Neutrino Mass
and
Grand Unification

Lecture XIX

21/12/2021

LMU
Winter 2021

Left-Right (LR) Symmetric Model: Rise and Fall of Parity

- Gauge theory: SSB a must
- $\underline{P}$ (LR): $\boxed{\text{SSB a hypothesis}}$

(i) $$G_{LR} = \overbrace{SU(2)_L \times SU(2)_R}^{\underline{P}} \times U(1)$$

$$g \equiv g_L = \underset{\times\,\underline{P}}{g_R} \qquad \bar{g}$$

(ii) matter = q, l

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R \equiv q_R$$

$$l_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \xrightarrow{\underline{P}} \begin{pmatrix} \nu \\ e \end{pmatrix}_R \equiv l_R$$

$$\Downarrow$$

$$\boxed{\exists\, \nu_R \Rightarrow m_\nu \neq 0}$$

Curse? $\Longrightarrow$ Blessing!

$$\boxed{Q_{em} = T_{3L} + T_{3R} + \frac{Y_{LR}}{2}}$$

$$\Downarrow$$

$$[Y_{LR}, \vec{T}_L] = [Y_{LR}, \vec{T}_R] = 0$$

$$Y_{LR}\, f_L = Y_{LR}\, f_{LR}$$

$$Y_{LR}\, \ell_L = (-1)\, \ell_L \Rightarrow Y_{LR}\, \ell_R = (-1)\, \ell_R$$

$$Y_{LR}\, q_L = \frac{1}{3}\, q_L \Rightarrow Y_{LR}\, q_R = \frac{1}{3}\, q_R$$

$$\Rightarrow \boxed{Y_{LR} = B - L}$$

$$\Uparrow$$

$$B q = \frac{1}{3} q, \quad L q = 0$$

$$B \ell = 0, \quad L \ell = \ell$$

- SM : B, L are accidental symmetries, (global)

$B - L =$ anomaly free

(iii) Higgs sector

(a) ϕ_{SM} (doublet) $\subseteq$?

- $\phi_{SM} \subseteq \Phi$ (minimal)

- $\mathcal{L}_Y = y_f \overline{f_L} \, \Phi \, f_R + h.c.$

$$f = q, l \Rightarrow f_{\underset{(R)}{L}} \to U_{\underset{(R)}{L}} \, f_{\underset{(R)}{L}}$$

$$\Downarrow$$

$$\boxed{\Phi \longrightarrow U_L \, \Phi \, U_R^{+}} \quad (B-L)\,\Phi = 0$$

$$\Uparrow$$

$$\bar{f}_L \Phi f_R \to \bar{f}_L \underbrace{U_L^{+} U_L}_{1} \Phi \underbrace{U_R^{+} U_R}_{1} f_R = \text{inv.} \; \checkmark$$

$SU(2)_L + SU(2)_R$ doublet

(bi - doublet)

$$\Downarrow$$

$$\boxed{\Phi = \begin{pmatrix} \phi_1 & \tilde{\phi}_2 \end{pmatrix}} \qquad \boxed{\phi_1 = \begin{pmatrix} \varphi_1^0 \\ \varphi_1^- \end{pmatrix}}$$

$$\left(\tilde{\phi} \equiv i\sigma_2 \phi^*\right)$$

$$\Updownarrow$$

$$\bar{f}_L \Phi f_R = \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} \begin{pmatrix} \varphi_1^0 & \varphi_2^+ \\ \varphi_1^- & -\varphi_2^{0*} \end{pmatrix} \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$= \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} \begin{pmatrix} \varphi_1^0 u_R + \varphi_2^+ d_R \\ \varphi_1^- u_R - \varphi_2^{0*} d_R \end{pmatrix}$$

$$= \bar{u}_L u_R \varphi_1^0 + \bar{u}_L d_R \varphi_2^+ +$$

$$+ \bar{d}_L \varphi_1^- u_R - \bar{d}_L d_R \varphi_2^{0*}$$

$$\Updownarrow$$

$$\Phi \longrightarrow U_L \Phi U_R^+$$

$$Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$$

$$\Rightarrow Q_{em} \Phi = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \Phi - \Phi \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

