Neutrino Mass and Grand Unification

Lecture XXIII

21/11/2022

LMU

Winter 2022

Topological defects:

(DW) Domain walls (II)

(i) Are dw real? $(E_{dw} > 0)$

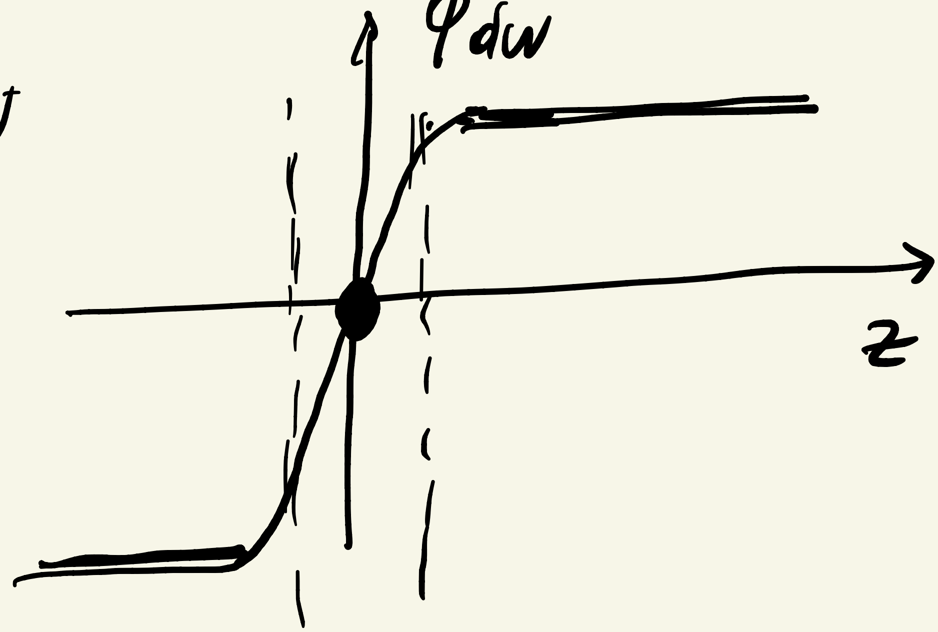
$\boxed{\text{YES}}$

(ii) If yes $\Rightarrow$ why they don't dissolve $\to$ vacuum?

$$E/_S \propto v^3 > 0 = E_{vac}$$

(iii) How to create them? <u>Cosmology</u>

(1) DW $(\phi \in R)$



$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi - \frac{\lambda^2}{2}\left(\phi^2 - v^2\right)^2$$

D: $\quad \phi \to -\phi \quad (Z_2)$

$$\boxed{\phi_{dw}(z) = v \tanh \lambda v z}$$

$\phi_{dw}(0) = 0$ & maximum of V

$$E/S = \int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\left(\frac{d\phi_{dw}}{dt}\right)^2 + V \right]$$

$$= \int_{-\infty}^{+\infty} dz\, 2V = \int_{-v}^{v} d\phi \left|\sqrt{2V}\right|$$

$$= \lambda \int_{-v}^{v} d\phi \left|(\phi^2 - v^2)\right| \propto \lambda v^3$$

YES, they are physical

(ii) conserved "charge"

(topological charge)

$$j^\mu = \epsilon^{\mu\nu} \partial_\nu \phi \qquad \mu, \nu = t, z \quad (0, 3)$$

$$(\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}) \qquad \Downarrow$$

$$\partial_\mu j^\mu = \epsilon^{\mu\nu}\partial_\mu \partial_\nu \phi = 0$$

$$\Downarrow$$

$$j^0 = \epsilon^{03}\partial_3 \phi = \frac{d\phi}{dz}(z)$$

$$\Downarrow$$

$$Q = \int_{-\infty}^{+\infty} dz\, j^0(z) = \int_{\phi(-\infty)}^{\phi(+\infty)} d\phi$$

