Neutrino Mass and Grand Unification

Lecture XVI

10/12/2021

LMU

Winter 2021

SU(5): Fermion masses revisited

$d=4$

$$\mathcal{L}_Y = \bar{5}_F^{\,i} \, Y_d \, 10_{F_{ij}} \, 5_H^{*j} +$$

$$+ \; 10_{F_{ij}}^T \, C \, Y_u \, 10_{F_{ke}} \, 5_{H_m} \, \epsilon_{ijkem}$$

$$+ h.c.$$

$$\langle 5_H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_W \end{pmatrix} \qquad M_W = \frac{g}{2} v_W$$

$$\Downarrow$$

$$\boxed{M_D = M_E^T}$$

$$M_U = M_U$$

$$\downarrow$$

$$5_F = \begin{pmatrix} d \\ e^c \\ -\nu^c \end{pmatrix}_R$$

d and e are buddies

Explanation:

$$SU(5) \xrightarrow[\langle 5_H \rangle]{} SU(4) \quad PS$$

$$\langle 5_H \rangle = \begin{pmatrix} \left.\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}\right\} SU(4) \\ v_W \end{pmatrix}$$

$$5_F = \begin{pmatrix} \left.\begin{matrix} d^\alpha \\ e^c \end{matrix}\right\} SU(4) \\ -\nu^c \end{pmatrix}$$

SU(4) accidental symmetry of $\mathcal{L}_Y$ <u>after</u> SSB

$\Downarrow$

$$\boxed{M_D = M_E^T}$$

NO PS symmetry in

SU(5)

$\rightarrow$ rather accidental PS in $\mathcal{L}_Y$

$$\boxed{\begin{array}{c}\text{How to save } SU(5)? \\ \Updownarrow \\ \text{How } M_D \neq M_E^T ?\end{array}}$$

$\Downarrow$

break SU(4) = PS

$$\Downarrow$$

inform $\mathcal{L}_Y$ that $\cancel{SU(4)}$

$$\Downarrow$$

bring in 24_H, since

$$\boxed{\langle 24_H \rangle = \mathrm{diag}\left(1 \;\; 1 \;\; 1 \;\; -3/2 \;\; -3/2\right) \times v_{GUT}}$$

$$\Downarrow$$

How to bring in 24_H?

"Just do it" (Nike)

$$\mathcal{L}_Y = \mathcal{L}_Y^{(d=4)} + \mathcal{L}_Y^{(d=5)}$$

$$\mathcal{L}_Y^{(d=5)} = \bar{5}_F \frac{\langle 24_H \rangle}{\Lambda} 10_F \; 5_H^*$$

$$\left(\because \; \Lambda \geq 10\, M_{GUT} \, , \quad \Lambda \leq M_{pl} \right)$$

$$+ \; 10_F \, C \frac{\langle 24_H \rangle}{\Lambda} \; 10_F \; 5_H \; + h.c.$$

$$\Downarrow$$

$$\boxed{SU(5) \text{ invariance ?}}$$

$$\Downarrow \quad \boxed{\begin{aligned} & 24_H = 24_H^\dagger \\ & \mathrm{tr}\, 24_H = 0 \end{aligned}}$$

$$\mathcal{L}_Y^{(d=5)} = \bar{5}_F^{\,i} \frac{\langle 24\rangle_i^j}{\Lambda} 10_{Fjk}\, 5_H^{*k} + \cdots \qquad (1)$$

$$+\, 10_{Fij} \frac{\langle 24^j_n\rangle}{\Lambda} 10_{lm}\, 5_{Hn}\, \varepsilon_{iklmn} \qquad (2)$$

Mass terms $\langle 5_H^* \rangle$

$$\bar{5}_F^{\,i} \frac{\langle 24\rangle_i^j}{\Lambda} 10_{Fj5}\, v_W =$$

$$= \bar{5}_F^{\,i} \frac{\langle 24\rangle_i^i}{\Lambda} 10_{Fi5}\, v_W \qquad \boxed{\begin{array}{c} 10_F^T = -10_F \\ \Downarrow \\ (10_F)_{55} = 0 \end{array}}$$

