Neutrino Mass and Grand Unification

Lecture XI

23 / 11 / 2021

LMU
Fall 21

SU(5): proton decay

Branching ratios?

Last lecture:

$$Y_f \longrightarrow M_f$$

Yukawa

$$\left(SM: \quad y_f = \frac{M_f}{v} \right)$$

SU(5): $\boxed{M_U^T = M_U}$ (up q)

$$\boxed{(\text{down q})\; M_D = M_E^T \qquad (\text{down q})}$$

<u>SM:</u>

$$V_{CKM} = U_{Lu}^{\dagger}\, U_{Ld}$$

$$\updownarrow$$

$$\frac{g}{\sqrt{2}} \left(\bar{u}\ \bar{c}\ \bar{t} \right)_L \gamma^{\mu}\, V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_{\mu}^{+}$$

$$M_u = \underline{U_{Lu}}^{\dagger}\, m_u\, U_{Ru}$$

diagonal

$$M_D = \underline{U_{Ld}}^{\dagger}\, m_d\, U_{Rd}$$

$$\vdots$$

$\Downarrow$ however

GUT offers hope at probing M_f

- (X, Y) int.

$$\mathcal{L}_{X,Y} = \frac{g}{\sqrt{2}}\Big[\overline{u^c_L}^{\,0}\gamma^\mu u_L^0 + \overline{d_R}^{\,0}\gamma^\mu e^{c\,0}_R + \overline{d_L}^{\,0}\gamma^\mu e^{c\,0}_L\Big]\bar{X}_\mu$$

$$+ \frac{g}{\sqrt{2}}\Big[\overline{u^c_L}^{\,0}\gamma^\mu d_L^0 + \overline{d_R}^{\,0}\gamma^\mu \nu^{c\,0}_R + \overline{u_L}^{\,0}\gamma^\mu e^{c\,0}_L\Big]\bar{Y}_\mu$$

$$e^c_R \equiv C\bar{e}_L^{\,T}\ ;\quad (\nu^c)_R \equiv C\bar{\nu}_L^{\,T}$$

notation

$$e_L^c \equiv C\bar{e}_R^T \sim e_R^*$$

$$M_U = U_L^\dagger m_u U_R$$

$$M_D = D_L^\dagger m_d D_R$$

$$M_E = E_L^\dagger m_\ell E_R$$

$$M_\nu = N_L^\dagger m_\nu N_R$$

$\uparrow$

neutrino

$$\Leftrightarrow \quad u^0_{L,R} \longrightarrow U_{L,R}\, u_{L,R}$$

$$d^0_{L,R} \longrightarrow D_{L,R}\, d_{L,R}$$

$$e^0_{L,R} \longrightarrow E_{L,R}\, e_{L,R}$$

$\nu_L^0 \longrightarrow N_L \, \nu_L$ (for later)

$$u_L \equiv \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \qquad e_L = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L$$

$$d_L \equiv \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad \ldots$$

$$\left.\begin{array}{l} u_R^0 \longrightarrow U_R \, u_R \quad \& \\ \Rightarrow (u^c)_L^0 \longrightarrow U_R^* \, (u^c)_L \end{array}\right\} \text{keep in mind}$$

$$(u_L^c) \equiv C \, \bar{u}_R^{\,T} = C \gamma_0 u_R^*$$

but:

$$\boxed{\begin{array}{ll} M_U^T = M_U & (1) \\ M_D = M_E^T & (2) \end{array}}$$

$\Downarrow$

$$M_U = M_U^T$$

(1) $\underline{M}_U = U_L^{+} m_u U_R$

$\Rightarrow M_U^T = U_R^T m_u U_L^{*}$

From (1) $\Rightarrow$ $\boxed{U_R = U_L^{*}}$

(2) $\underline{M}_D = D_L^{+} m_d D_R$

$M_E^T = E_R^T m_e E_L^{*}$

$m_d = m_e$

From (2) $\Rightarrow$ $\boxed{\begin{gathered} E_L^{*} = D_R \\ E_R = D_L^{*} \end{gathered}}$

