Neutrino Mass
and
Grand Unification

Lecture XX

11/1/2022

LMU
Winter 2022

Chiral Anomalies

'64

Adler
Bell, Jackiw

① gauge theories are

renormalisable

$\Updownarrow$

$\boxed{\text{current conservation}}$

$\Downarrow$

Ward identities in QED

| true in unbroken and |

Spontaneously broken theories

① Chiral gauge theories

$$T_L \neq T_R$$

<u>example</u> SM

$\downarrow$

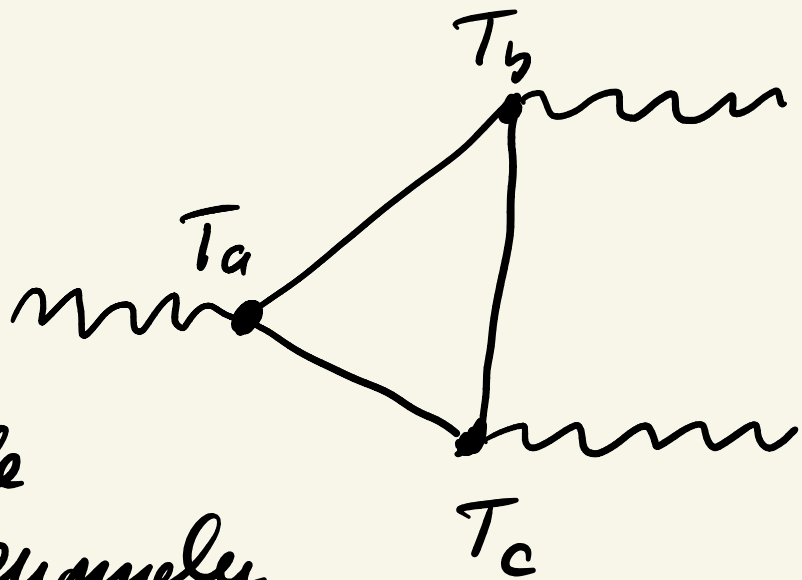
weak $T_L = \sigma/2$, $T_R = 0$

$\Rightarrow$ $\boxed{\text{currents with } \gamma_5}$

e.g. $\frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma^\mu e_L$

$\Downarrow$

anomalies



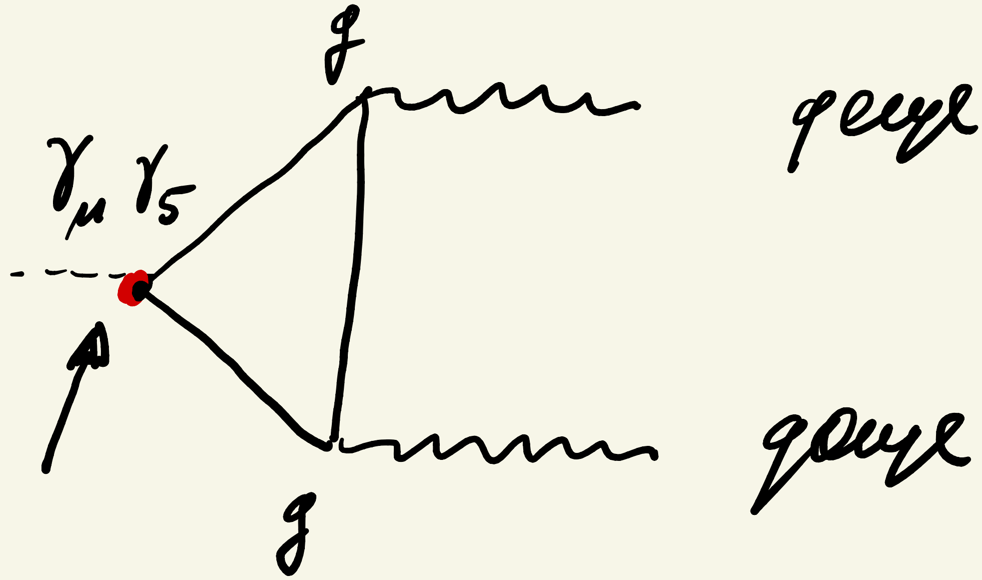
$= 0$

Triangle anomaly

triangle diagrams are no-no!