$$\boxed{Q = \phi(+\infty) - \phi(-\infty)}$$

$$\left(\frac{dQ}{dt} = 0\right)$$

$$\Rightarrow Q_{vac} = 0 \quad (\phi_0 = v \; (-v))$$

$$\{ \phi_0(+\infty) = \phi_0(-\infty)$$

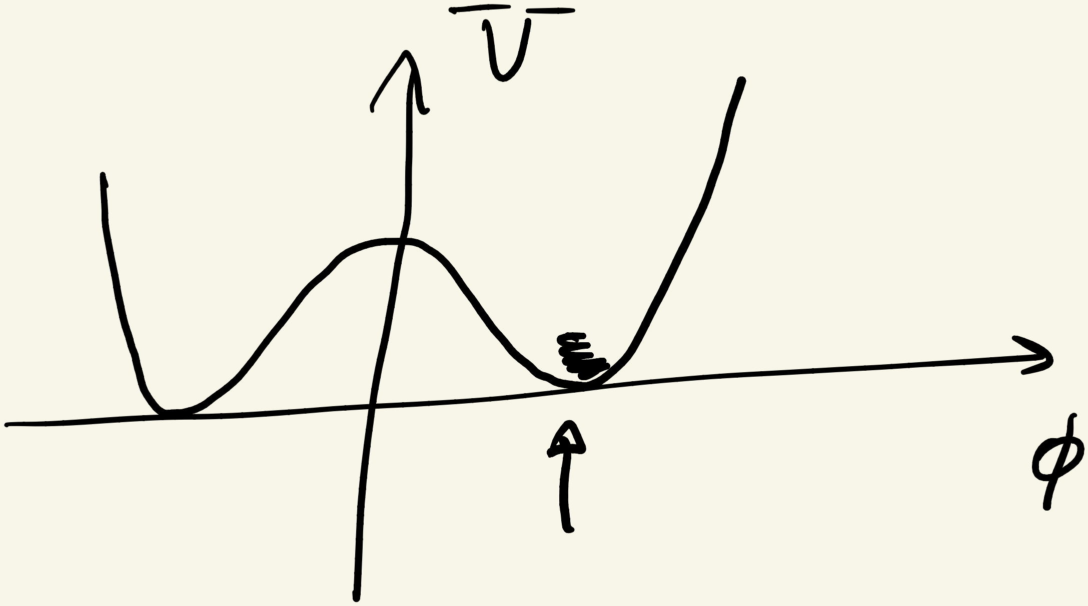
$$\boxed{\begin{aligned} Q_{dw} &= v - (-v) = 2v \\ Q_{anti\,dw} &= -v - v = -2v \end{aligned}}$$