$\downarrow$

$$\mathcal{L}_X \to \bar{X}_\mu \frac{g}{\sqrt{2}} \Big[\overline{u^c_L}\, U_R^T U_L \gamma^\mu u_L \; +$$

$$+ \overline{d_R}\, D_R^+ E_L^* \gamma^\mu e^c_R + \overline{d_L}\, D_L^+ E_R^* e^c_L \Big]$$

$$= \bar{X}_\mu \frac{g}{\sqrt{2}} \Big[\overline{u^c_L}\, \underbrace{U_L^+ U_L}_{1} \gamma^\mu u_L \; +$$

$$+ \overline{d_R}\, \underbrace{D_R^+ D_R}_{1} \gamma^\mu e^c_R + \overline{d_L}\, \underbrace{D_L^+ D_L}_{1} \gamma^\mu e^c_L \Big]$$

$$\Rightarrow \boxed{\text{No mixing in } X}$$

$$\Downarrow$$

From X: $\boxed{\begin{aligned} &B(p \to e^+ \pi^0) \\ &= B(p \to \mu^+ K^0) \end{aligned}}$

partial

$\Downarrow$

Need Y:

$$\mathcal{L}_Y = \frac{g}{\sqrt{2}} \bar{Y}_\mu \Big[\overline{u^c_L} \gamma^\mu V_R^T D_L \, d_L +$$

$$+ \overline{d_R} D_R^\dagger N_L^* \gamma^\mu \nu^c_R + \overline{u_L} U_L^\dagger D_L \gamma^\mu e^c_L \Big]$$

$$= \frac{g}{\sqrt{2}} \bar{Y}_\mu \Big[\overline{u^c_L} \gamma^\mu \overbrace{V_n^T U_n^*}^{1} U_R^T D_L \, d_L$$

$$+\bar{d}_R\, D_R^{\dagger}\, \underbrace{E_L^{*}\, E_L^{T}}_{1}\, N_L^{*}\, \nu_R^{c} + \bar{u}_L\, V_{CKM}\, \gamma^{\mu} e_L \Big]$$

CKM!

$$= \frac{g}{\sqrt{2}}\, \bar{W}_{\mu} \Big[\bar{u}_L^{c}\, \gamma^{\mu}\, \underbrace{U_L^{\dagger} D_L}_{V_{CKM}\,!!}\, d_L +$$

$$+\bar{d}_R\, \underbrace{\bar{V}_{PMNS}^{T}}_{\text{lepton mixing}}\, \nu_R^{c} + \bar{u}_L\, V_{CKM}\, \gamma^{\mu} e_L \Big]$$

Mohapatra '79

$\Downarrow$

$$\mathcal{B}(p \to e^+\pi^0) \neq \mathcal{B}(p \to \mu^+ K^0)$$

$$\mathcal{B}(p \to e^+ K^0), \ \mathcal{B}(p \to \mu^+\pi^0)$$

$\boxed{\text{2 gen}}$ $$V_{CKM} \simeq V_C \simeq \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c \equiv \cos\theta_C, \ s \equiv \sin\theta_C$$

$\boxed{\text{Message}}$

Same θ_C from W sect.

$\Rightarrow$ x, y out!

$$\boxed{\theta_c = \theta_u - \theta_d}$$

$B^0 \longrightarrow$ sets p decay branching ratios !

- weak.

$$V_{CKM}^{(L)} = U_L^{+} D_L$$

analog $V_{\text{"CKM"}}^{(R)} = U_R^{+} D_R = ?$

free in SM

$\Rightarrow$ $\boxed{\text{"}V_{CKM}^{(R)}\text{"} = \text{remains arbitrary}}$

$\boxed{\text{RH (right-handed) quark mixing} = \text{arbitrary}}$

$LH \longleftrightarrow RH$

$$\frac{g}{\sqrt{2}} W_{\mu L}^{+} \left[\bar{u}_L \, V_{CKM}^{(L)} \gamma^\mu d_L \right] + h.c.$$

$\updownarrow$ LR

$$\frac{g}{\sqrt{2}}\, W^{+}_{\mu_R} \left[\bar{u}_R \, V^{(R)}_{\text{"CKM"}} \, \gamma^\mu d_R \right] + h.c.$$

heavy RH W boson!

