrinos are massless as in the SM.

Possible higher dimensional operators are not enough: neutrino mass comes out too small ($\lesssim 10^{-4} eV$) and the threshold effects do not cure the lack of unification.

It is important to know then what the minimal consistent realistic extensions are. There are two. You can

- add a symmetric complex scalar field (and higher dimensional operators for charged fermion masses) [14] $15_H = (1_C, 3_W) + (6_C, 1_W) + [(3_C, 2_W) = leptoquarks]$, with $(1_C, 3_W)$ being the usual Higgs triplet behind the type II seesaw [15]. The leptoquarks $(3_C, 2_W)$ may remain light (but not necessarily), and a rather interesting prediction is a fast proton decay, on the edge of experimental limits.
- add an adjoint fermion field $24_F = (8_C, 1_W) + (1_C, 3_W) + (1_C, 1_W) + (3_C, 2_W) + (\bar{3}, 2_W)$. The fields $(1_C, 1_W) + (1_C, 3_W)$ are responsible for type I [17] + III [18] hybrid seesaw. The model requires higher dimensional operators both for charged femions and for realistic neutrino Dirac Yukawa couplings It predicts a light fermion triplet $(1_C, 3_W)$, with a mass below TeV so that the running of the SU(2) gauge coupling is slowed down and meets U(1) above 10^{15} GeV. It's phenomenology is quite interesting for it leads to lepton number violation at colliders in the form of same sign di - leptons as suggested originally in seesaw a long time ago [19]. For the relevant studies in the context of the type III seesaw see [20].

The two theories have in common a 'fast' proton decay, with $\tau_p \leq 10^{35}$ yr, to keep in mind in what follows.

3 Minimal supersymmetric SU(5)

The underground rush continued, or better to say, got boosted with the success of the minimal supersymmetric SU(5). Low energy supersymmetry, suggested in order to stabilize the Higgs mass hierarchy, predicted correctly $sin^2\theta_W = 0.23$ in 1981 [21] [22] [23] [24] ten years before its confirmation at LEP. It actually did even better: the prediction of $sin^2\theta_W = 0.23$ was tied to the prediction of the heavy top quark, with $m_t \simeq 200$ GeV. Namely, in 1981 the low indirect measurements gave $sin^2\theta_W = 0.21$, with the assumed value $\rho = 1$. In order to make a case for low energy supersymmetry, Marciano and I [24] had to say that ρ was bigger, which required loops, which required at least one large coupling, and a natural SM candidate was the top quark, with $y_t \simeq 1$. It is remarkable that both the $sin^2\theta_W = 0.23$ and the heavy top would turn out to be true. It should be stressed that heavy top quark played also a crucial role in the radiative Higgs mechanism [25] [26]. Thus heavy top is an integral part of low energy supersymmetry.

The GUT scale was predicted: $M_{GUT} \simeq 10^{16}$ GeV and in turn $\tau_p(d = 6) \simeq 10^{35\pm1}$ yr, which would have rendered proton decay out of experimental reach. However, supersymmetry leads to a new contribution: $d = 5$ operators [27] through the exchange of heavy color triplet Higgsino (T and $\bar{T}$). A rough estimate gives

$$G_T \simeq \frac{\alpha}{4\pi} y_u\, y_d \frac{m_{\text{gaugino}}}{M_T m_{\tilde{f}}^2} \simeq 10^{-30}\ \text{GeV}^{-2}$$

which for $y_u \simeq y_d \simeq 10^{-4}$, $m_{\text{gaugino}} \simeq 100$ GeV, $m_{\tilde{f}} \simeq$TeV and $M_T \simeq 10^{16}$ GeV gives $\tau_p(d = 5) \simeq 10^{30-31}$ yr. It would seem that today this theory is ruled out. It was actually proclaimed dead in 2001 when the triplet mass was carefully computed to give $M_T^0 = 3 \times 10^{15}$GeV [28] (for the superscript 0 explanation, see below). Caution must be raised however for two important reasons: i) the uncertainty in sfermion masses and mixings [29] and ii) uncertainty in M_T [30] due to necessity of higher dimensional operators [31] to correct bad fermion mass relations $m_d = m_\ell$ [32]. The $d = 4$ operators, besides correcting these relations also split the masses m_3 and m_8 of weak triplet and color octet, respectively, in the adjoint 24_H Higgs super multiplet and one gets

