Neutrino Mass

and

Grand Unification

Lecture XVIII

17/12/2021

LMU

Winter 2021

Seesaw Mechanism

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$; e_R , ν_R $\leftarrow$ new

$$N_L = C \bar{\nu}_R^T \Rightarrow \boxed{L(N_L) = -1}$$

$\Downarrow$

$$\bar{\nu}_R M_D \nu_L = N_L^T C M_D \nu_L$$

Dirac neutrino mass matrix

ν_R = lepton

$L(\nu_R) = 1$

$\Downarrow$

$$\frac{1}{2} \nu_R^T C M_R \nu_R + h.c. =$$

$$= \frac{1}{2} N_L^T C M_N N_L N_L + h.c.$$

$$\underbrace{M_N = M_R^* = M_R^\dagger}_{\text{Majorana mass matrix}}$$

$$\Downarrow \qquad \boxed{C^T = -C}$$

$$\underbrace{\psi_L^T C M_M \psi_L}_{\text{Majorana}} = -\psi_L^T C^T M_M^T \psi_L$$

$$= \psi_L^T C M_M^T \psi_L$$

$$\Rightarrow \boxed{M_M^T = M}$$

$$\Downarrow$$

$$M_{\nu N} = \begin{array}{c} \\ \nu_L \\ \\ N_L \\ \\ \end{array} \begin{pmatrix} 0 & M_D^T \\ & \\ M_D & M_N \end{pmatrix} \Rightarrow M_{\nu N}^T = M_{\nu N}$$

$$\qquad\qquad\qquad\quad \nu_L \qquad\quad N_L$$

$$\underbrace{M_N \gg M_D}$$

(gauge invariant $\gg M_W$)

SSB of gauge SM symmetry $< (\ll)\ M_W$

$$\begin{pmatrix} \nu \\ N \end{pmatrix} \longrightarrow U \begin{pmatrix} \nu \\ N \end{pmatrix} \quad \therefore$$

$$\boxed{U^T M_{\nu N} U = \text{Diagonal}} \quad (1)$$

$$U = \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix} \quad (\theta \ll 1)$$

$$U U^+ = \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix} \begin{pmatrix} 1 & -\theta^+ \\ \theta & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+\theta^+\theta & \theta^+ - \theta^+ \\ -\theta+\theta & 1+\theta\theta^+ \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \theta^+\theta \to 0 \\ \theta\theta^+ \to 0 \end{pmatrix}$$

✓

$$(1)\quad \begin{pmatrix} 1 & -\theta^T \\ \theta^* & 1 \end{pmatrix} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$\simeq \begin{pmatrix} -\theta^T M_D & M_D^T - \theta^T M_N \\ M_D & \cancel{\theta^* M_D^T} + M_N \end{pmatrix} (-''-)$$

$$= \begin{pmatrix} -\theta^T M_D - \theta(M_D^T - \theta^T M_N) & \cancel{-\theta^T M_D \theta^+} + (M_D^T - \theta^T M_N) \\ \underbrace{M_D - M_N \theta}_{= 0} & \cdots + M_N \end{pmatrix}$$

$$M_D = M_N \theta$$

$$\Rightarrow \boxed{\theta = \frac{1}{M_N} M_D}$$

$$M_D^T - \Theta^T M_N = M_D^T - M_D^T \frac{1}{\cancel{M_N}} \cancel{M_N} = 0$$

$$\Downarrow$$

$$U^T \mathcal{M}_{\nu N}\, U = \begin{pmatrix} \mathcal{M}_\nu & 0 \\ 0 & \mathcal{M}_N \end{pmatrix}$$