$$\Uparrow$$

$$U = e^{i\theta_i T_i} \qquad U^+ = e^{-i\theta_i T_i}$$

$$e^{i\theta_i \overset{\shortparallel}{(-T_i)}}$$

$$\Phi = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$Q_{em}\, \Phi = \begin{pmatrix} \overset{=0}{(\tfrac{1}{2}-\tfrac{1}{2})}a & \overset{=\boxed{+1}}{(\tfrac{1}{2}+\tfrac{1}{2})}b \\ \underset{\boxed{-1}\,\shortparallel}{(-\tfrac{1}{2}-\tfrac{1}{2})}c & \underset{\overset{\shortparallel}{0}}{(-\tfrac{1}{2}+\tfrac{1}{2})}d \end{pmatrix} \quad \checkmark$$

- Φ = two doublets (ϕ_1, ϕ_2) ?

- Reminder: SU(5) $\phi \subseteq 5_H$

$$\overset{\shortparallel}{\begin{pmatrix} T \\ \phi \equiv D \end{pmatrix}}$$

T is heavy

$\Downarrow$

Expect: only one ϕ_i light in LR — other heavy

(a) new things

$$\begin{tikzcd} G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \arrow[d, "\Phi_{new}"] \\ SU(2)_L \times U_Y \end{tikzcd}$$

$$\left.\begin{aligned} Q_{em} &= T_{3L} + \frac{Y}{2} \\ &= T_{3R} + T_{3R} + \frac{B-L}{2} \end{aligned}\right\} \quad \frac{Y}{2} = T_{3R} + \frac{B-L}{2}$$

$\Rightarrow \quad \Phi_{new} = \chi_R$ ∴

$\langle \chi_R \rangle \neq 0$ breaks $SU(2)_R$

<u>not</u> $SU(2)_L$

$\boxed{\text{repr. of } SU(2)_R}$

(<u>not</u> of $SU(2)_L$)

$$\Downarrow$$

$$\boxed{\chi_R \Longrightarrow \chi_L}$$

(P)

$\Vert$

(LR)

$$\text{SSB} : G_{LR} \longrightarrow G_{SM}$$

$$\Updownarrow \quad (v_R \gg v_L)$$

$$\underbrace{\langle \chi_R \rangle = v_R \neq 0 \, , \quad \underbrace{\langle \chi_L \rangle = 0}_{SU(2)_L}}_{\boxed{\text{SSB of parity}}}$$

let's ignore the quantum number $\chi_{L,R}$

$$\Downarrow \quad \chi_{L,R} = \text{real scalars}$$

$$V = -\frac{\mu^2}{2}\left(\chi_L^2 + \chi_R^2\right) + \frac{\lambda}{4}\left(\chi_L^4 + \chi_R^4\right)$$

$$+\frac{\lambda'}{2}\chi_L^2\chi_R^2$$

$$= -\frac{\mu^2}{2}\left(\chi_L^2+\chi_R^2\right)+\frac{\lambda}{4}\left(\chi_L^2+\chi_R^2\right)^2$$

$$+\frac{\lambda'-\lambda}{2}\chi_L^2\chi_R^2$$

determines who gets a vev

- $\lambda'=\lambda \Rightarrow \underbrace{\boxed{\langle\chi_L\rangle^2+\langle\chi_R\rangle^2=\mu^2/\lambda}}$

who gets a vev,

$\langle\chi_L\rangle$ ~~or~~ and $\langle\chi_R\rangle$?

$$\Downarrow$$

$$\boxed{\text{Cannot know!}}$$

- physically $\lambda'\neq\lambda$

$$\Downarrow$$

$$\boxed{\begin{array}{l} (a)\ \lambda' > \lambda \\ (b)\ \lambda' < \lambda \end{array}}$$

$$\boxed{\begin{array}{l} (a) \Rightarrow \langle\chi_L\rangle\langle\chi_R\rangle^2 = 0 \ (\text{minimum}) \\ \qquad \Rightarrow \langle\chi_L\rangle = 0,\ \langle\chi_R\rangle \neq 0 \end{array}}$$