$$\Rightarrow \boxed{DW = \text{stable}}$$

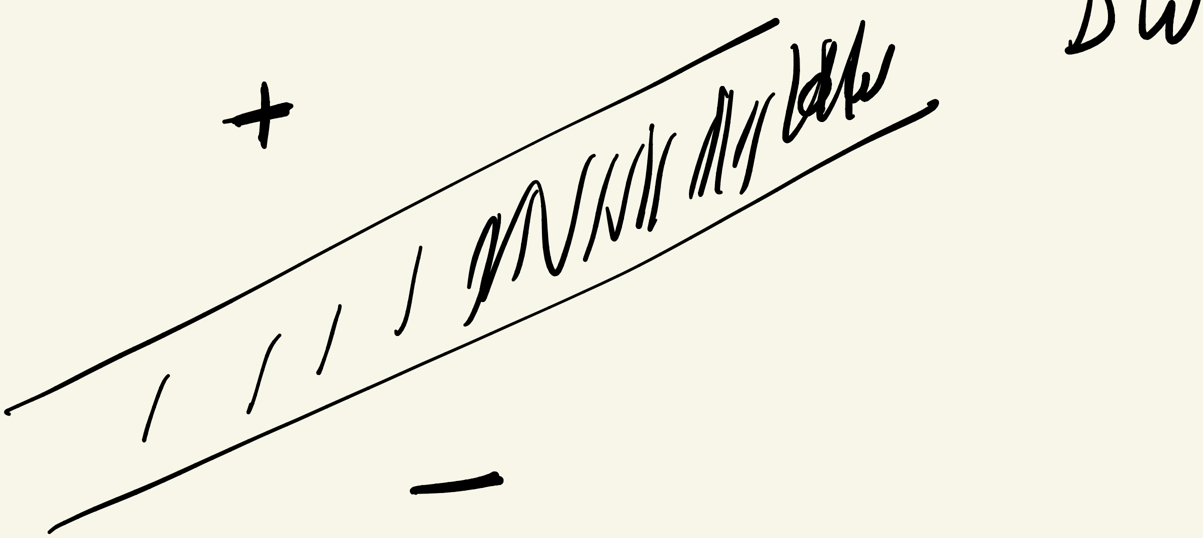
(iii) <u>Wall creation</u>

$$V = \frac{\lambda^2}{2}(\phi^2 - v^2)^2$$

$$\Downarrow$$



<u>while</u>



<u>Comment on j_0</u>

$$j_0 = \frac{d\phi}{dz}$$

$$\rho(dw) = \frac{d\phi_{dw}}{dz} = v^2\lambda \frac{1}{\cos^2 v\lambda z}$$

around zero

$\boxed{\text{charge} - \text{lives near } z=0}$

$$\boxed{\delta = \frac{1}{v\lambda}}$$

width Dw

Evolution of universe

universe was hot → getting cold

$$\boxed{\text{mass scale} = v}$$

(a) $T \gg v$

(b) $T \ll v$

(b) $T \to 0$

$$\boxed{\begin{aligned} T_\nu &\simeq 3^\circ K \\ &\simeq 10^{-4}\, eV \end{aligned}}$$

$$V \equiv V_0 = \frac{\lambda^2}{2}\left(\phi^2 - v^2\right)^2$$

$$= \lambda^2/_2\, \phi^4 + \frac{\lambda^2}{2} v^4 \boxed{- \lambda^2 v^2 \phi^2} \leftarrow$$

(a) $\underline{T \gg v}$

1972 <u>Kirzhnits</u>

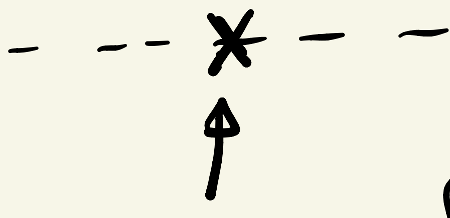
(1974

Dolan, Jackiw

Weinberg)

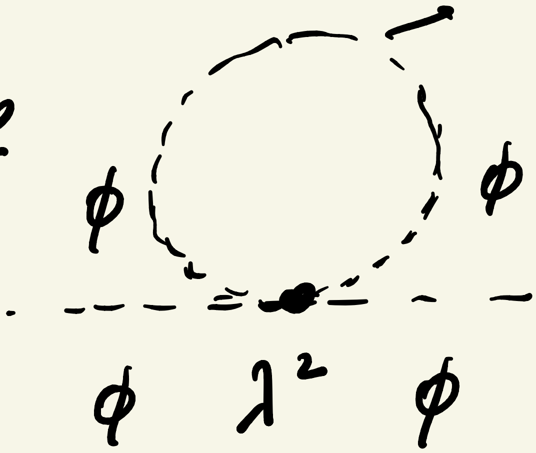
· $\mu^2 \equiv \lambda v^2 \Rightarrow -\mu_0^2 \phi^2 \in V$

$$\boxed{m^2 = 2\mu^2}$$



$$\int d^4k \, \frac{1}{k^2 - m^2} \qquad (T=0)$$

<u>loop</u>



$$\to \lambda^2 \Lambda^2$$

cut-off

$$\boxed{\underbrace{(-\mu_0^2 + \lambda^2 \Lambda^2)}_{-\mu^2} \phi^2}$$

renormalization of mass

high $T \Rightarrow \dfrac{1}{k^2-\cancel{m^2}} \to \dfrac{1}{k^2-T^2}$

expected

$\boxed{c = \hbar = k = 1}$

$d(T) = d(E) = d(m)$

$$\Downarrow$$

$$V(T \gg v) = V_0 + \lambda^2 T^2 \phi^2$$

$$= \lambda^2 (T^2 - \cancel{v^2}) \phi^2 + \frac{\lambda^2}{2} \phi^4 + \cdots$$