$$\mathcal{M}_\nu = -\Theta^T \mathcal{M}_D$$

$$\Downarrow$$

$$\boxed{\mathcal{M}_\nu = -M_D^T \frac{1}{\mathcal{M}_N} M_D}$$

seesaw formula

$$E^2 = \vec{p}^{\,2} + m^2$$

mass

<u>Dirac</u>

$\cdot$ $m\bar{f}f \to -m\bar{f}f$

<u>digression</u>

$$f \to e^{i\alpha\gamma_5} f$$

$\cdot$ $m\bar{f}_L f_R + h.c.$

$$f_R \to e^{i\alpha} f_R, \quad f_L \to f_L$$

$$\longrightarrow m e^{i\alpha} \bar{f}_L f_R$$

<u>Neutrino mass</u>

<u>Dirac</u> $\quad M_N = 0 \Rightarrow M_\nu = M_D$

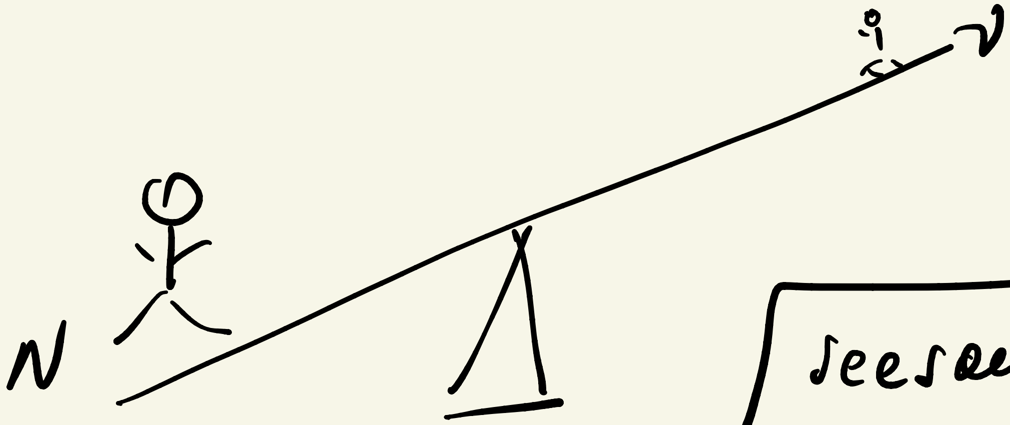
$$M_\nu \ll M_e \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}$$

why?

<u>Majorana</u> $\quad M_\nu \simeq M_D \, \theta$

$$\theta = \frac{1}{M_N} M_D$$

$$\Rightarrow \quad M_\nu \ll M_D \; (\sim M_e \, ?)$$



seesaw

How to test it?

$$\begin{pmatrix} \nu \\ N \end{pmatrix}^0 \longrightarrow U\begin{pmatrix} \nu \\ N \end{pmatrix} = \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \nu + \theta^+ N \\ -\theta\nu + N \end{pmatrix}$$

$$\nu \longrightarrow \nu + \theta^+ N$$

$$\Downarrow$$

$$\frac{g}{\sqrt{2}}\, \bar{\nu}_L \gamma^\mu e_L W_\mu^+ \Rightarrow$$

$$= \frac{g}{\sqrt{2}} \left(\bar{\nu}_L \gamma^\mu e_L W_\mu^+ + \bar{N}_L \Theta \gamma^\mu e_L W_\mu^+ \right)$$

th.c.

$\boxed{\text{new}}$

$$h.c. = \bar{e}_L \gamma^\mu \Theta N_L W_\mu^-$$

$\Downarrow$

$\boxed{N \to e + W^+}$ $\boxed{\text{through } \Theta}$

$$\cdot\ N_L^T C M_N N_L + h.c.$$

$\Downarrow$

$$N_M = N_L + C\bar{N}_L^T$$
$$= N_L + (N^c)_R$$

$\circledast$ $\bar{e}_L \gamma^\mu N_L W_\mu^- = \overline{(N^c)}_R \gamma^\mu (e^c)_R W_\mu^-$ $\boxed{\text{Prove}}$

e goes out $\quad e_R^c \equiv C\bar{e}_L^T \quad$ e^c enters

$$\boxed{N_M : \; N_M = N_M^c}$$

$\Downarrow$ Majorana

$$\bar{N}_R^c \gamma^\mu e_R^c W_\mu^- = \bar{N}_R \gamma^\mu e_R^c W_\mu^- + h.c.$$

$+ h.c.$

$$\boxed{\overline{e^c_R}\,\gamma^\mu N_R W_\mu^+}$$

Keung, G.S. 1983

θ (keep in mind)