$$\left.\begin{array}{l} M_{GUT} = M_{GUT}^0 \left(\frac{M_{GUT}^0}{2m_8}\right)^{1/2} \\ M_T = M_T^0 \left(\frac{m_3}{m_8}\right)^{5/2} \end{array}\right\} \begin{array}{l} M_{GUT}^0 \simeq 10^{16}\ \mathrm{GeV} \\ M_T^0 = 3\times 10^{15}\ \mathrm{GeV} \end{array}$$

where the superscript 0 denotes the predictions for $m_3 = m_8$ at the tree level with $d = 5$ operators neglected. The fact that M_{GUT} goes up with m_8 below M_{GUT} was noticed quite some time ago [33]. Imagine that $d = 4$ terms dominate for small cubic Yukawa self coupling, in which case one has $m_3 = 4m_8$ and thus $M_T = 32M_T^0 \simeq 10^{17}$ GeV $\simeq M_{GUT}$ ($m_8 \simeq 10^{15}$ GeV). In turn a strong suppression of proton decay with $\tau_p \simeq 10^3 \tau_p^0(d = 5) \simeq 10^{33-34}$ yr. In principle the ratio of the triplet and octet masses can be as large as one wishes, so at first glance the proton lifetime would seem not to be limited from above at all. However, all this makes sense if the theory remains perturbative and thus predictive. Increasing M_{GUT} would bring it too close to the Planck scale, so it is fair to conclude that the proton lifetime is below 10^{35} yr. This prediction is not hard, though. The d=4 operators not only cure the bad mass relations, but also also split the Yukawa couplings of the SM doublet Higgs and the color triplet. if you allow for cancellation between the d=3 and d=4 couplings, one can in principle make the color triplet couplings as small as one wishes and thus suppress the proton decay. In principle, it is even possible to make a color triplet mass at the electro-weak scale [34]. The cancellation of matrices is rather unnatural, so I will not pursue it here. However, it may emerge in more complex models, such as SO(10) [35], so this should be kept mind as a serious possibility.

In short, the minimal supersymmetric SU(5) is still a perfectly viable theory, and the $d = 5$ proton decay is expected close to the present limit. The theory is crying for a new generation of proton decay experiments. It would seem that B-L remains an accidental global symmetry of nucleon decay and one expects the dominant mode $p \to K^+ \bar\nu_\mu$ characteristic of $d = 5$ operators. However, this minimal theory must account for neutrino masses and mixings which implies that R-parity is broken. The first important implication of not assuming R-parity is that the lightest neutralino cannot be dark matter, for it decays too fast with the collider signature of lepton number violation. Thus, the only dark matter candidate is an unstable gravitino. It decays into the neutrino and the photon through the neutrino-gaugino mixing

$$\Theta_{\nu\,\mathrm{gaugino}} \simeq \sqrt{\frac{m_\nu}{M_{\mathrm{gaugino}}}}$$

The decay is suppressed by the Planck scale [36]

$$\begin{aligned} \Gamma(3/2 \to \gamma\nu) &= \frac{1}{32\pi}\frac{m_{3/2}^3}{M_{Pl}^2}\Theta^2_{\nu\,\mathrm{gaugino}} \\ &\simeq \frac{1}{32\pi}\frac{m_{3/2}^3}{M_{Pl}^2}\frac{m_\nu}{M_{\mathrm{gaugino}}} \\ &\leq 10^{-50}\ \mathrm{GeV} \quad (\tau \geq 10^{26} sec). \end{aligned}$$

The lower limit on gravitino lifetime comes from requiring that the flux of produced photons does not exceed the observed diffuse photon background, and it gives an upper limit on gravitino mass, $m_{3/2} \leq 1 - 10$ GeV [36], reasonable for the LSP. If one relaxes the neutrino-gaugino mixing, gravitino can be heavier; see e.g. [37]. For a review and references on gravitino dark matter, see [38].