$$m_T^2(\phi) = \lambda^2 T^2 > 0$$

$$\Downarrow$$

$$\boxed{\langle\phi\rangle_T = 0 \qquad (T \gg v)}$$

· gauge theory

$$m_T^2(\phi) = a\,T^2|\phi|^2$$

$$a = \lambda^2 + g^2 > 0$$

$\Rightarrow$ $\boxed{\text{SM symmetry is restored at } T \gg M_W}$

SM: $T_c \simeq M_W \simeq 100\,\text{GeV} \simeq 10^{11}\,\text{eV}$
$\simeq 10^{15}\,{}^{\circ}\text{K}$

$\boxed{\text{Universe}}$ today R_u^0 = visible universe

$$\underset{\substack{\uparrow \\ \text{today}}}{R_U^0} \simeq 10^{10}\,\mathrm{yr} \simeq 10^{28}\,\mathrm{cm} = \underset{\substack{\uparrow \\ \text{horizon today}}}{\underline{d_H(t_0)}}$$

$$d_H(t) = t = \frac{1}{T}\frac{M_{pe}}{T}$$

$$\boxed{\boxed{t \simeq \frac{M_{pe}}{T^2}}\ \text{big bang model}}$$

$$\boxed{ds^2 = dt^2 - R^2(t)\,d\vec{x}^{\,2}}$$

$\uparrow$

size

$$\underset{\text{today}}{R(t=t_0)} = R_U^0 \simeq 10^{28}\,\text{cm}$$

- <u>nothing is lost</u>

$$N = Vn = R^3 T^3 = \text{const.}$$

$\wr$

\# of states

$\boxed{\text{density at high } T}$

$$n = T^3$$

dimension argument

$\Downarrow$

$$R_U = \frac{c}{T_U} \qquad c = \text{const.}$$

$$c = R_U T_U \simeq 10^{28}\,\text{cm}\; 10^{-4}\,\text{eV}$$

$$\simeq 10^{28}\, 10^{-13}\,\text{cm GeV}$$

$$\boxed{\text{cm GeV} \simeq 10^{14}}$$

$$\boxed{\text{GeV}^{-1} \simeq 10^{-14}\,\text{cm}}$$

size of proton

$$\boxed{c \simeq 10^{29}}$$

$$t = \frac{M_{pl}}{T^2} \leftarrow \boxed{\underbrace{\rho_r \simeq T^4}}$$

true for high T $\Uparrow$ dimensional grounds

$$\boxed{\cdot\ \frac{n_B}{n_\gamma} = \frac{\text{baryon}}{\text{photon}} = \frac{\overset{\simeq 10^{80}}{N_B}}{\underset{\simeq 10^{90}}{N_\gamma}} \simeq 10^{-10}}$$

$$\boxed{m_B \simeq \text{GeV}}$$

(today) $E_B \simeq m_B N_B \simeq N_B \text{ GeV}$

$E_\gamma \simeq T_\gamma N_\gamma \simeq 10^{-13} \text{ GeV } N_\gamma$

$T_\gamma \simeq 10^{-4}\,\text{eV}$

$$\Rightarrow \quad E_B \simeq N_B\,\text{GeV} \simeq 10^{80}\,\text{GeV}$$

$$E_\gamma \simeq 10^{-13}\,10^{10}\,N_B\,\text{GeV} \simeq 10^{-3}\,N_B\,\text{GeV}$$

$$\simeq 10^{77}\,\text{GeV}$$

$$\boxed{\frac{E_\gamma}{E_B} = 10^{-3} \qquad \frac{N_\gamma}{N_B} \simeq \frac{n_\gamma}{n_B} = 10^{10}}$$