~~$(b) \Rightarrow \langle\chi_L\rangle \neq 0 \neq \langle\chi_R\rangle$~~

~~$(\langle\chi_L\rangle = \langle\chi_R\rangle)$~~

$$\Downarrow$$

$$\boxed{\mu^2 > 0,\quad \lambda > 0,\quad \lambda' - \lambda > 0}$$

Parameter space $\Rightarrow$

$$G_{LR} \longrightarrow G_{SM}$$

$$\langle \chi_R \rangle = v_R \neq 0$$

$$\langle \chi_L \rangle = 0$$

Choose: $\chi_L, \chi_R = ?$

<u>Hint:</u> SM is a good, correct theory

$\Leftrightarrow$ <u>no</u> new light states

$\langle \chi_R \rangle = v_R \neq 0 \Rightarrow$ all non SM particles should have a mass $\propto v_R$

$G_{LR} \Rightarrow \underset{\substack{\uparrow \\ \text{direct}}}{\nu}$ masses ($\Leftarrow G_{SM}$)

$\Leftrightarrow$ all states: mass from SSB

$$\boxed{\text{all non-SM states: } m_{new} \propto v_R}$$

$$\begin{cases} W_R^{\pm},\ Z_R \ \therefore\ m_{(W,Z)_R} \propto v_R \\ \nu_R \ \therefore\ m_{\nu_R} \propto v_R \end{cases}$$

$$\Downarrow$$

fixes : $\chi_R \equiv \Delta_R$ = adjoint (triplet) of $SU(2)_R$

$\Rightarrow$ $\chi_L \equiv \Delta_L$ = adjoint of $SU(2)_L$

$\Leftrightarrow$ $\mathcal{L}_Y(\Delta) = l_R\, l_R\, \Delta_R$ (symb.)

$$\begin{tikzcd} l_R\, l_R\, \Delta_R \arrow[d, "P"] \\ l_L\, l_L\, \Delta_L \end{tikzcd}$$

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group theory : $\boxed{\Delta_{L,R} \to U_{L,R}\, \Delta_{L,R}\, U^{\dagger}_{L,R}}$

$$\downarrow$$

$$Y_\Delta\left(l_R^T\, C\, i\sigma_2\, \Delta_R\, l_R + l \to R\right)$$

$\downarrow \qquad U_R = e^{i\theta T}, \; U_R^T = e^{-i\theta T^T}$

$$\ell_R^T C\, i\sigma_2\, \Delta_R\, \ell_R \rightarrow \ell_R^T C\, U_R^T\, i\sigma_2\, U_R\, \Delta_R \underbrace{U_R^\dagger U_R}_{1} \ell_R$$

$$= \ell_R^T C\, i\sigma_2 \underbrace{U_R^\dagger U_R}_{1} \Delta_\ell\, \ell_R$$

$$= \ell_R^T C\, i\sigma_2\, \Delta_R\, \ell_R \;\checkmark$$

$$\boxed{U_R^T \sigma_2 = \sigma_2 U_R^\dagger}$$

$$\boxed{\Delta_R \equiv (\alpha) T_i\, \delta_i = \frac{\sigma_i}{2} \delta_i} \quad (L \leftrightarrow R)$$

- $(B-L)\, \Delta_R = 2\, \Delta_R \Rightarrow (B-L)\, \Delta_L = 2\, \Delta_L$

$$\ell_R\, \ell_R\, \Delta_R$$

B-L: -1 -1 2

- $Q_{em}\,\Delta_R = (T_{3R} + 1)\,\Delta_R$

$$= \left[\frac{\sigma_3}{2}, \Delta_R\right] + \Delta_R$$

$(\Delta_R \to U_R\,\Delta_R\,U_R^{+})$

$$\Downarrow$$

$$Q_{em} : \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$\Leftrightarrow \boxed{\Delta_R \propto \begin{pmatrix} \delta_R^{+}/\sqrt{2} & \enclose{circle}{\delta_R^{++}} \\ \delta_R^{0} & -\delta_R^{+}/\sqrt{2} \end{pmatrix}} \quad \boxed{\text{Prove}}$$

$$\Updownarrow$$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad \langle \Delta_L \rangle = 0$$

non-trivial

- <u>gauge boson masses</u>

$$(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R) \longrightarrow$$

$$\boxed{\begin{aligned} M_{W_R}^2 &= g^2 v_R^2 \\ M_{Z_R}^2 &= 2(g^2+\bar{g}^2) v_R^2 \end{aligned}}$$

