Neutrino Mass and Grand Unification

Lecture XXIV

25/1/2022

LMU
Winter 2022

(Cosmic) Strings

$U(1)$ $\quad \phi \in \mathbb{C} \therefore \phi \to e^{i\alpha(x) Q} \phi$

$$\mathcal{L} = \frac{1}{2} (D_\mu \phi)^* (D^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\boxed{V(\phi) = \frac{\lambda^2}{2} \left(|\phi|^2 - v^2 \right)^2}$$

- $Q\phi = \phi \quad (q_\phi = 1)$

$$D_\mu \phi = (\partial_\mu - ig A_\mu) \phi$$

- SSB $\Leftrightarrow |\phi_0|^2 = v^2$

$$\mathcal{M}_0 = \{ \phi_0 \therefore V = 0 \} = \{ \phi_0 \therefore \}$$

$$\Rightarrow \mathcal{M}_0 = \{ (\phi_0 = \phi_{01} + i\,\phi_{02}) \Rightarrow \phi_{01}^2 + \phi_{02}^2 = v^2 \}$$

$\boxed{\text{\underline{Static}, finite energy (per length) classical solution}}$

· <u>digression</u> Domain Wall

$$\mathcal{M}_\infty = \{ +z, -z; \; z \to \infty \}$$

$$\text{map}: \quad \phi_{cl}(z) : \mathcal{M}_\infty \to \mathcal{M}_0$$

$$\mathcal{M}_0 = \{ \pm v \}$$

<u>String</u> $\quad \mathcal{M}_\infty = S_1$ (infinitely long)

$$E/L = \int d\varsigma \left[\tfrac{1}{2} |D_i \phi|^2 + V(\phi) \right]$$

$\shortparallel$

finite $\Rightarrow$ $\left.\begin{array}{l} D_i \phi \to 0 \\ V(\phi) \to 0 \end{array}\right\}$ at ∞

$$\Rightarrow \boxed{\begin{array}{l} |\phi| \xrightarrow[\infty]{} v \\ D_\mu \phi \xrightarrow[\infty]{} 0 \end{array}} \quad \Rightarrow \mathcal{M}_\infty = S_1$$

(a) vacuum $\quad \phi_0 = v$ (solution)

$\frac{1}{2} |D_\mu \phi|^2 \to \frac{1}{2} g^2 A_\mu A^\mu v^2 \quad \downarrow \quad m_h = \lambda v$

$\boxed{m_A = gv} \Leftrightarrow$ trivial map

$$\boxed{\begin{matrix} \phi_0(\infty) \longrightarrow v \\ \| \\ S_1 \end{matrix}} \qquad (E(\phi_0) = 0)$$

(b) string: $\underbrace{\phi_s^{(1)}(\infty) \longrightarrow v e^{i\theta}}_{\text{at infinity}}$

(ρ, θ, z)

$\Downarrow$

$$\underbrace{\phi_s(0) = 0}_{\text{maximum of } V}$$

side comment

most general map $\phi_s^{(n)}(\infty) \to v e^{in\theta}$

string: $D_\mu \phi = (\partial_\mu - i g A_\mu)\phi \xrightarrow[\infty]{} 0$

$$\phi \to v e^{i\theta}$$

$$\Rightarrow \boxed{A_\mu \xrightarrow[\infty]{} \frac{1}{g} \partial_\mu \theta}$$

$\boxed{\text{pure gauge ?}}$

$\boxed{\text{NO}}$ $\boxed{\text{Nielsen, Olesen '71}}$

$$S_1^\infty : \quad \oint A_\mu dx^\mu = \frac{1}{g} \oint dx^\mu \partial_\mu \theta$$

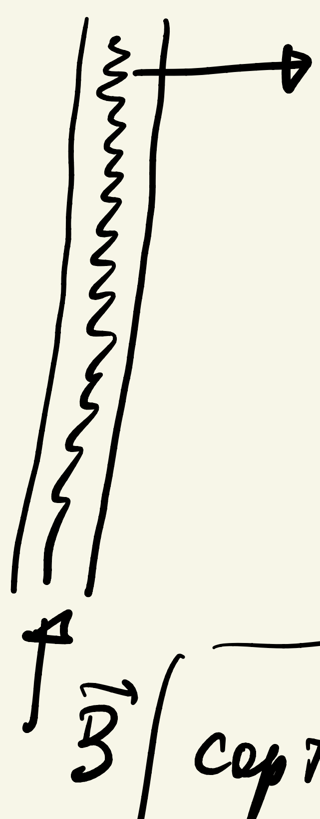
$$= \frac{1}{g} \int_{S_1} d\theta = \frac{2\pi}{g}$$

$$\Downarrow$$

$$\oint A_\mu dx^\mu = \int \vec{B}\, d\vec{S} \qquad (\vec{B} = \nabla \times \vec{A})$$