$$\boxed{N \to e^c + W^-}$$

Due to L violation

$$\Downarrow$$

$$\boxed{\Gamma(N \to e W^+) = \Gamma(N \to e^c W^-)}$$

$$\boxed{\Leftrightarrow N = \text{Majorana}}$$

argument: $\Gamma(N \to e^c w^-) \to 0$

$$m_N^M \to 0$$

so $\Rightarrow \Gamma(N \to e w^+) \neq$

$$\Gamma(N \to e^c w^-)$$

WRONG! $\quad m_N \to 0 \Rightarrow$

$$N = \text{stable}$$

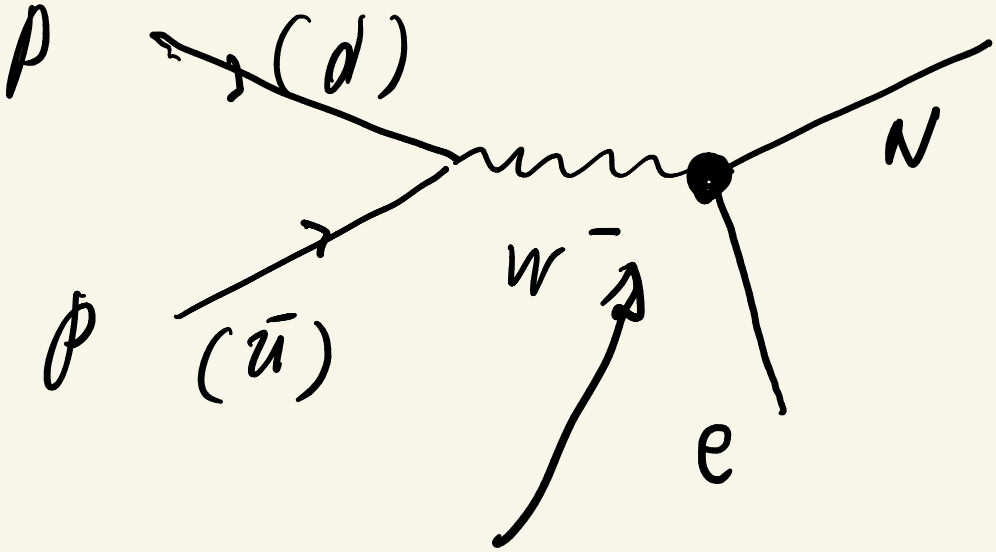
$$\Gamma(N \to e) = N\,(N \to \bar{e})$$