Triangle anomalies must vanish in a chiral

gauge theory



$$\mathcal{L}_D = i\,\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi$$

$$= i\,\bar{\psi}\gamma_\mu \partial_\mu \psi - m\bar{\psi}\psi + \mathcal{L}_{int}$$

$\Rightarrow$ $\psi \to e^{i\alpha}\psi$ = symmetry

$$\Downarrow$$

$$\boxed{\begin{aligned} \partial_\mu j^\mu &= 0 \\ j_\mu &= \bar{\psi}\gamma^\mu\psi \end{aligned}}$$

$$\psi \to e^{i\alpha}\psi \iff \begin{aligned} \psi_L &\to e^{i\alpha}\psi_L \\ \psi_R &\to e^{i\alpha}\psi_R \end{aligned}$$

- $m = 0 \Rightarrow$

$$\psi_L \to e^{i\alpha}\psi_L, \quad \psi_R \to e^{i\beta}\psi_R$$

$$\Rightarrow \quad \psi \to e^{i\beta\gamma_5}\psi$$

$$\Rightarrow \boxed{\partial_\mu j_5^\mu = 0}$$

$$\boxed{j_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi}$$

Quantum level (loops)

$$\boxed{\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} F_{\mu\nu} F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta}}$$

symmetry breaking by loops !

$\Rightarrow$ disaster for chiral gauge theory :

loss of c.c. $\Rightarrow$ loss of Ward identities

$\Uparrow$ $\Downarrow$

current conservation

$\boxed{\text{loss of renormalizability}}$

$\Downarrow$

Anomalies (chiral) must cancel:

(i) enough at 1-loop level

(ii) anomaly does not depend

on fermion mass in the loop

Anomaly cancellation

- $\mathcal{A}_L = \mathcal{A}_R$

  e.g. Q. C. D.

  $$T_L = T_R$$

  $$\boxed{\Rightarrow \text{ no } \gamma_5 \text{ int.}}$$

- Anomaly : $T_L \neq T_R$

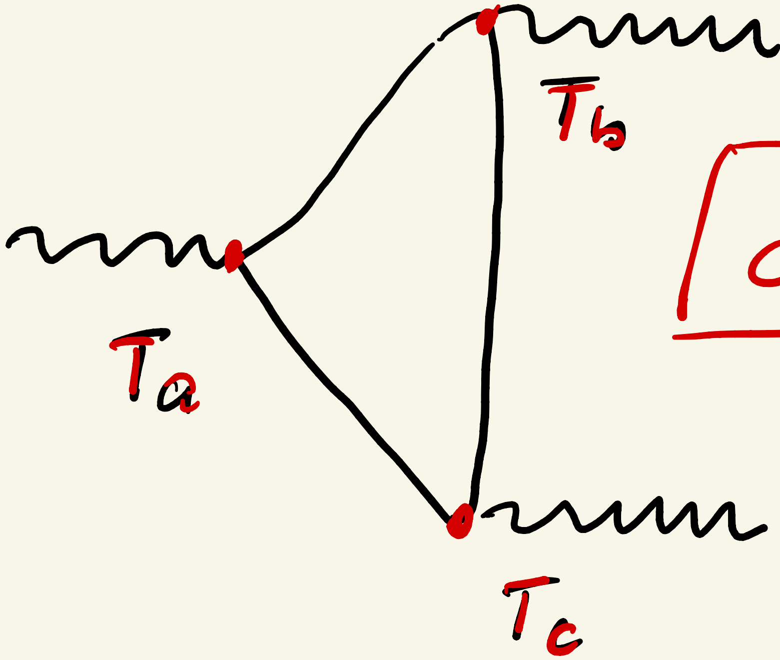
e.g. ew SM

$$\left\{ \underline{\text{digression}} \quad \text{i)}\ f_L,\ f_R \Rightarrow \mathcal{A}_L = \mathcal{A}_R \right.$$

$$\left. \text{ii)}\ f_L,\ (f^c)_L \equiv c\,\bar{f}_R^T \Rightarrow \mathcal{A}_L + (\mathcal{A}^c)_L = 0 \right\}$$