$$\boxed{\frac{E_{DM}}{E_B} \simeq 10 \quad \text{(Dark Matter)}}$$

(Kolb, Turner)

| Cosmology |

- $R = \frac{10^{29}}{T}$ $\quad t = \frac{M_{pe}}{T^2} = d_H$

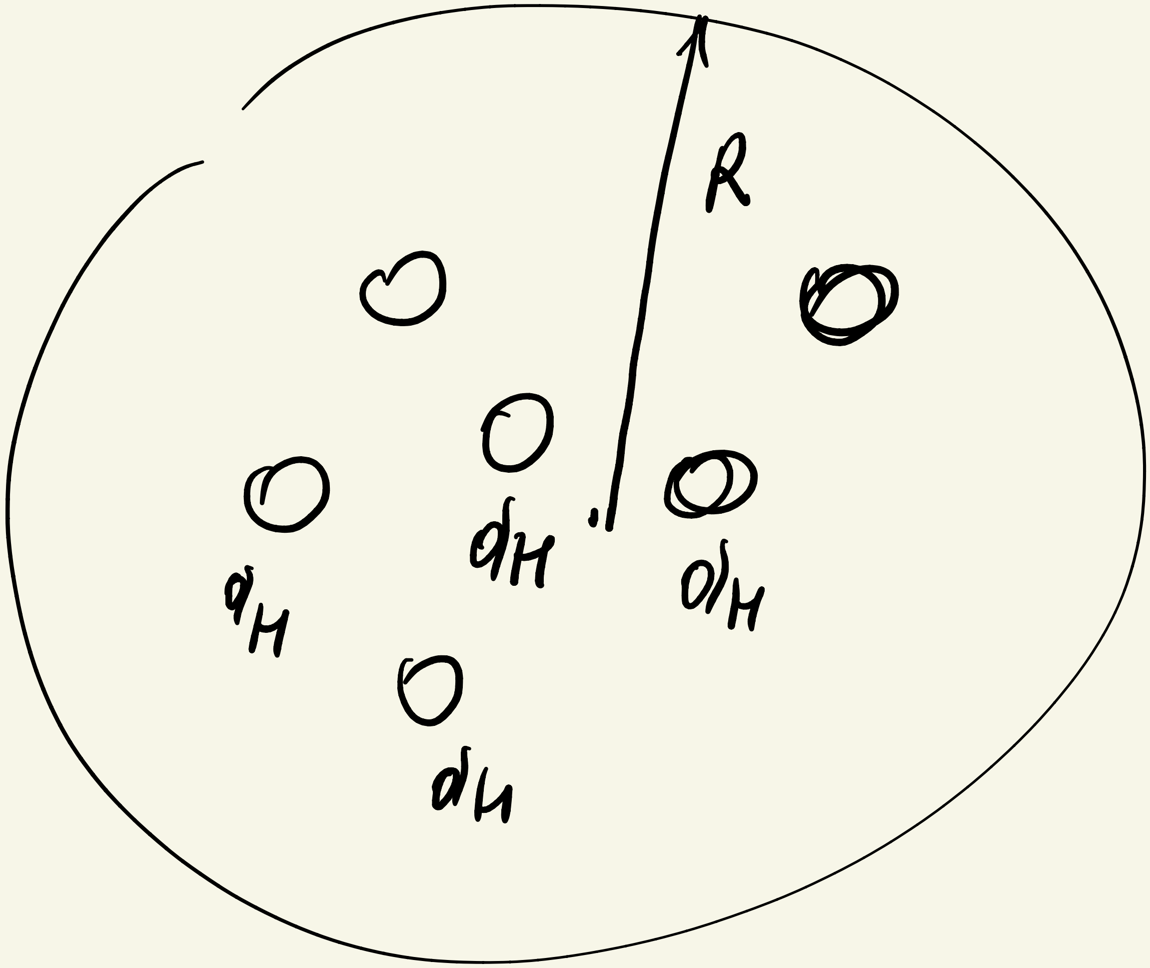
today $R_0 = t_0$ (def.)