Once R-parity is not assumed ad hoc, one has a new source of proton decay too. Due to allowed couplings in the superpotential $\lambda_1\, u^c\, d^c\, d^c + \lambda_2\, q\, \ell\, d^c + \lambda_3\, \ell\, \ell\, e^c$, $\tilde d^c$ mediates $d = 6$ proton decay, which implies the limit $\lambda_1\, \lambda_2 \leq 10^{-25}$ (=?) for $m_{\tilde d^c} \simeq TeV$. The question mark indicates an interesting possibility of these couplings causing proton decay. It can be shown that the parameter space allows for a B+L violating mode $n \to e + K^+$ [39]. As I show below, this decay mode cannot come from a conventional picture of grand unification with a desert.

4 SO(10)

Although SU(5) is the minimal theory of grand unification and as such deserves maximal attention as a laboratory for studying proton decay, SO(10) has important merits

- it unifies a fermion family in a spinorial 16_F representation and as such is a minimal unified theory of matter and interactions
- it automatically contains right-handed neutrinos N
- it gives naturally $M_N \gg M_W$ and so neutrino has a tiny mass through the see-saw mechanism
- in supersymmetry R-parity is a gauge symmetry [40]
- in the renormalizable version R-parity remains exact [41] and the lightest supersymmetric partner (LSP) is stable, and thus becomes a natural dark matter candidate.

While ordinary, non supersymmetric SO(10) was studied at length over the years, no predictive realistic model of fermion masses and mixings ever emerged.

All the fermion masses and mixings can be accounted for with the 10_H and 126_H representations, the latter providing a large mass to the right-handed neutrinos. In the Pati- Salam $SU(2)_L \times SU(2)_R \times SU(4)_C$ language

$$10_H = (2,2,1) + (1,1,6)$$

and

$$126_H = (2,2,15) + (3,1,10) + (1,3,\bar{1}0) + (1,1,6)$$

and one can see that in principle no bad mass relations come out. If 10_H is taken to be real, a minimal scenario, the predictions turn out to be wrong [43], and making it complex kills the predictions. This is why ordinary SO(10) does not do the job. The theory deserves attention for it allows for an intermediate L-R symmetry [44]; for recent attempts to revive ordinary SO(10) see [43], [45].

This situation improves in supersymmetry: the couplings being holomorphic simplifies things. Although a single 10_H is necessarily complex, still, analyticity guarantees a single Yukawa, and similarly for 126_H. This minimal supersymmetric version, coined renormalizable, although suggested already in 1982 [46], and revisited ten years later [47], has been studied at length only in recent years [48] [49]. A boost was provided by an observation that $b - \tau$ unification can be naturally tied with the large atmospheric mixing angled in the type II seesaw [50]. This means that small quark and large lepton weak mixing angles follow naturally from the common Yukawa couplings without any need for flavor symmetries. This is an important result which shows that the much talked about issue of rather different quark and lepton mixings is not a problem as normally argued. In the SO(10) grand unified theory it is simply a product of a broken quark-lepton or Pati-Salam symmetry. In other words, although at the GUT scale quarks and leptons are completely equivalent with the same Yukawa couplings, at the SM energies, their different masses lead to different mixing angles.

After a great initial success, when pinned down, the theory ran into tension between fast proton and neutrino masses. It was revisited recently [51] and shown to work with the so called split supersymmetry spectrum of heavy sfermions. It is important to note that the nucleon decay branching ratios are determined. This is an example for a kind of theory of proton decay we are searching for. It remains to be shown that the solution found is unique.

Instead of a large 126_H Higgs, one may choose a 16_H and then build 126_H Higgs effectively as 16_H^2. This induces a proliferation of couplings and one must introduce extra flavor symmetries, and thus go beyond a simple GUT picture. For a discussion of this approach, see [52]. In any case, it is clear that no accepted minimal model has yet emerged. In this sense it is really important to keep in mind that the minimal supersymmetric SU(5) theory is not ruled out. It should be viewed as the laboratory for studying grand unification and related issues, such as proton decay and magnetic monopoles, the way the minimal ordinary SU(5) used to be when it worked.

5 Effective operator analysis of nucleon decay and the desert picture

Since we do not have **the** theory of grand unification it is worthwhile to study the generic features of high scale theories of proton decay. This is done through the effective operator expansion [53]. In this program one expands the baryon violating operators in M_W/M_B or m_p/M_B, where M_B is the scale responsible for proton decay. If M_B is very large, one can safely assume the SM gauge symmetry unbroken at that scale. This is surely true in conventional grand unification with a desert where $M_B = M_{GUT}$.