(a) $i = \alpha$ (1,2,3) colr

/ (b) $i = 4$ ~~(c) $i = 5$~~

$$= \bar{d}_R \, \frac{v_{GUT}}{\Lambda} \, d_L v_W + \bar{e}^c_R \left(-\frac{3}{2} \frac{v_{GUT}}{\Lambda} \right) e^c_L$$

$$v_W$$

$$\Uparrow$$

$$10_F = \left(\begin{array}{c|cc} u^c & u & d \\ \hline & 0 & e^c \\ & & 0 \end{array} \right)_L$$

$$\Downarrow$$

$$\mathcal{L}_Y^{(d=4)} + \mathcal{L}_Y^{(d=5)} \Rightarrow$$

$$\boxed{\begin{aligned} M_D &= v_W \left(Y_D + \frac{v_{GUT}}{\Lambda} \right) \\ M_E^T &= v_W \left(Y_D - \frac{3}{2} \frac{v_{GUT}}{\Lambda} \right) \end{aligned}}$$

$$\boxed{M_D \neq M_E}$$

$$E_L^* \neq D_R$$

$$E_R \neq D_L^*$$

e.g. $X \quad \bar{d_L} \; \underbrace{D_L^+ E_R^*}_{\text{arbitrary}} \; e_L^c + \cdots$

$\Rightarrow$ $\boxed{\text{lose predictions for proton decay}}$

$$y_D^{\text{eff}} = y_D + \frac{v_{OUT}}{\Lambda}$$

$$y_E^{\text{eff}} = y_D - \frac{3}{2}\,\frac{v_{OUT}}{\Lambda}$$

$$\boxed{\frac{v_{OUT}}{\Lambda} \ll 1/10 \neq \text{small}}$$

$$\boxed{\text{because } \; y_D < 1/10}$$

$$\Downarrow \quad \begin{aligned} y_d &\simeq 10^{-4} \\ y_e &\simeq 10^{-5} \end{aligned}$$

Summary:

• $\boxed{d=5 \text{ in } \mathcal{L}_Y \text{ saves } SU(5)}$

$\therefore \quad M_D \neq M_E$

- $\boxed{\text{lose predictions for } p \text{ decay}}$

$m_{\text{"}d\text{"}} \neq m_{\text{"}e\text{"}} \implies$

$\boxed{m_b \neq m_\tau} \quad m_s \neq m_\mu \,, \; m_e \neq m_d$

$$\left. \begin{array}{l} m_b \simeq 4\,\text{GeV} \\ m_\tau \simeq 2\,\text{GeV} \end{array} \right\} \; y_b \simeq 1/25$$

- $d = 5$ in $V(24_H, 5_h)$

$d = 4 \implies m_3 = 4\, m_8 \leftarrow$ color octet weak triplet

$$\boxed{d=4+d=5 \Rightarrow m_3 \neq m_8}$$

bottom line

- SU(5) = ok for fermion mass relations

- SU(5) unification fails
- $m_\nu = 0$

Effective theory

may be $\longrightarrow$ great guide

(1) "V-A was the key"

Weinberg ?

Fermi $\longrightarrow$ V-A Marshak, Sudarshan

1934 1957

$\Downarrow$

$\gamma_\mu(--)$

$\Downarrow$

could be a gauge theory

$\Downarrow$

$$\boxed{SM}$$

Glashow ---

(2) $d=6$ p decay

Weinberg '79

$\Downarrow$

$$\boxed{qqql} \Rightarrow B-L \,!$$

$$\Rightarrow n \not\to K + l$$

$$\boxed{SU(5) = \text{perturbative}}$$

$\alpha_U \simeq 1/40$

ordinary

$\alpha_U \simeq 1/25$

susy