$\Rightarrow$ magnetic flux !!!

$$\boxed{\Phi_{magnetic} = \frac{2\pi}{\delta}}$$



$\rightarrow$ width $\delta \sim \frac{1}{v}$

$\vec{B}$ $\boxed{\text{captured inside}}$

Digression   width of DW

$$\frac{E}{S} = \int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\left(\frac{d\phi_{dw}}{dz}\right)^2 + V \right]$$

$$\phi_{dw}(z) = v \tanh \lambda v z$$

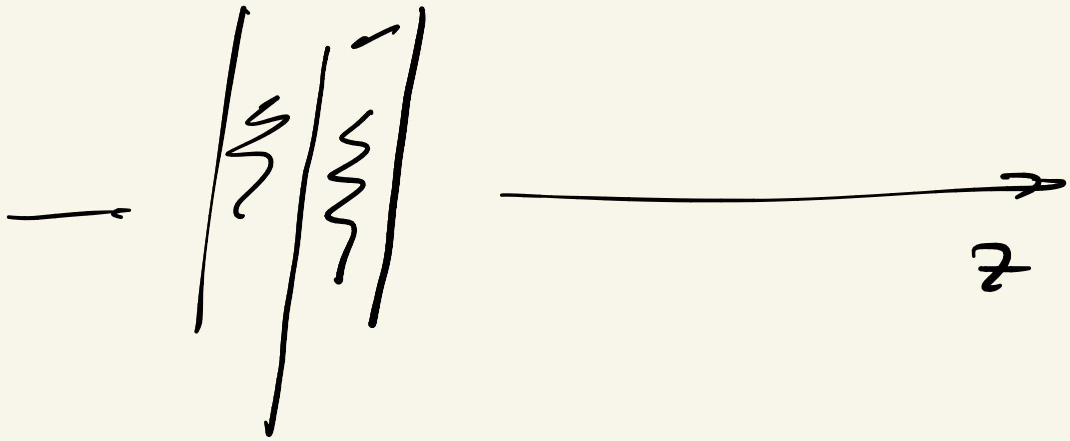
$$\delta \simeq \frac{1}{\lambda v}$$

width

$$|z| \gg \delta \Rightarrow \phi_{dw} \to \pm v$$

$$\boxed{DW = \text{concentrated at the origin}}$$

$\uparrow \phi$



$$|z| < \delta : \quad \phi_{dw} = 0$$

$$|z| > \delta : \quad |\phi_{dw}| = v$$

$$\Downarrow$$

$$E/S = \int_{-\delta}^{\delta} dz \left[\frac{1}{2} \left| \frac{\Delta\phi}{2\delta} \right|^2 + V(\phi_{dw}=0) \right]$$

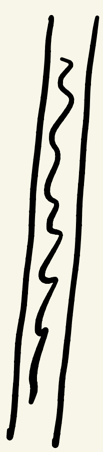
$$= \int_{-\delta}^{\delta} dz \left[\frac{1}{2} \frac{\cancel{4} v^2}{\cancel{4}\delta^2} + \frac{\lambda^2}{2} v^4 \right]$$

$$E/S = \underset{\delta \to \infty}{\frac{v^2}{\delta}} + \underbrace{\lambda^2 v^4 \delta}_{\delta \to 0}$$

$$0 = \frac{\partial\, E/S}{\partial \delta} = -\frac{v^2}{\delta^2} + \lambda^2 v^4 = 0$$

$$\Rightarrow \boxed{\delta = \frac{1}{\lambda v}}$$

$$E/L = \int dS \left[\frac{1}{2}\left(\vec{B}^2 + \underset{\substack{\| \\ 0}}{\vec{E}^2}\right) + V \right]$$