There are only four leading $d = 6$ operators [54]

$$O_1 = (u_R\, d_R)\,(q_L\, \ell_L) \qquad O_2 = (q_L\, q_L)\,(u_R\, e_R)$$
$$O_3 = (q_L\, q_L)\,(q_L\, \ell_L) \qquad O_4 = (u_R\, d_R)\,(u_R\, e_R)$$

The gauge meson dominance would imply only the first two which can lead to a number of predictions. Even in general one has an immediate prediction of an accidental B - L symmetry in any theory with a high scale M_B, which explains why any GUT model was predicting it.

There are also a number of immediate isospin relations

$$\Gamma(p \to \ell_R^+\, \pi^0) = \tfrac{1}{2}\Gamma(n \to \ell_R^+\, \pi^-) = \tfrac{1}{2}\Gamma(p \to \bar{\nu}\pi^+) = \Gamma(n \to \bar{\nu}\, \pi^0)$$

$$\Gamma(p \to \ell_L^+\, \pi^0) = \tfrac{1}{2}\Gamma(n \to \ell_L^+ \pi^-)$$

Also, it is evident that $\bar{s}$ goes out, such as in the allowed decay $p \to K^+\bar{\nu}$, characteristic of d=5 operators in supersymmetry.

One conclude in turns that there can be no two body neutron decay into kaons and charged leptons

$$n \not\to K^+\, \ell \qquad n \not\to K^-\, \ell^+$$

This is why the R-parity violating mode $n \to K^+\, e$ discussed in section 3. is so important: if discovered, it would point out immediately towards a low scale source of proton decay. It results from the operator

$$d\, s\, d\, \bar{\ell}\, \langle H \rangle / \tilde{m}^3$$

where the Higgs vev is needed to break the SM symmetry which otherwise guarantees the conservation of B - L. The low scale $\tilde{m}$ of supersymmetry breaking allows for this operator not being suppressed. In GUT $\tilde{m}$ becomes M_{GUT}, implying the suppression M_W/M_{GUT}. Furthermore, the necessary presence of s quark for symmetry reasons implies the absence of the pionic mode: $n \not\to \pi^+\, e$, which makes the it even more predictive [39].

This leads to an important message: the decay modes $n \to K^+\, e$ and $n \to K^-\, e^+$ imply a low energy source of proton decay. The limits are roughly 10^{31} yr and I urge the experimentalists to improve them.

6 Proton decay: matrix elements

Due to the shortage of space, I will be very brief here. There have been many attempts of computing the matrix elements using the non-relativistic quark model, the bag model, the lattice and the chiral Lagrangian techniques. In recent years, the focus has been on the lattice and chiral Langrangians, especially the combination of the two. For the work on lattice see e.g. [55]. A lot of progress was made on the lattice computation of chiral Lagrangian coefficients, see e.g. [56]. The trouble is that chiral perturbation theory works great for soft pions, while the pions from the proton decay would have the momentum on the order of proton mass. Thus, one needs to go beyond the leading term of the original classic [57] .

7 Summary and Outlook

I would conclude with the following messages:

- there is no universally accepted grand unified theory of nucleon decay
- most models give$\tau_p \leq 10^{35}$ yr, especially with low energy supersymmetry
- minimal SU(5) is ruled out and its minimal extension leads to possible LHC physics through the light fermion triplet
- minimal supersymmetric SU(5) still perfectly viable with gravitino being the only possible (unstable) dark matter
- test of GUT desert picture provided by decay modes $n \to K^+ \ell$ and $n \to K^- \ell^+$ which would indicate low energy physics as a source of p decay, such as R-parity violating couplings

Thus, a new generation of experiments is badly needed. Here is a list of proposals

Cherenkov detector - MEGATON (hopefully): good for the pion modes

- HYPER-Kamiokande
- 3M -(Megaton, Modular, Multipurpose)

  Homestake - DUSEL (Deep Underground Science and Engineering Lab)
- MEMPHYS - MEgaton Mass PHYSics

  Fréjus - LAGUNA (Large Apparatus Grand Unification and Neutrino Astrophysics) project

Liquid Argon Detector - 100 kT: good for the kaon modes

- LANNDD (Liquid Argon Neutrino Nucleon Decay Detector)

  Homestake - DUSEL?
- GLACIER (Giant Liquid Argon Charge Imaging ExpeRiment)

  Europe - Laguna

Liquid scintillator - 50 kT: good for the kaon modes

- LENA (Low Energy Neutrino Astronomy)

  Europe - Laguna

Hopefully they all will be funded and some will reach 10^{35} yr in 10-20 years? One cannot overemphasize the importance of trying to reach this scale, generic of grand unification. Without experiment, a beautiful field of proton decay and grand unification is bound to turn into metaphysics.