$SU(2)_L \times SU(2)_R$

$g = g_L = g_R$

$\bar{g} - U(1)_{B-L}$

$$\Delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta_R^+ & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+\frac{1}{\sqrt{2}} \end{pmatrix}$$

$\swarrow$ $\qquad\qquad$ $\uparrow$ eaten by W_R^+

$$v_R + h_R + i\,G_R$$

$\uparrow$ eaten by Z_R

$\boxed{\text{new Higgs}}$

$h_R \equiv \delta_R$

$$\Delta_R^{un} = \begin{pmatrix} 0 & \delta_R^{++} \\ v_R + \delta_R & 0 \end{pmatrix}$$

$$\Downarrow$$

$$Y_\Delta (\nu\ e)_R^T\, C \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \delta_R^{++} \\ v_R+\delta_R & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$= (\nu e)_R^T\, C \begin{pmatrix} v_R+\delta_R & 0 \\ 0 & -\delta_R^{++} \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_R Y_\Delta$$

$$= \nu_R^T\, C\, (v_R+\delta_R)\, \nu_R\ Y_\Delta$$

$$- e_R^T\, C\, \delta_R^{++}\, e_R\ Y_\Delta$$

$$\Downarrow$$

$$\boxed{M_{\nu_R} = Y_\Delta v_R \longleftarrow \text{large scale}}$$

$$\Downarrow$$

$$\boxed{\text{seesaw}}$$

$$\boxed{\delta_R^{++} \, e_R^T C e_R \, \frac{M_{\nu_R}}{v_R}}$$

Central results

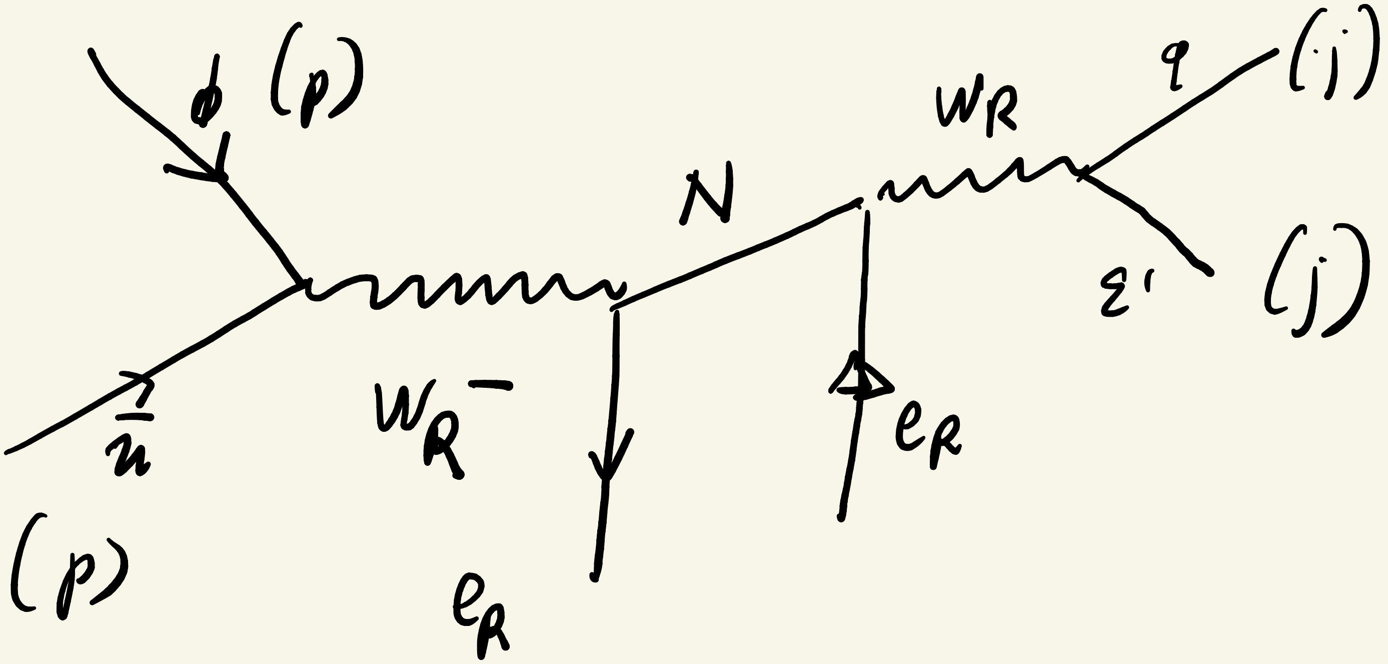
1. $\exists \, \nu_R \Rightarrow M_{\nu_R} \propto M_{W_R}$