$$\boxed{R/d_H = 10^{29} \frac{T}{M_{pe}}}$$

<u>nucleosynthesis</u> : $T \simeq MeV$

$$R/d_H = 10^{29} \frac{10^{-3}}{10^{19}} \simeq 10^{7}$$

$$\left(R/d_H\right)^3 \simeq 10^{21}$$

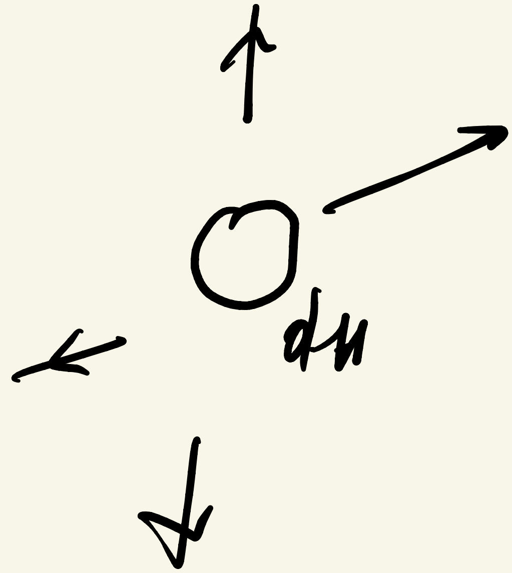


$\boxed{d_H \text{ pieces} \longleftrightarrow \text{never in contact}}$

$\boxed{\text{HORIZON PROBLEM}}$

$\Downarrow$

INFLATION

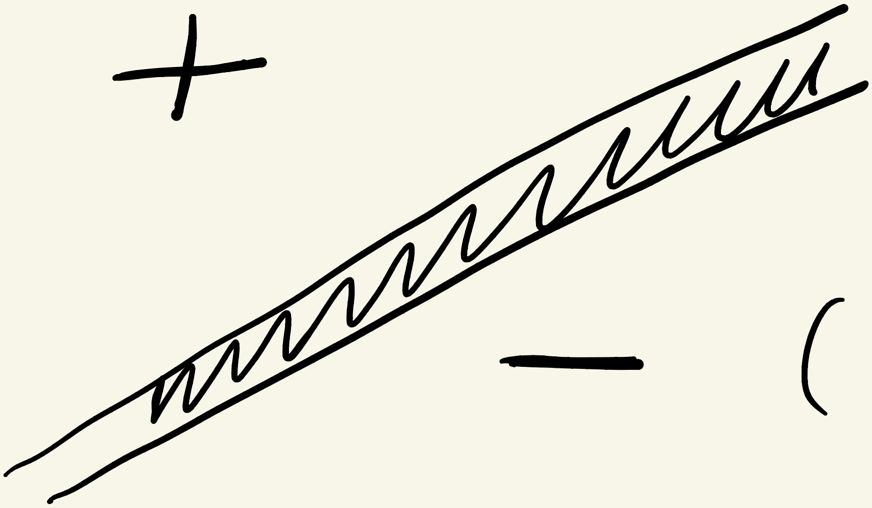


exponential growth !

$\boxed{\cdot\, T \gg v}$ $\quad \langle \phi \rangle = 0$

$T \ll v \quad \langle \phi \rangle = v \ (-v)$

$\Downarrow$ Kibble mechanism



$$\left(E/S = v^3 \right)$$

<u>one wall in universe</u>

$$E_{wall} = S\, v^3 = R_U^2\, v^3$$

$$\left(E_{univer} = 10^{80}\,\text{GeV} \cdot \underset{\uparrow \atop DM}{10} \simeq 10^{81}\,\text{GeV} \right)$$

$$\text{GeV cm} \simeq 10^{14}$$

$$E_{wall} = 10^{56}\,\text{cm}^2\, v^3$$

$(v > 10^3\,\mathrm{GeV})$ $\boxed{\text{LHC}}$

$$\boxed{\begin{aligned} E_{wall} &> (10^{28}\,\mathrm{cm} \cdot 10^3\,\mathrm{GeV})^2 \cdot 10^3\,\mathrm{GeV} \\ &\Rightarrow (10^{31} \cdot 10^{14})^2\, 10^3\,\mathrm{GeV} \\ &\Rightarrow 10^{93}\,\mathrm{GeV} > 10^{12}\, E_{universe} \end{aligned}}$$