8 Acknowledgements

I wish to thank Pran Nath and other organizers of SUSY 09 for an excellent conference. I also wish to thank Wilfried Buchmüller and Laura Covi and others at DESY for another great meeting, PASCOS 09. This talk was ordered by the organizers of both as a review of proton decay and grand unification. I am deeply grateful to my collaborators on the topics discussed here Charan Aulakh, Borut Bajc, Pavel Fileviez Pérez, Bill Marciano, Alejandra Melfo, Rabi Mohapatra, Miha Nemevšek, Andrija Rašin (Andrija, vrati se, sve ti je oprošteno) and Francesco Vissani.

I am grateful to Ilja Doršner, Gia Dvali, Alejandra Melfo and Enkhbat Tsedenbaljir for useful discussions. Thanks are also due to Vladimir Tello for his help with this manuscript. I am deeply grateful to Borut Bajc, Svjetlana Fajfer and other members of the theory group at "Jožef Stefan" Institute in Ljubljana for their warm hospitality during this write up. This work was partially supported by the EU FP6 Marie Curie Research and Training Network "UniverseNet" (MRTN-CT-2006-035863) and by the Senior Scientist Award from the Ministry of Science and Technology of Slovenia.

References

[1] H. Weyl, Z. Phys. **56** (1929) 330 [Surveys High Energ. Phys. **5** (1986) 261].

[2] E.C..G Stuekelberg, Helv. Phys. Acta. 11 (1939) 299

[3] E.P. Wigner, Proc. Am. Philos. Soc. 93 (1949) 521;

[4] F. Reines, C. L. Cowan and M. Goldhaber, Phys. Rev. **96** (1954) 1157.

[5] E. Majorana, Nuovo Cim. **14**, 171 (1937).

[6] G. Racah, Nuovo Cim. **14**, 322 (1937)

[7] W. H. Furry, Phys. Rev. **56**, 1184 (1939).

[8] J. C. Pati and A. Salam, Phys. Rev. D **10** (1974) 275.

[9] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438.

[10] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[11] M. Goldhaber, In **Boston 1988, Proceedings, Neutrino physics and astrophysics* 486-489.*

[12] P. Nath and P. Fileviez Pérez, Phys. Rept. **441** (2007) 191 [arXiv:hep-ph/0601023].

[13] R. N. Mohapatra, Phys. Rev. Lett. **43** (1979) 893.

[14] I. Doršner and P. Fileviez Pérez, Nucl. Phys. B **723**, 53 (2005) [arXiv:hep-ph/0504276].

[15] M. Magg and C. Wetterich, Phys. Lett. B **94** (1980) 61.

R. N. Mohapatra and G. Senjanović, Phys. Rev. D **23** (1981) 165.

G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181** (1981) 287.

[16] B. Bajc and G. Senjanović, JHEP **0708**, 014 (2007) [arXiv:hep-ph/0612029].

B. Bajc, M. Nemevšek and G. Senjanović, Phys. Rev. D **76**, 055011 (2007) [arXiv:hep-ph/0703080].

[17] P. Minkowski, Phys. Lett. B **67** (1977) 421.

T. Yanagida, proceedings of the *Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, 1979, eds. A. Sawada, A. Sugamoto, KEK Report No. 79-18, Tsukuba.

S. Glashow, in *Quarks and Leptons, Cargèse 1979*, eds. M. Lévy. et al., (Plenum, 1980, New York).

M. Gell-Mann, P. Ramond, R. Slansky, proceedings of the *Supergravity Stony Brook Workshop*, New York, 1979, eds. P. Van Niewenhuizen, D. Freeman (North-Holland, Amsterdam).

R. Mohapatra, G. Senjanović, Phys.Rev.Lett. **44** (1980) 912

[18] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C **44** (1989) 441.