Direct Probe of Majorana

$\boxed{\text{How to produce } N?}$

$$\frac{g}{\sqrt{2}} \bar{N}_L \, \Theta \, \gamma^\mu \, e_L \, W_\mu^+$$

<u>Seesaw</u> $\left(\Theta = \frac{1}{M_N} m_D \ll 1 \right)$



$$(\Theta \ll 1)$$

1 gev ! $m_N \simeq \text{TeV}$

$$|m_\nu| = m_D^2 / m_N \Rightarrow$$

$$m_D^2 = m_\nu m_N \qquad \theta = m_D / m_N$$

$$\Rightarrow m_D^2 / m_N^2 \simeq m_\nu / m_N$$

$$\sigma(pp \to N) \propto \theta^2 \simeq \frac{m_D^2}{m_N^2}$$

$$\propto \frac{m_\nu}{m_N} \lesssim \frac{10^{-9}}{10^{+3}}$$

$$\lesssim 10^{-12}$$

$$N \to e + W$$

$$\nu + Z \quad , \quad \nu + h$$

- $|m_\nu| = -\frac{m_D^2}{m_N} = +\frac{y_D^2 v^2}{m_N}$

  seesaw

- $m_\nu = \frac{v^2}{\Lambda}$ $\qquad d=5$ effective

$$\boxed{+\frac{y_D^2}{m_N} = \frac{1}{\Lambda}}$$

$$m_N = \frac{\Lambda}{y_D^2}$$

- $m_D = m_e \simeq \text{MeV}$

$\Downarrow$

$$m_\nu \simeq \frac{m_e^2}{m_N} \Rightarrow m_N \simeq \frac{m_e^2}{m_\nu}$$

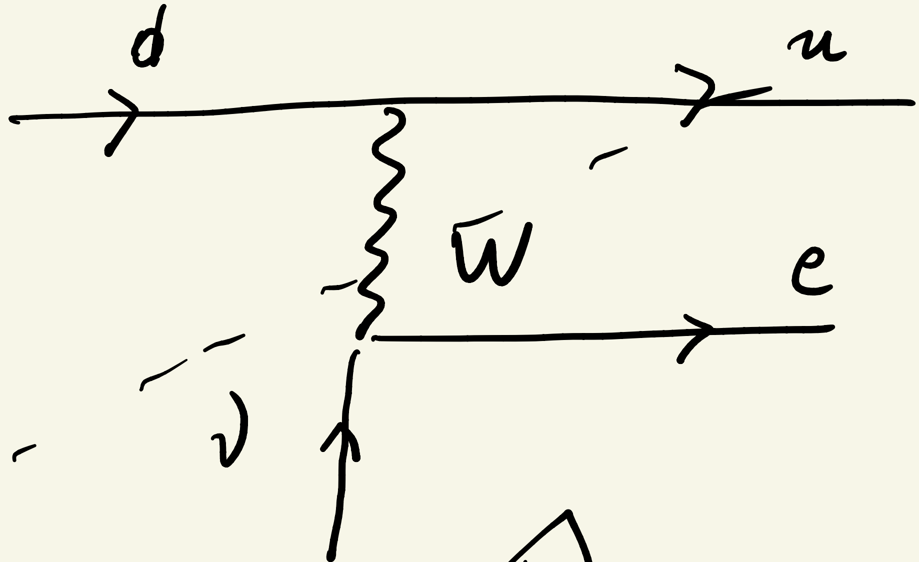
$$\simeq \frac{10^{-6}}{10^{-9}} \text{GeV}$$