$\boxed{\text{SM}}$ (minimal)

$$\left.\begin{array}{ll} \begin{pmatrix} u \\ d \end{pmatrix}_L & u_R,\ d_R \\ \begin{pmatrix} \nu \\ e \end{pmatrix}_L & e_R \\ & Q = T_3 + \frac{Y}{2} \end{array}\right\} \text{stay tuned}$$



$\boxed{\text{chiral}}$

$$A_{abc} \propto \mathrm{Tr}\, T_a T_b T_c + \text{perm.}$$

$$\propto \mathrm{Tr}\, \{T_a, T_b\} T_c \qquad \boxed{\text{PROVE}}$$

$$\propto d_{abc}$$

universal

$\Updownarrow$

- $[T_a, T_b] = i f_{abc} T_c$

$$\boxed{\begin{array}{l} \mathcal{A}_{abc}(R) = \mathcal{A}(R)\, d_{abc} \\ \quad \therefore \quad \mathcal{A}(F = \text{fundamental}) = 1 \end{array}}$$

- an anomaly free group?

$$\Rightarrow \boxed{SU(2)}$$

F : $\quad T_a = \sigma_a/2$

$$\mathrm{Tr}\{\sigma_a, \sigma_b\} = 2\,\delta_{ab}$$

$$\Rightarrow \mathcal{A}_{abc}(F) = \mathrm{Tr}\{T_a, T_b\}\, T_c$$

$$= \frac{1}{8} \mathrm{Tr} \{ \sigma_a, \sigma_b \} \sigma_c =$$

$$= \frac{1}{4} (\mathrm{Tr}\, \sigma_c) \, \delta_{ab} = 0$$

$$\boxed{\Rightarrow \ \mathcal{A}_{abc} (R) = 0}$$

Q. E. D.

- Fundamental rep. $^{(F)}$ $\quad F \to U F$

$$U = e^{i \theta_a T_a} \qquad SU(N)$$

$$[T_a, T_b] = i f_{abc} T_c$$

$$T_a = T_a^{\dagger}, \quad \mathrm{Tr}\, T_a = 0$$

$\Downarrow$

anti F !

$$F^* \longrightarrow U^* F^* = e^{-i\theta_a T_a^*} F^*$$

$$= e^{i\theta_a(-T_a^*)} F^*$$

$$\hat{T}_a F^* = (-T_a^*) F^*$$

$$= (-T_a^T) F^*$$

$$\mathcal{A}_{abc}(\bar{F}) = -\mathrm{Tr}\,\{T_a^T, T_b^T\} T_c^T$$

$$= -\mathrm{Tr}\,\{T_b, T_a\}^T T_c^T$$

$$= -\mathrm{Tr}\,\left(T_c \{T_b, T_a\}\right)^T$$

$$= -\operatorname{Tr} T_c \{T_b, T_a\}$$

$$= -\operatorname{Tr} \{T_b, T_a\} T_c$$

$$= -\operatorname{Tr} \{T_a, T_b\} T_c = -A_{abc}(F)$$

$$\boxed{A(\bar{F}) = -A(F)}$$

$$\boxed{\begin{aligned} A_{abc}(R) &= A(R)\, A_{abc}(F) \\ &= A(R)\, d_{abc} \end{aligned}}$$

anomaly for repr. R

$$A_{abc}(F) = d_{abc}$$

$$\mathcal{A}(F) = 1$$

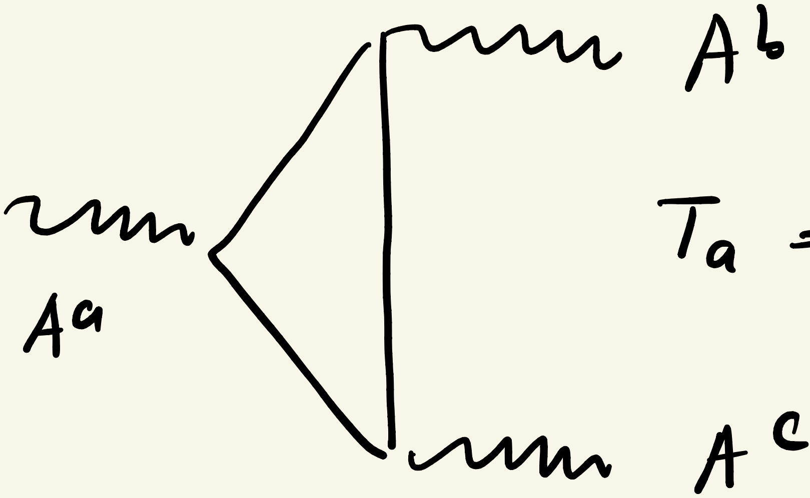
Standard Model

$(Y_q)\begin{pmatrix} u \\ d \end{pmatrix}_L$ $(Y_U)\; u_R,\; d_R\; (Y_D)$

$(Y_\ell)\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ $e_R\;(Y_E)$

- $SU(2)$ anomaly

?