$$\Updownarrow$$

$$\boxed{SO(10) \ \text{GUT}}$$

$$\boxed{M_f = Y_f \, v} \quad \text{basic}$$

$$M_U = M_U^T \Leftarrow Y_U^T = Y_U$$

$$M_D = M_E^T \Leftarrow Y_D = Y_E^T$$

$\Rightarrow y_f = \frac{m_f}{v}$ minimal SM

$\Rightarrow \Gamma(h \to f\bar{f}) \propto \left(\frac{m_f}{m_W}\right)^2$

<u>one</u> Higgs doublet

$\Rightarrow$ $\underbrace{b,\ t,\ \tau,\ \mu\,(?),\ W,\ Z}$

get mass from
Higgs – Weinberg
mechanism

$$\boxed{B, L \text{ symmetries}}$$

$$\bar{X}\,[\,\underbrace{\bar{u}^c u + \bar{d} e^c}\,]$$

$$\cancel{B},\ \cancel{L} \Rightarrow B - L \quad \text{good!}$$

$$\bar{y}\,[\,\bar{u}^c d + \bar{u} e^c \cdots\,]$$

$$T\,[\,\bar{u}^c d + \cdots\,]$$

$$5_H = \begin{pmatrix} T^\alpha \\ \Phi \end{pmatrix} \begin{array}{l} \leftarrow \text{color triplet} \\ \leftarrow \text{Higgs} \end{array}$$

$B-L$ = probe of $SU(5)$?

$\Downarrow$ 1979

Effective

Weinberg

$\cancel{B}_L \cancel{L} \nearrow$

$\uparrow$

learning from Fermi

<u>Fermi</u> $\mathcal{H}_{eff} = G_F\, \bar{p}\, n\, \bar{e}\, \nu$

weak int. $= \dfrac{1}{\Lambda_F^2}\, \bar{p}\, n\, \bar{e}\, \nu$

$$\Downarrow$$

Weinberg $\cancel{B}, \cancel{L}$:

$$\frac{1}{\Lambda_{\cancel{B}}^2} \, q \, q \, q \, l \qquad (d = 6)$$

$\boxed{\text{4 fermions}}$

- Assume that $\Lambda_{\cancel{B}}$ ($\equiv$ $\equiv$ scale of new $\cancel{B}, \cancel{L}$) $\gg M_{W}$

$\Rightarrow$ $\boxed{\text{valid in GUT}}$

$$\Downarrow$$

$$\underbrace{SU(3) \times SU(2) \times U(1)}$$

symmetric

$$\Uparrow$$

effective int. $(d=6)$

$$\boxed{\text{precise notation}}$$

$$\boxed{\begin{array}{ll} q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L & l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \\ u_R, \; d_R, \; e_R & \end{array}}$$

- Lorentz inv.

$$q_L^T C \, q_L \, , \quad l_L^T C \, l_L$$

$$q_L^T C \, l_L \, , \quad u_R^T C \, u_R \, ,$$

$$u_R^T C \, d_R \, , \quad u_R^T C \, e_R \; \cdots$$

$$\Downarrow \qquad 1/\Lambda_{\cancel{B}}^2 \text{ out}$$

$$O_1 \overset{?}{=} (q_L{}^T C\, q_L)\,\underbrace{(q_L{}^T C\, l_L)}_{}\,(?)$$

$\boxed{\text{Lorentz inv.}}$ one quark, one lepton

but $SU(2)_L$ inv. ! ?

$\Rightarrow$ NO!

$q_L{}^T\, C\; i\sigma_2\, q_L \Leftarrow$

$SU(2)_L$ inv.

$\Downarrow$

$$\cdot \; O_1 = (q_L^{\alpha\,T} C\, i\sigma_2\, q_L^{\beta})\,(q_L^{\gamma\,T} C\, i\sigma_2\, l_L)$$

$$\varepsilon_{\alpha\beta\gamma} \qquad \alpha, \beta, \gamma = r, y, b$$

$$= \underbrace{(u_L^T C\, d_L)\,(u_L^T C}_{P}\; \underline{e_L} - d_L^T C \nu_L)$$

$$O_2 = (q_L^T C\, i\sigma_2\, q_L)(u_R^T C\, e_R)$$

$$O_3 = (q_L^T C\, i\sigma_2\, l_L)\,(u_R^T C\, d_R)$$

$$O_4 = (u_R^T C\, e_R)\,(u_R^T C\, d_R)$$

Only 4!

Abbott, Wise '84

$\Downarrow$

$$\mathcal{O}_{1,2,3,4} = qqq\,l$$

(all)

lepton

NOT $\bar{l}$

$$\Downarrow$$

$$\underbrace{p \to \bar{\ell} + \cdots\cdots}_{B-L}$$

$\uparrow$ goes out

Q. Why not $p \to e + \pi^+ + \pi^+$?

A. $\cancel{B-L} \Rightarrow \boxed{\text{Impossible}}$