$\leftarrow$ $B =$ at the origin $\to$ up to δ

$V = 0$

$$\Phi_m = \frac{2\pi}{g}$$

$\Downarrow$

$$B \cdot \cancel{\pi} \delta^2 = \frac{2\cancel{\pi}}{g} \Rightarrow \boxed{B = \frac{2}{g\delta^2}}$$

$$\Rightarrow E/L = \int_0^\delta d\delta \left[\frac{1}{2} \frac{4}{g^2\delta^4} + \frac{\lambda^2}{2} v^4 \right]$$

$$= \pi \left[\frac{2}{g^2\delta^2} + \frac{\lambda^2}{2} v^4 \delta^2 \right]$$

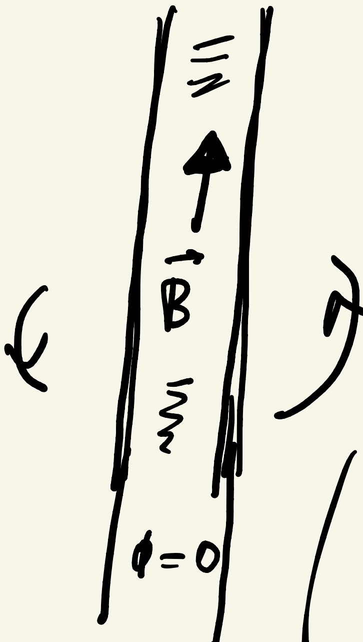
$$\Rightarrow \frac{\partial(E/L)}{\partial\delta} = 0 \Longrightarrow$$

$$\boxed{\frac{2}{g^2\delta^3} = \frac{\lambda^2}{2} v^4 \delta}$$

$$\Downarrow$$

$$\delta^4 = \frac{4}{v^4} \frac{1}{g^2\delta^2}$$

$$\Rightarrow \boxed{\delta \simeq \frac{1}{v}\frac{1}{\sqrt{g\lambda}}}$$



$$\boxed{\phi_s(\infty) = v e^{i\Theta}}$$

$$\boxed{\rho > \delta \longrightarrow \text{infinity}}$$

$$\phi_s(0) \stackrel{?}{=} v e^{i\Theta} \Leftarrow \underline{\text{impossible!}}$$

not well defined

$$\boxed{\phi_s(\rho) = f(\rho)\, e^{i\Theta}} \;\therefore$$

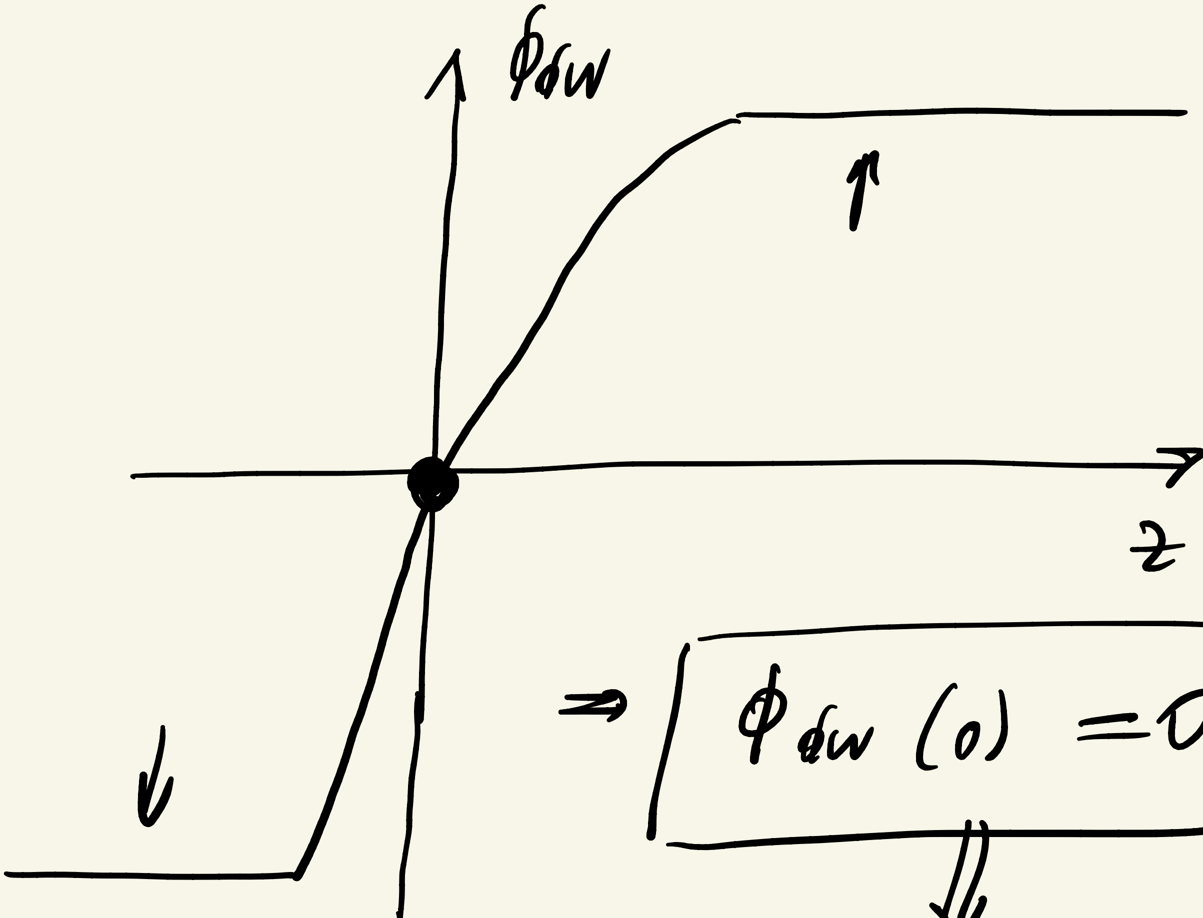
$$\boxed{\begin{aligned} f(0) &= 0 \\ f(\infty) &= v \end{aligned}}$$