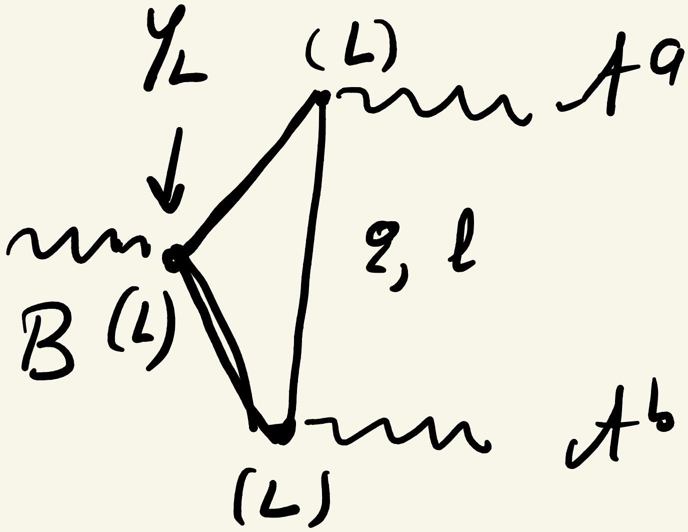


$T_a = \sigma_a/2$

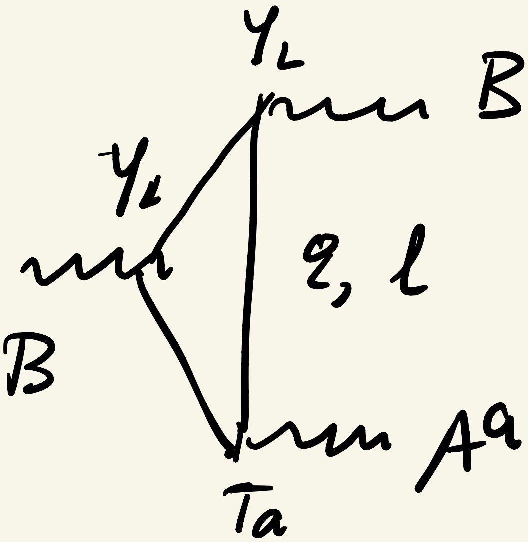
$$\left.\begin{aligned} A^a &= SU(2) \\ B &= U(1)_Y \end{aligned}\right\} \text{gauge boson}$$

$$\Rightarrow \boxed{\mathcal{A}\,(SU(2)) = 0}$$

- mixed $SU(2) - U(1)$ anomaly



(i)



(ii)

(i) $\mathrm{Tr}\, Y_L \{T_a, T_b\} = 0$

$= \mathrm{Tr}\, Y_L\, d_{ab} = 0$ (over all fermions)

$\Downarrow$

$$\boxed{\mathrm{Tr}\, Y_L = 0}$$

MUST

$\Downarrow$ $Y_L \Rightarrow$ LH fermions

$$\boxed{\underset{\text{color}}{3}\, Y_q + Y_e = 0} \qquad (1)$$

(ii) $\quad \mathrm{Tr}\, Y_L^2 T_a = 0 \qquad$ (must be)

