

Neutrino Mass
and
Grand Unification

Lecture XI

23 / 11 / 2021

LMU
Fall 21



SU(5) : proton decay

Breeding ratios?

Last lecture:

$$Y_f \longrightarrow H_f$$

Yahawa

$$(\text{JH} : \quad g_f = \frac{M_f}{\sigma})$$

SU(5) : $\boxed{M_D^T = M_D}$ (up 2)

$(\text{down 2}) M_D = M_E^T$ (down 2)

$$\underline{SM}: \quad V_{CKM} = U_{Ld}^+ \bar{U}_{Ld}$$



$$\frac{g}{\sqrt{2}} (\bar{u} \bar{c} \bar{t})_L \gamma^\mu V_{C\mu u} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \psi_\mu^+$$

$$M_u = \underline{U_{Lu}^+} m_u \bar{U}_{Qu}$$

↑
diagonal

$$M_D = \underline{U_{Ld}^+} m_d \bar{U}_{Rd}$$

. . .

↓ however

GUT offers hope at probing M_f

- (x, y) int.

$$\mathcal{L}_{x,y} = \frac{g}{\sqrt{2}} \left[\bar{u}_L^c \gamma^\mu u_L^c + \bar{d}_R^c \gamma^\mu e_R^c + \bar{d}_L^c \gamma^\mu e_L^c \right] \bar{x}_\mu$$

$$+ \frac{g}{\sqrt{2}} \left[\bar{u}_L^c \gamma^\mu d_L^c + \bar{d}_R^c \gamma^\mu v_R^c + \bar{u}_L^c \gamma^\mu e_L^c \right] \bar{y}_\mu$$

$$\bar{e}_R^c \equiv C \bar{e}_L^T; (v^c)_R \equiv C \bar{v}_L^T$$

notation

$$e_L^c \equiv C \bar{e}_R^{-\top} \sim e_R^*$$

$$M_U = U_L^+ m_u U_R$$

$$M_D = D_L^+ m_d D_R$$

$$M_E = E_L^+ m_e E_R$$

$$M_\nu = N_L^+ m_\nu N_R$$

↑
neutrino

$$\Leftrightarrow n_{L,R}^o \rightarrow U_{L,R} u_{L,R}$$

$$d_{L,R}^o \rightarrow D_{L,R} d_{L,R}$$

$$e_{L,R}^o \rightarrow E_{L,R} e_{L,R}$$

$\nu_L^0 \rightarrow N_L \bar{J}_L$ (for later)

$$u_L = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \quad e_L = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L$$

$$d_L = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \quad \dots$$

$$\begin{aligned} u_R^0 &\rightarrow U_R u_R + \quad \} \quad \text{keep} \\ \Rightarrow (u^e)_L^0 &\rightarrow U_R^* (u^c)_L \quad \} \quad \text{in mind} \end{aligned}$$

$$(u^c)_L \equiv C \bar{u}_R^\top = C \gamma_0 u_R^*$$

but: // $M_U^\top = M_U$ (1)

$$M_D = M_E^\top \quad (2)$$

$$(1) \quad M_V = U_L^T m_u U_R \quad M_V = M_V^T$$

$$\Rightarrow M_V^T = U_R^T m_u U_L^*$$

$$From (1) \Rightarrow \boxed{U_R = U_L^*}$$

$$(2) \quad M_D = D_L^+ m_d D_R$$

$$M_E = E_R^T m_e E_L^*$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} m_d = m_e$$

$$From (2) \Rightarrow \boxed{E_L^* = D_R \quad \leftarrow} \\ \boxed{E_R = D_L^*}$$



$$\mathcal{L}_x \rightarrow \bar{x} \frac{g}{\sqrt{2}} \left[\bar{u}_L^c U_R^\top U_L \gamma^\mu u_L + \right.$$

$$\left. + \bar{d}_R D_R^+ E_L^* \gamma^\mu e_R^c + \bar{d}_L D_L^+ \bar{E}_R^* e_L^c \right]$$

$$= \bar{x} \frac{g}{\sqrt{2}} \left[\bar{u}_L^c \underbrace{U_L^+}_{1} U_L \gamma^\mu u_L + \right.$$

$$\left. + \bar{d}_R \underbrace{D_R^+}_{\perp} D_R \gamma^\mu e_R^c + \bar{d}_L \underbrace{D_L^+}_{1} D_L \gamma^\mu e_L^c \right]$$