$$E/L = \int ds \left[\frac{1}{2} B^2 + \underbrace{V}_{v^4 \delta} \right]$$

$$\boxed{E/L \simeq v^4 \delta^2 \simeq v^2} \qquad \left(\delta \simeq \frac{1}{v}\right)$$

Domain wall

$$\boxed{\phi_{dw}(z) = v \tanh \lambda v z}$$



$$\Rightarrow \boxed{\phi_{dw}(0) = 0}$$

$$\Downarrow$$

$$\boxed{\text{Energy} > 0}$$

- $j_\mu = \epsilon_{\mu\nu}\partial^\nu \phi \frac{1}{2v}(t,z)$

$$\Rightarrow \partial^\mu j_\mu = 0 \Rightarrow Q = \int dz\, j^0$$

$$= \int dz \frac{d\phi}{dz}\frac{1}{2v} = \int d\phi \frac{1}{2v}$$

$$Q = \frac{1}{2v} \left[\phi(\infty) - \phi(-\infty) \right]$$

$$Q(dw) = 1 \qquad Q(vac) = 0$$

$$Q(anti) = -1$$

<u>String</u> $(t; x, y)$

$$j^\mu = \epsilon^{\mu\nu\rho} \, \partial_\nu \phi_a \partial_\rho \phi_b \, \epsilon_{ab} \, C$$

$$\phi = \phi_1 + i \phi_2 \qquad \begin{pmatrix} \varepsilon_{12} = -\varepsilon_{21} = 1 \\ \varepsilon_{11} = \varepsilon_{22} = 0 \end{pmatrix}$$

$$\partial_\mu j^\mu = \epsilon^{\mu\nu\rho} \left[\partial_\mu \partial_\nu \phi_a \partial_\rho \phi_b \, \varepsilon_{ab} \right.$$

$$\left. + \partial_\nu \phi_a \, \partial_\mu \partial_\rho \phi_b \, \varepsilon_{ab} \right] = 0$$

$$j^0 = \varepsilon_{ij}\,\varepsilon_{ab}\,\partial_i \phi_a \,\partial_j \phi_b \; C$$

$$= \varepsilon_{ij}\,\varepsilon_{ab}\Big[\partial_i\left(\phi_a\,\partial_j \phi_b\right)$$

$$- \cancel{\phi_a\,\partial_i \partial_j \phi_b}\Big]\; C$$

$$\boxed{\begin{aligned} j^0 &= \varepsilon_{ij}\,\partial_i\left(V_j\right) C \\ V_i &= \varepsilon_{ab}\,\phi_a\,\partial_i \phi_b \end{aligned}}$$

$$\int j^0\, dS = C \oint_\infty dx_i \; V_i$$

$$= C \oint_\infty dx_i \left(\varepsilon_{ab}\,\phi_a\,\partial_i \phi_b\right)$$

