Neutrino Mass and Grand Unification

Lecture XII

26/11/2021

LMU
Fall 2021

Nucleon Decay:

Effective Theory

Weinberg 1979

- $E \gg M_W$

$\Downarrow$

- $SU(3) \times SU(2) \times U(1)$ inv.

$\Downarrow$ $(M_W \to 0)$

$$\frac{1}{\Lambda_{\not{B}}^2} \underbrace{q\ q\ q}_{\text{color (proton)}}\ l \Leftarrow \text{lepton}$$

$\not{B}$, $\not{L}$

$\Downarrow$

NOT **B − L**

~~$q\,q\,q\,\bar{l}$~~

$q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad l \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L$

$u_R \qquad d_R \qquad e_R$

$\Downarrow$

- $\underbrace{q_L^T C\, i\sigma_2\, q_L}_{u_L^T C d_L - d_L^T C u_L \;\sim\; u_L^T C d_L} \qquad \underbrace{q_L^T C\, i\sigma_2\, l_L}_{(u_L^T C e_L - d_L^T C \nu_L)}$

- $\underbrace{q_L^T C\, i\sigma_2\, q_L}$ $\quad u_R^T C\, e_R$

  NOT $\cancel{e_R^c}$

- $(u_R^T C\, d_R)(q_L^T C\, i\sigma_2\, l_L)$

- $u_R^T C\, d_R \quad u_R^T C\, e_R$

$$\underbrace{\qquad\qquad\qquad\qquad}$$

$\times \quad 1/\Lambda_{\cancel{B}}^2$

$$GUT = SU(5) \Rightarrow \frac{1}{\Lambda_{\cancel{B}}^2} = \frac{g^2}{\underline{M}_{X,Y}^2}$$

/ GOOD

iff $\Lambda_B \gg M_W$

$$\Gamma(p \to \pi^0 e^+) = \Gamma(n \to \pi^0 \nu)$$

$$\Gamma(p \to \pi^0 e^+) = \frac{1}{2}\,\Gamma(n \to \pi^- e^+)$$

~~$= \frac{1}{2}\,\Gamma(n \to \pi^+ e^-)$~~

why ?

l in $\Leftrightarrow$ $\bar{l}$ out

$\frac{1}{2}$: $\pi^- = \bar{u} d$ $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

- **kaon final state**

$\underbrace{q\,q\,q}\,\ell$

$\Downarrow$

$\underbrace{u\,u\,s\,\ell}$ (for K final)

$\Downarrow$

$\bar{s}$ goes out

$K^+ = u\bar{s}$ $\quad\quad$ $\bar{K}^0 = d\bar{s}$

$K^0 = d\bar{s}$ $\quad\quad$ $K^- = \bar{u}s$

$\Downarrow$

K^+, K^0 can go out

$\Downarrow$

$n \to K^+ \ell$ $\qquad$ $n \to K^- \bar{\ell}$

only 2-body K channels

$\Delta(B-L) = 0$ forbids it $\qquad \Leftarrow \bar{\ell}$ goes out

$\Downarrow$

$n \not\to$ 2 body K final

$\Downarrow$

Find $n \to K^+ e,\ K^+ \mu$

$n \to K^- \bar{e},\ K^- \bar{\mu}$

$\Downarrow$

you eliminate all GUT

(all theories where new physics at high energy)

$\Downarrow$

Physics at nuclear decay = low energy ($\lesssim M_W$), small couplings

<u>Weinberg</u> $\Lambda_{\not{B}} \gg M_W$

$\Downarrow$

$$\frac{1}{\Lambda_{\not{B}}^2}\,(q\,q\,q\,l)$$

$d = 6$ leading

+ $d = 7$ operators $\frac{M_W}{\Lambda_{\not{B}}^3}$ ---

but if $\Lambda_{\not{B}} \gtrsim M_W \Rightarrow$

even $d=7$ (8?) can

contribute

Q. Can you find $d=7$

∴ ~~B-L~~ ?

Other operators?

$$q_L^T C \, i\sigma_2 \, \gamma^\mu \gamma^\nu q_L \quad q_L^T C \, i\sigma_2 \, \gamma_\mu \gamma_\nu l_2$$

?