$\boxed{\text{DW = disaster}}$

$\boxed{\text{DW Problem}}$

Solutions

① <u>inflation</u>

② <u>symmetry non-restoration</u>

(high T)

Weinberg '74

Mohapatra, G.S. '79

Dvali, G.S. '94

③ $V = -\phi^2 + \phi^4 \quad (\phi \to -\phi)$

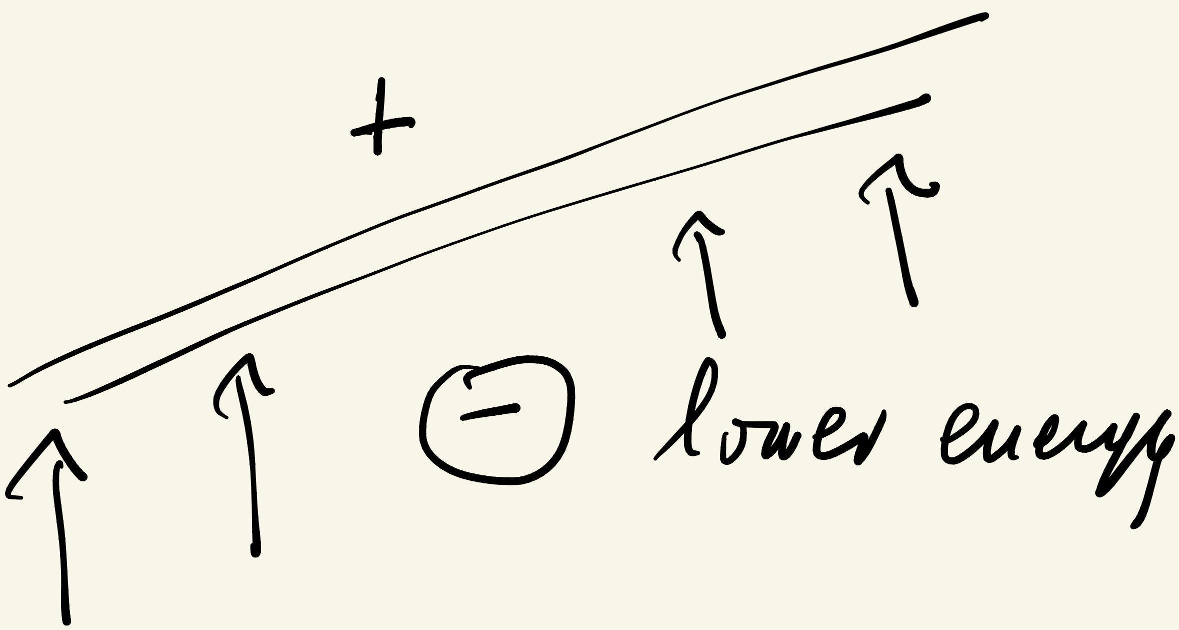
$$+ \frac{\phi^5}{M_{pl}} \quad (\text{gravity})$$

$\downarrow$

breaks $\phi \to -\phi$

$$\simeq \underbrace{\frac{\langle \phi \rangle}{M_{pl}}}_{(\varepsilon \ll 1)} \phi^4$$