[19] W. Y. Keung and G. Senjanović, Phys. Rev. Lett. **50**, 1427 (1983).

[20] R. Franceschini, T. Hambye and A. Strumia, Phys. Rev. D **78**, 033002 (2008) [arXiv:0805.1613 [hep-ph]].

F. del Aguila and J. A. Aguilar-Saavedra, arXiv:0808.2468 [hep-ph].

F. del Aguila and J. A. Aguilar-Saavedra, arXiv:0809.2096 [hep-ph].

A. Arhrib, B. Bajc, D. K. Ghosh, T. Han, G. Y. Huang, I. Puljak and G. Senjanović, arXiv:0904.2390 [hep-ph].

T. Li and X. G. He, arXiv:0907.4193 [hep-ph].

[21] S. Dimopoulos, S. Raby, F. Wilczek, Phys. Rev. D **24** (1981) 1681.

[22] L.E. Ibáñez, G.G. Ross, Phys. Lett. B **105** (1981) 439.

[23] M. B. Einhorn and D. R. T. Jones, Nucl. Phys. B **196**, 475 (1982).

[24] W. J. Marciano and G. Senjanović, Phys. Rev. D **25**, 3092 (1982).

[25] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. **68** (1982) 927 [Erratum-ibid. **70** (1983) 330].

[26] L. Alvarez-Gaume, J. Polchinski and M. B. Wise, Nucl. Phys. B **221** (1983) 495.

[27] N. Sakai and T. Yanagida, Nucl. Phys. B **197**, 533 (1982).

S. Weinberg, Phys. Rev. D **26** (1982) 287.

[28] H. Murayama and A. Pierce, Phys. Rev. D **65**, 055009 (2002) [arXiv:hep-ph/0108104].

[29] B. Bajc, P. Fileviez Pérez and G. Senjanović, Phys. Rev. D **66**, 075005 (2002) [arXiv:hep-ph/0204311].

[30] B. Bajc, P. Fileviez Pérez and G. Senjanović, arXiv:hep-ph/0210374.

[31] J. R. Ellis and M. K. Gaillard, Phys. Lett. B **88**, 315 (1979).

[32] M. S. Chanowitz, J. R. Ellis and M. K. Gaillard, Nucl. Phys. B **128**, 506 (1977).

A. J. Buras, J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B **135** (1978) 66.

[33] C. Bachas, C. Fabre and T. Yanagida, Phys. Lett. B **370**, 49 (1996) [arXiv:hep-th/9510094].

[34] G. R. Dvali, Phys. Lett. B **287** (1992) 101.

[35] G. R. Dvali, Phys. Lett. B **372**, 113 (1996) [arXiv:hep-ph/9511237].

[36] See e.g. F. Takayama and M. Yamaguchi, Phys. Lett. B **485**, 388 (2000) [arXiv:hep-ph/0005214].

[37] K. Ishiwata, S. Matsumoto and T. Moroi, Phys. Rev. D **78** (2008) 063505 [arXiv:0805.1133 [hep-ph]].

[38] W. Buchmuller, arXiv:0910.1870 [Unknown].

[39] F. Vissani, Phys. Rev. D **52**, 4245 (1995) [arXiv:hep-ph/9503227].

[40] R. N. Mohapatra, *Phys. Rev.* **D 34**, 3457 (1986).

A. Font, L. E. Ibáñez and F. Quevedo, *Phys. Lett.* **B228**, 79 (1989).

S. P. Martin, *Phys. Rev.* **D46**, 2769 (1992).

[41] C. S. Aulakh, B. Bajc, A. Melfo, A. Rašin and G. Senjanović, Nucl. Phys. B **597**, 89 (2001) [arXiv:hep-ph/0004031].

See also, C.S. Aulakh, K. Benakli and G. Senjanović, Phys. Rev. Lett. **79** (1997) 2188.

[42] C. S. Aulakh, A. Melfo and G. Senjanović, Phys. Rev. D **57**, 4174 (1998) [arXiv:hep-ph/9707256].

C. S. Aulakh, A. Melfo, A. Rašin and G. Senjanović, Phys. Lett. B **459** (1999) 557.

[43] B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Rev. D **73**, 055001 (2006) [arXiv:hep-ph/0510139].

[44] J. C. Pati and A. Salam, Phys. Rev. D **10** (1974) 275.