$\Downarrow$

NOT new

$$\propto \left(q_L^T C \, i\sigma_2 \, q_L\right)\left(q_L^T C \, i\sigma_2 \, l_2\right)$$

$\Downarrow$ **prediction**

- only $q\,q\,q\,\ell$ $\therefore$ $\Delta(B-L)=0$

$\not\exists$ $p \to \bar{\ell} + \cdots$

$u \to \bar{\ell} + \text{---}$

- $q_L^T C\, i\sigma_2\, q_L \quad q_L^T C\, i\sigma_2\, \ell_L$
- $\quad$ —‖— $\quad u_R^T C\, e_R$
- $u_R^T C\, d_R \quad$ —‖—
- $u_R^T C\, d_R \quad u_L^T C\, i\sigma_2\, \ell_L$

⇓

$n \not\to$ kam final (2 body)

Supersymmetry

$$f \longleftrightarrow b$$

↑ fermion

↑ boson

$$\underbrace{\Lambda_{susy}}_{\text{supersymmetry}} \simeq \text{TeV}$$

$$\Downarrow$$

$$d \longleftrightarrow \tilde{d} \quad (Q_{\tilde{d}} = -1/3)$$

down quark $\longleftrightarrow$ down squark

$$5_H = \begin{pmatrix} T \\ D = \Phi \end{pmatrix}$$

T — color triplet (↔ down squark)

$-1/3 = Q_T$

$$\Downarrow$$

$$\tilde{d} \sim T$$

$$\uparrow$$

$\Delta B \neq 0 \neq O$

(susy int.)

- $m_{\tilde{d}} \leq \text{TeV} \Rightarrow$

Weinberg expansion is wrong!

$\Rightarrow d = 7$ operator $\therefore$

$\Delta(B-L) \neq 0$

UNIFICATION CONSTRAINTS

Georgi, Quinn, Weinberg
'74

⇓

theory of unification
of couplings

SU(5):

$$\tau_p \gtrsim 10^{34}\, yr$$

$$\Longrightarrow M_X \equiv M_{GUT} > 10^{15}\, GeV$$

α is E dependent

$$\frac{1}{\alpha(E_2)} = \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln E_2/E_1$$

⇑

coupling run with E

↓

Gross, Wilczek; Politzer

'1973'

$$b = \frac{11}{3} T_{GB} - \frac{2}{3} T_F - \frac{1}{3} T_S$$

gauge bosons — fermions — scalars

$$T \delta_{ab} = \mathrm{Tr}\, T_a T_b$$

$$D_\mu = \partial_\mu - ig\, T_a A_\mu$$

$$T = T(R)$$

depends on repr.

$$b > 0 \Rightarrow \alpha(E_2) < \alpha(E_1)$$

$$E_2 > E_1$$

Asymptotic Freedom

(AF)

$\Uparrow$

comes from T_{GB}

$\uparrow$

comes from gauge boson self-int.

$T_{GB} \leftarrow$ adjoint

SU(2)

- $s = 1/2$ = doublet (fund.)
  (F)