$$\simeq 10^3 \text{GeV}$$

seesaw $\Big\Downarrow$ $d = 5$

$\nu =$ Majorana

$\Downarrow$



$$\Uparrow$$

$$\cancel{Ge \longrightarrow Ar + e + \bar{\nu}}$$

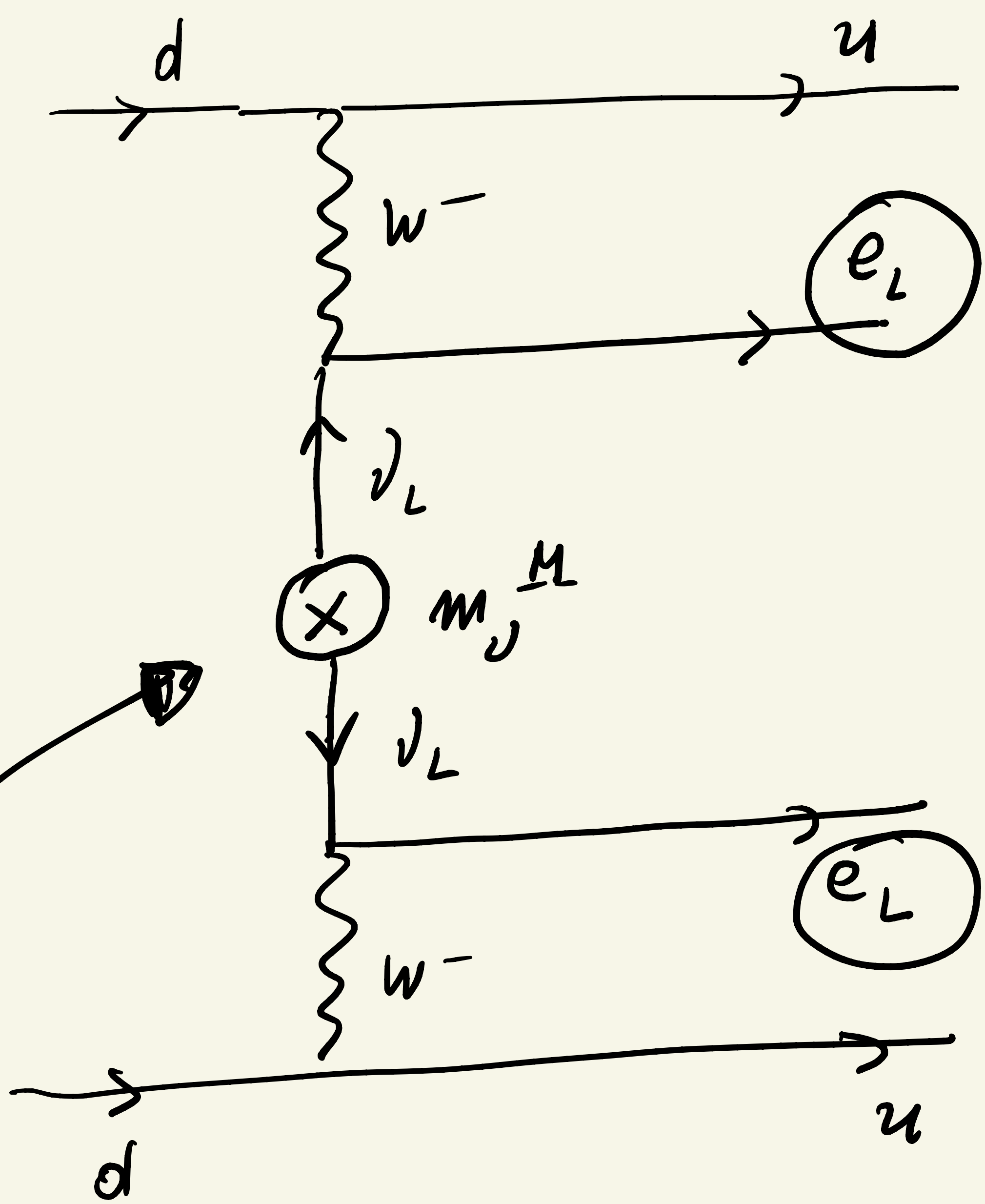
$$M_{Ar} > M_{Ge}$$

but

$$Ge \longrightarrow Se + e + e + \bar{\nu} + \bar{\nu}$$

$$M_{Ge} > M_{Se}$$ Goepert-Mayer

$$T_{2\beta} \simeq 10^{21}\ yr$$ 1935



$$m_\nu^M \left(\nu_L^T C \nu_L + \underline{\underline{\nu_L^\dagger C^\dagger \nu_L^*}} \right)$$

$0\nu 2\beta$

Neutrinoless double beta decay

$$\boxed{E_{2e} = \text{fixed!}}$$

$$\tau_{0\nu 2\beta} \gtrsim 10^{25}\,\text{yr} \qquad \boxed{\text{GERDA}}$$

2020

$$A_{\nu}^{0\nu 2\beta} \simeq G_F^2 \, \frac{m_\nu}{p^2}$$

$$\boxed{p \simeq 100\,\text{MeV}}$$

$$\frac{m_\nu + \cancel{\not p}}{\cancel{m_\nu^2} - p^2}$$

$$\tau_{0\nu 2\beta} \propto m_\nu^2$$

$$\Rightarrow \boxed{m_\nu \leq 1\,\text{eV}}$$

$\nu =$ Majorana $\Rightarrow$ $0\nu 2\beta$

$0\nu 2\beta \Rightarrow \nu =$ Majorana ?

$\downarrow$

Does $0\nu 2\beta \Rightarrow$ caused by m_ν^M ?

$\Downarrow$

NO

• $m_\nu{}^{M} \Rightarrow e = e_L$

$\Downarrow$

$$\boxed{\begin{array}{c} \text{if } e = e_R \text{ (one) in } 0\nu 2\beta \\ \Rightarrow \text{New Physics} \end{array}}$$

• Problem ! we added ν_R by hand
(singlet)

but that is not how
it happened

SM: • $\cancel{P}$ = maximal (β decay)

• $\cancel{P}$ = all reasons

'70s

• LR Theory:

P = restored at fund. level

$\cancel{P}$ in nature $\Leftarrow$ 1956 Lee, Yang

but P = restored

$$\boxed{L \leftrightarrow R}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \longrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \longrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\Downarrow$$

$$\boxed{\begin{gathered} G_{LR} = SU(2)_L \times SU(2)_R \times U(1) \\ Q = T_{3L} + T_{3R} + \frac{B-L}{2} \end{gathered}}$$

$L - R \equiv P$ is Spont. Broken

$$\Downarrow$$

$$\boxed{\exists\ \nu_R} \Rightarrow \boxed{m_\nu \neq 0}$$

$$\Downarrow$$

seesaw !

$$N_L = C\,\bar{\nu}_R^{\,T} = \text{pleqial}$$

$$W_R$$ directly

$$\left(\bar{\nu}_L \gamma^\mu e_L W^+_{\mu L} \to \bar{\nu}_R \gamma^\mu e_R W^+_{\mu R}\right)$$

$$\begin{array}{ccc} (r=4) & & (r=4) \\ SM & \xrightarrow[\text{GUT}]{} & SU(5) \end{array}$$

LR $\longrightarrow$ SO(10)

(r=5) (r=5)

Gauge theory

1. Group
2. matter = q, ℓ
3. Higgs $\longrightarrow$ m masses

<u>SM</u> $\overline{f_L}\, f_R$ $-\overline{f_L\, f_R}$

charged f $\longrightarrow$

D S'

$\Downarrow$

only Higgs $\rightarrow$ m_f

- if ν_R $\Rightarrow$ $\boxed{\nu_R\,\nu_R \text{ allowed}}$