2. $M_\nu = - M_D^T \frac{1}{M_N} M_D$

(seesaw)

$$M_N \equiv M_{\nu_R}^*$$

3. ν_R (N) — coupled to W_R



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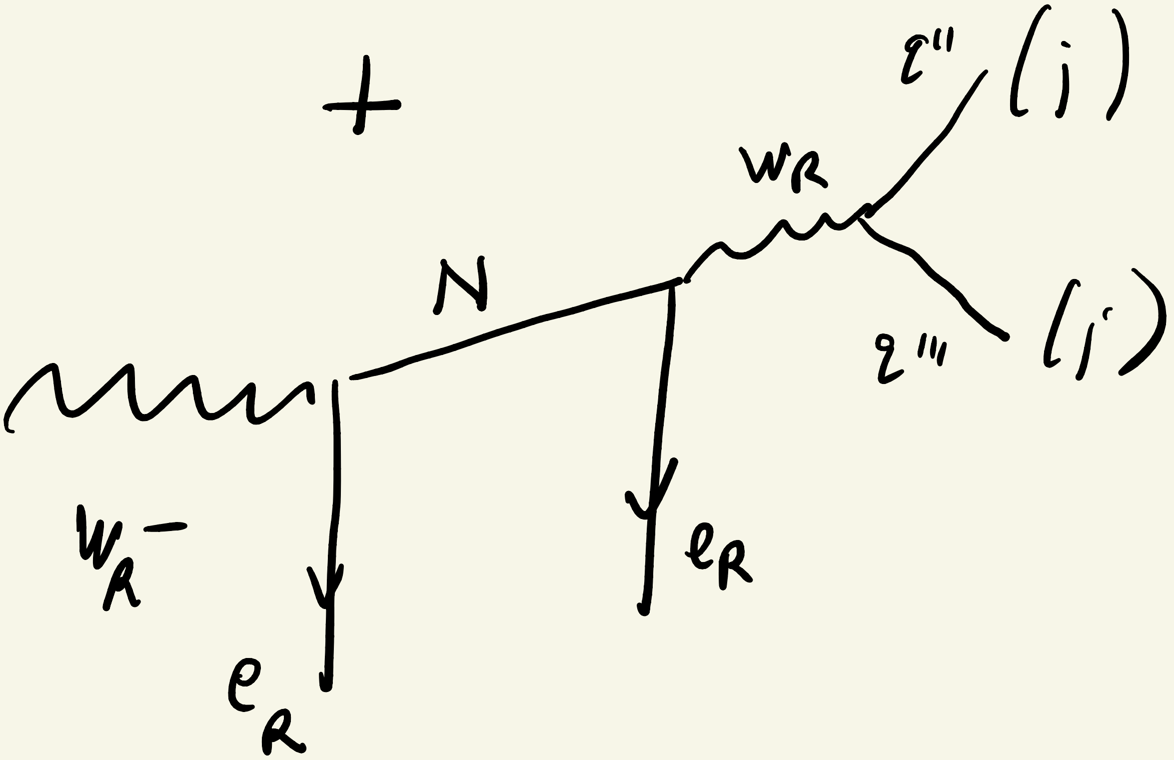

$\Rightarrow \boxed{\Delta L = 2 \text{ at LHC (?)}}$

N = Majorana!

↓

$$\boxed{\left.\begin{array}{l} \tfrac{1}{2}\; e\,e \\ \tfrac{1}{2}\; e\,\bar{e} \end{array}\right\}\ \text{final states}}$$

LR theory:

(a) rational for ν_R

(b) $m_\nu \neq 0$

(c) see saw

(d) potential discovery of N at LHC $\therefore$

$\Rightarrow N =$ Majorana, $\Delta L = 2$