$$T_a = \sigma_a/2$$

$$T(1/2)\, \delta_{ab} = \mathrm{Tr}\, T_a T_b$$
$$= \frac{1}{2}\, \delta_{ab}$$

$$\boxed{T(F) = 1/2}$$

$$\Downarrow$$

$$\boxed{T(F) = \frac{1}{2} \text{ for any } SU(N)}$$

- $s = 1 \qquad T_3 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}$

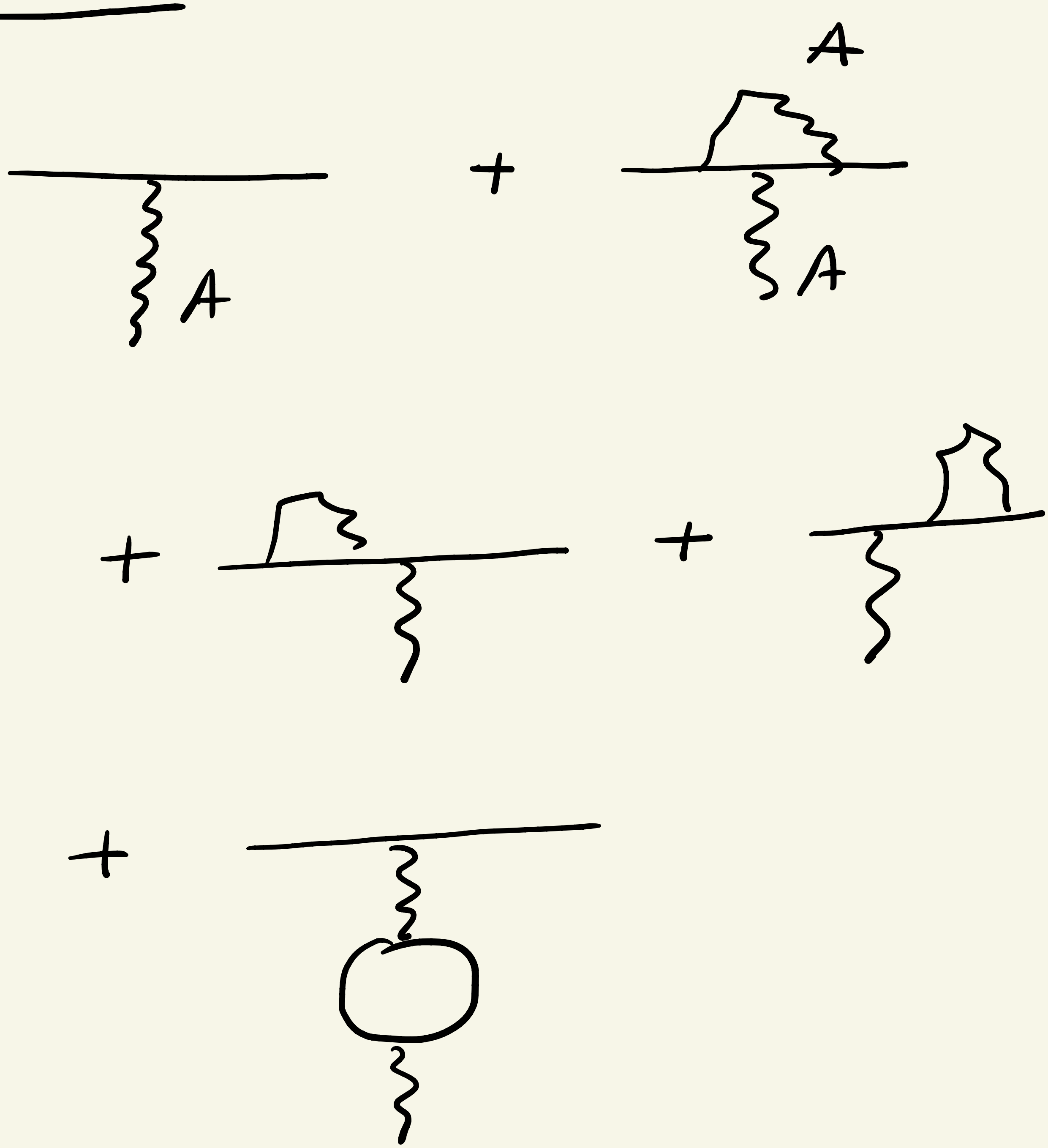
$$T_V T_3^2 = 2$$

$$\boxed{T(s=1) = T(\text{Adjoint}) = 2}$$

$$\boxed{T(\text{Adjoint in } SU(N)) = ?}$$

Hint: math. induction

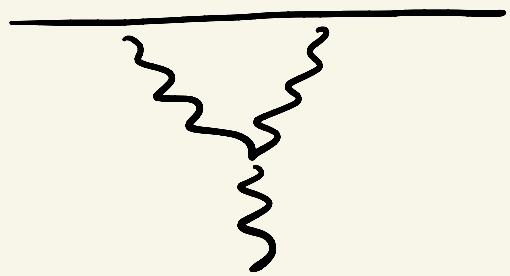
QED

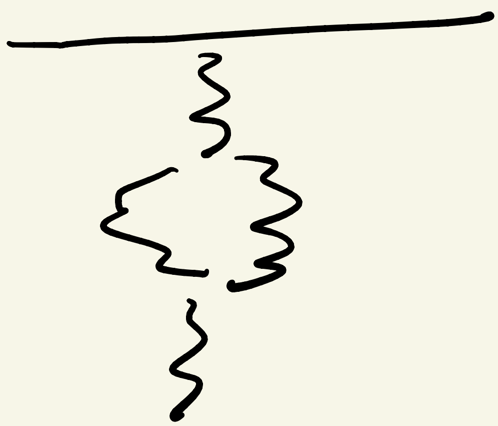


$$e(E) = e_0\left(1 + c_1 e_0^2 \ln \frac{\Lambda}{E}\right)$$

(ln)

YM

+ 

+ 

$$g(E_2) = g_0\left(1 + c_N g_0^2 \ln \frac{\Lambda}{E_2}\right)$$

(c_N ← SU(N))

$$g(E_1) = g_0\left(1 + c_N g_0^2 \ln \frac{\Lambda}{E_1}\right)$$

$\Downarrow$

$$g(E_2) - g(E_1) = c_N^2 g_0^3 \ln \frac{E_1}{E_2}$$

$$= c_N^2 g^3 \ln E_2/E_1$$

running comes from

renormalization

$M_{in} \longrightarrow M_{out}$

ns running

$$\alpha_3(M_W) = \alpha_s(M_W) \simeq 1/10$$

$$\alpha_2(M_W) = \alpha_W(M_W) \simeq 1/30$$

$$\alpha_1(M_W) \; \left(\simeq \alpha_{em}(M_W)\right) \simeq 1/100$$

(HW) $T(\text{Adjoint}) = ?$

$SU(N)$