$$\phi \longrightarrow v e^{i\theta}$$

$$\frac{1}{v}\phi_a = \frac{x_a}{\rho} \Leftarrow \begin{cases} \phi_1 \xrightarrow{\ } v\cos\theta = v\, x/\rho \\ \phi_2 \xrightarrow{\ } v\sin\theta = v\, y/\rho \end{cases}$$

$$Q = \int j^0 dS = C v^2 \oint dx_i\, \varepsilon_{ab} \frac{x_a}{\rho} \partial_i \frac{x_b}{\rho}$$

$$= C v^2 \oint dx_i\, \varepsilon_{ab} \frac{x_a}{\rho} \left(\delta_{ib} \frac{1}{\rho} + x_b \partial_i \frac{1}{\rho} \right)$$

$$= C v^2 \oint dx_i \left(\varepsilon_{ai} \frac{x_a}{\rho^2} + \cancel{\varepsilon_{ab} \frac{x_a x_b}{\rho} \partial_i \frac{1}{\rho}} \right)$$

$$\Downarrow$$

$$\boxed{Q = C v^2 \oint dx_i\, \varepsilon_{ai} \frac{x_a}{\rho^2}}$$

$$\varepsilon_{12} = 1 = -\varepsilon_{21}$$

$$= C v^2 \int \left(-dx \frac{y}{\rho^2} + dy \frac{x}{\rho^2} \right)$$

$$x = \rho\cos\theta, \quad y = \rho\sin\theta$$

$$\Rightarrow Q_s = c v^2 \int_0^{2\pi} \left(\frac{\cancel{\rho^2} \sin^2\theta}{\cancel{\rho^2}} + \frac{\cancel{\rho^2} \cos^2\theta}{\cancel{\rho^2}} \right) d\theta$$

$$= c\, v^2\, 2\pi \neq 0$$

$$\boxed{Q_s = 1}$$

$$\boxed{2\pi c v^2 = 1}$$

$$\Rightarrow Q_{vac} = 0$$

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anti-string:

$$\phi_s \xrightarrow[\rho \to \infty]{} v e^{i\theta}$$

$$\phi_{as} \xrightarrow[\rho \to \infty]{} v e^{-i\theta}$$

$$\Longrightarrow \boxed{Q_{as} = -1}$$

[ensures string stability]

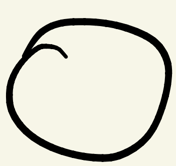
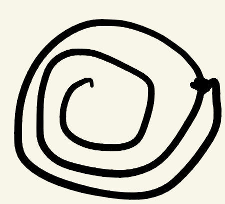
$$\phi \to e^{i\alpha}\phi \quad \Rightarrow \quad \boxed{\alpha(2\pi) = 2\pi n}$$

$$\phi(2\pi) = \phi(0)$$

single-valued field f-n

<u>Summary</u>

$$\underbrace{\phi^{(n)} \xrightarrow[\rho\to\infty]{} v\, e^{in\theta}}_{\text{most general single-valued } \phi}$$

M_∞  M_0  n

$$*** \quad \boxed{Q(\phi^n) = n\, Q(\phi_1) = n} \quad **$$

$$\boxed{\text{PROVE}}$$

long string (over universe)

$L = R_U$ ← universe radius

$$E_s = v^2 L = v^2 R_U$$

$$\boxed{\text{Mass}_{\text{universe}} = (E_u) \simeq 10\, E_{\text{visible}} \simeq 10 \cdot 10^{80}\,\text{GeV} \simeq 10^{81}\,\text{GeV}}$$

$$M_s = E_s \simeq v^2\, 10^{28}\,\text{cm} \quad (v \gtrsim 10^3\,\text{GeV})$$

$$\gtrsim 10^{6}\,\text{GeV}\,(10^{28}\,\text{GeV cm}) \longrightarrow 10^{14}$$

$$\gtrsim 10^{6}\, 10^{28}\, 10^{14}\ \text{GeV} \simeq 10^{48}\ \text{GeV}$$

$$\simeq 10^{-33}\ \text{Mass universe}$$

UNIVERSE

$$\underbrace{R = c/T}_{\text{universe}} \qquad \underbrace{d_H = \frac{M_{pl}}{T^2}}_{\text{horizon}}$$

$$\boxed{R/d_H\ (T \gg T_0) = \text{huge}}$$

$D = C$ (charge conjugation)