$SU(2)_L \times U(1)_Y$

$\Longleftrightarrow [T_a, Y] = 0$

$\Rightarrow Y_L =$ fixed for a doublet

$$= \mathrm{Tr}\, Y_L^2(F) \frac{\sigma_a}{2} = y_L^2(F)\, \mathrm{Tr} \frac{\sigma_a}{2} = 0$$

$$\Rightarrow \mathrm{Tr}\, Y_L = 0 \qquad (3Y_q + Y_e) = 0$$

$$Q = T_3 + \frac{Y}{2}$$

$$\boxed{\text{QED} \Leftrightarrow Q_L = Q_R}$$

$$\mathrm{Tr}\, Y_L = 0 \Rightarrow \mathrm{Tr}\, Q_L = \mathrm{Tr}\left(T_3 + \frac{Y_L}{2}\right)$$

$$= \mathrm{Tr}\, T_3 + \frac{1}{2}\mathrm{Tr}\, Y_L$$

$$= 0 + 0 = 0$$

$$\Rightarrow \boxed{\mathrm{Tr}\, Q_L = 0 \Rightarrow \mathrm{Tr}\, Q_R = 0}$$

$$Q_R = Y_R \qquad (T_3 f_R = 0)$$

$$\Rightarrow \boxed{T_1\, Y_R = 0} \qquad \boxed{\text{MUST}}$$

$$\left(Q = T_3 + \frac{Y}{2} \right)$$

- $Q_L = Q_R$

$$u: \quad \underbrace{\frac{1}{2} + Y_q \frac{1}{2}}_{Q_L} = \underbrace{\frac{1}{2}\, Y_U}_{Q_R}$$

$$\Downarrow$$

$$\boxed{Y_U = 1 + Y_q}$$

$$d: \quad -\frac{1}{2} + \frac{1}{2} Y_q = \frac{1}{2} Y_D$$

$$\Rightarrow \boxed{Y_D = -1 + Y_q}$$

$$e: \quad \underbrace{-\frac{1}{2} + Y_e}_{Q_L} = \underbrace{\frac{1}{2} Y_E}_{Q_R}$$

$$\Rightarrow \boxed{Y_E = -1 + Y_e}$$

$$\cdot \ \mathrm{Tr}\, Y_R \overset{\downarrow}{=} 0$$

$$\Downarrow$$

$$\underset{\uparrow}{3}\left[(\cancel{1} + Y_a) + (\cancel{-1} + Y_a)\right] + (-1 + Y_e) \overset{\downarrow}{=} 0$$

colour

$$3 \cdot 2 Y_a - 1 + Y_e \overset{\downarrow}{=} 0$$

but: $\quad 3 Y_a + Y_e = 0 \quad (\mathrm{Tr}\, Y_L = 0)$

$$\Downarrow$$

$$\boxed{3Y_q - 1 = 0}$$

$$\Rightarrow \quad Y_q = 1/3$$

$$Y_e = -1$$

Charge quantization between q and e

$$\Downarrow$$

$$\left(\begin{aligned} Q_{e_L} &= -\frac{1}{2} - \frac{1}{2} = -1 \\ Q_{e_R} &= Q_{e_L} = -1 \end{aligned} \right)$$

$$Q_{u_R} = \frac{2}{3}$$

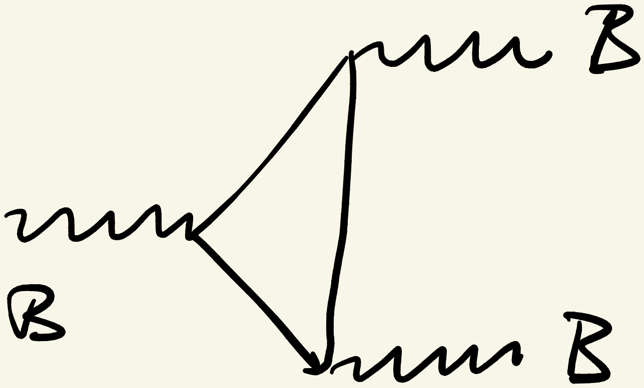
$$Q_{u_L} = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$$

$$Q_{d_L} = -\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} = -\frac{1}{2} \cdot \frac{2}{3} = -\frac{1}{3}$$

$$-\frac{1}{3} = Q_{d_L}$$

$$\boxed{Q_{\nu_L} = \frac{1}{2} - \frac{1}{2} = 0} \qquad (m_\nu \doteq 0)$$

(iii)



$$\Rightarrow \boxed{\operatorname{Tr} Y_L^3 = \operatorname{Tr} Y_R^3}$$

$(1+\gamma_5) \qquad (1-\gamma_5)$

$\boxed{\text{PROVE}}$