Rai, G.S. '95



if $\varepsilon \simeq \dfrac{\text{TeV}}{M_{pl}} \simeq 10^{-16}$

$\Downarrow$

$\boxed{\text{dw pushed away}}$

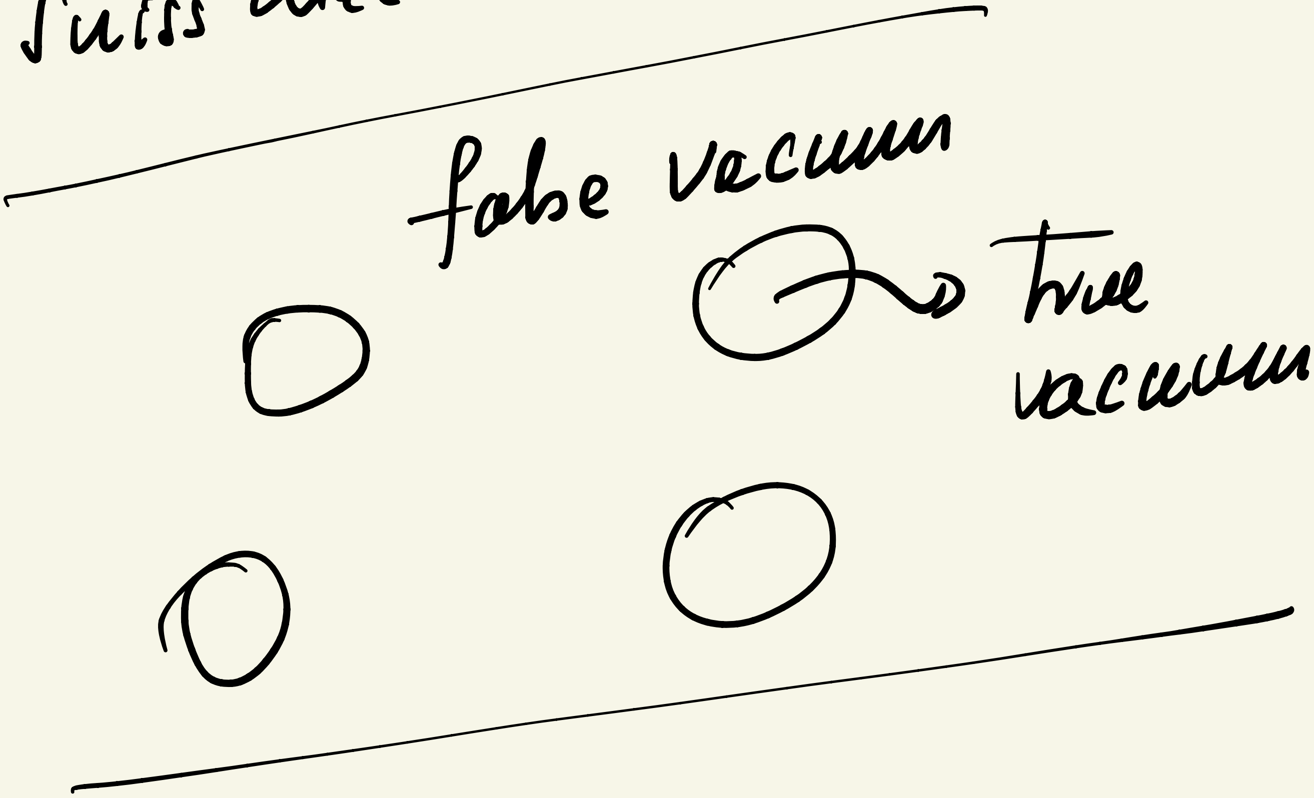
<u>Inflation</u> Guth '80

24_H of $SU(5)$

$SU(3) \times SU(2) \times U(1)$

- $SU(4) \times U(1)$ ← false vacuum

Swiss cheese



true vacuum grows →

SM vacuum

Guth, E. Weinberg
'1982

Linde - -

add by hand

ϕ : $V = \frac{\lambda}{4}\phi^4$, $\lambda \simeq 10^{-14}$

DW: $\phi \to -\phi$

example LR : $\cancel{P}$ sport.

$$\varphi_L \xleftrightarrow{P} \varphi_R$$

$\langle \varphi_L \rangle = 0$ $\langle \varphi_R \rangle = v_R$

$$\phi \longrightarrow (\varphi_L - \varphi_R) \longrightarrow -(\varphi_L - \varphi_R)$$

Dvali, Melfo, G.S. '95

Post: Guth '80

Guth, Weinberg '82

Weinberg '79