R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11** (1975) 2558.

G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12** (1975) 1502.

G. Senjanović, Nucl. Phys. B **153** (1979) 334.

[45] S. Bertolini, L. Di Luzio and M. Malinsky, Phys. Rev. D **80**, 015013 (2009) [arXiv:0903.4049 [hep-ph]].

[46] C.S. Aulakh, R.N. Mohapatra, Phys. Rev. D **28** (1983) 217.

T. E. Clark, T. K. Kuo and N. Nakagawa, Phys. Lett. B **115** (1982) 26.

[47] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993). See also D. G. Lee and R. N. Mohapatra, Phys. Rev. D **51**, 1353 (1995) [arXiv:hep-ph/9406328]. L. Lavoura, Phys. Rev. D **48** (1993) 5440 [arXiv:hep-ph/9306297]. B. Brahmachari and R. N. Mohapatra, Phys. Rev. D **58** (1998) 015001 [arXiv:hep-ph/9710371].

[48] K. Matsuda, Y. Koide and T. Fukuyama, Phys. Rev. D **64** (2001) 053015.

T. Fukuyama and N. Okada, JHEP **0211** (2002) 011.

[49] An incomplete set of references

H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Lett. B **570**, 215 (2003) [arXiv:hep-ph/0303055].

C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Lett. B **588**, 196 (2004) [arXiv:hep-ph/0306242].

H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Rev. D **68**, 115008 (2003) [arXiv:hep-ph/0308197].

B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Rev. D **70** (2004) 035007 [arXiv:hep-ph/0402122].

C. S. Aulakh and A. Girdhar, Nucl. Phys. B **711**, 275 (2005) [arXiv:hep-ph/0405074].

T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, J. Math. Phys. **46**, 033505 (2005) [arXiv:hep-ph/0405300].

T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, Phys. Rev. D **72**, 051701 (2005) [arXiv:hep-ph/0412348].

C. S. Aulakh and S. K. Garg, Nucl. Phys. B **757** (2006) 47 [arXiv:hep-ph/0512224].

B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Lett. B **634**, 272 (2006) [arXiv:hep-ph/0511352].

S. Bertolini, T. Schwetz and M. Malinsky, Phys. Rev. D **73**, 115012 (2006) [arXiv:hep-ph/0605006].

For reviews see G. Senjanović, Talk given at SEESAW25: International Conference on the Seesaw Mechanism and the Neutrino Mass, Paris, France, 10-11 Jun 2004 (Published in *Paris 2004, Seesaw 25* 45-64) [arXiv:hep-ph/0501244],

C. S. Aulakh, arXiv:hep-ph/0506291,

G. Senjanović "Theory of neutrino masses and mixings", lectures in the Course CLXX of the International School of Physics "Enrico Fermi", Varenna, Italy, June 2008. Published in Measurements of Neutrino Mass, Volume 170 International School of Physics Enrico Fermi Edited by: F. Ferroni, F. Vissani and C. Brofferio, September 2009.

[50] B. Bajc, G. Senjanović and F. Vissani, Phys. Rev. Lett. **90**, 051802 (2003) [arXiv:hep-ph/0210207].

B. Bajc, G. Senjanović and F. Vissani, Phys. Rev. D **70** (2004) 093002 [arXiv:hep-ph/0402140].

See also B. Bajc, G. Senjanović and F. Vissani, arXiv:hep-ph/0110310.

[51] B. Bajc, I. Doršner and M. Nemevšek, JHEP **0811**, 007 (2008) [arXiv:0809.1069 [hep-ph]].

[52] See the talk of Babu at SUSY09.

[53] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

F. Wilczek and A. Zee, Phys. Rev. Lett. **43**, 1571 (1979).

[54] L. F. Abbott and M. B. Wise, Phys. Rev. D **22**, 2208 (1980).

[55] Y. Aoki, C. Dawson, J. Noaki and A. Soni, Phys. Rev. D **75**, 014507 (2007) [arXiv:hep-lat/0607002].

[56] Y. Aoki *et al.* [RBC-UKQCD Collaboration], Phys. Rev. D **78**, 054505 (2008) [arXiv:0806.1031 [hep-lat]].

[57] M. Claudson, M. B. Wise and L. J. Hall, Nucl. Phys. B **195**, 297 (1982).