\Rightarrow No mixing in X



$$F_{\text{full}} \propto : \boxed{\begin{aligned} & B(\rho \rightarrow e^+ \pi^0) \\ & = B(\rho \rightarrow \mu^+ \mu^0) \end{aligned}}$$

partial



Need \mathcal{L} :

$$\begin{aligned} \mathcal{L}_q &= \frac{g}{\sqrt{2}} \bar{y}_\mu \left[\bar{u}_L^c \gamma^\mu V_R^T D_L d_L + \right. \\ &+ \bar{d}_R D_R^+ N_L^+ \gamma^\mu \bar{v}_R^c + \bar{u}_L^c U_L^+ D_L \gamma^\mu \bar{e}_L^c \left. \right] \\ &= \frac{g}{\sqrt{2}} \bar{y}_\mu \left[\bar{u}_L^c \gamma^\mu \underbrace{\bar{V}_n^T}_{1} \underbrace{\bar{U}_n^*}_{L} \underbrace{\bar{U}_R^T}_{1} D_L d_L \right. \end{aligned}$$

$$+ \bar{d}_R D_R^+ \underbrace{E_L E_L^T}_{\perp} N_L^* V_R^c + \bar{u}_L V_{CKM} \gamma^\mu e_L]$$

ChM!

$$= \frac{g}{\sqrt{2}} \bar{f}_\mu \left[\bar{u}_L^\mu \gamma^\mu \underbrace{U_L^+ D_L}_{} d_L + V_{CKM} \right]$$

$$+ \bar{d}_R \underbrace{\bar{V}_{PHNS}^T}_{\text{lepton mixing}} V_R^c + \bar{u}_L V_{CKM} \gamma^\mu e_L]$$

Mshapatin '79



$$\mathcal{B}(p \rightarrow e^+ \bar{\pi}^0) \neq \mathcal{B}(p \rightarrow \mu^+ \bar{\nu})$$

$$\mathcal{B}(p \rightarrow e^+ \bar{\nu}), \mathcal{B}(p \rightarrow \mu^+ \bar{\nu})$$

2 glu

$$V_{\text{cav}} \approx V_C \approx \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c \equiv \cos \theta_c, \quad s \equiv \sin \theta_c$$

Message

Some θ_c from W act.

$\Rightarrow x, y$ int !

$$\Theta_c = \Theta_u - \Theta_d$$

)

$B^0 \rightarrow \pi^+ \rho^-$ decay

branching ratios !

weak.

$$V_{CKM}^{(L)} = V_L^+ D_L$$

analog $V_{CKM}^{(A)} = V_R^+ D_R = ?$

free in SM

$\Rightarrow \bar{V}_{CKM}^{(R)}$ = remains arbitrary

R H (right-handed) quark
mixing = arbitrary

LH \leftrightarrow RH

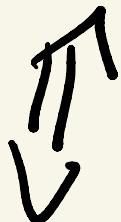
$$\frac{g}{\sqrt{2}} W_{\mu L}^+ \left[\bar{u}_L \bar{V}_{CKM}^{(L)} \gamma^\mu d_L \right] + h.c.$$

$\uparrow \downarrow LR$

$$\frac{g}{\sqrt{2}} W_{\mu_R}^+ \left[\bar{u}_R \gamma_{\text{cusp}}^{(R)} \gamma^\mu d_R \right] + \text{h.c.}$$

$\not\rightarrow$

heavy RH w boson!



SO(10) GUT

$$-M_f = Y_f \varrho$$

basic

$$M_0 = M_0^T \Leftarrow Y_0 = Y_0^T$$

$$M_E = M_E^T \Leftarrow Y_E = Y_E^T$$

$$\Rightarrow y_f = \frac{m_f}{v} \quad \text{minimal SM}$$

$$\Rightarrow \Gamma(h \rightarrow f\bar{f}) \propto \left(\frac{m_f}{m_W}\right)^2$$

one Higgs doublet

$\Rightarrow b, t, \tau, \mu(?), W, Z$

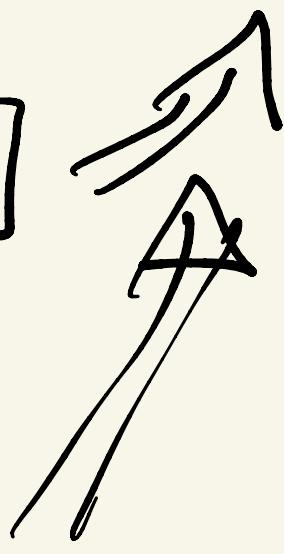
get mass from

Higgs - Weinberg
mechanism

B, L symmetries

$$\bar{X} \left[\bar{u}^c u + \bar{d}^c d + \bar{e}^c e \right]$$

~~B, L~~ \Rightarrow B-L good!

$$\bar{g} \left[\bar{u}^c d + \bar{u}^c e^- \right]$$


$$T \left[\bar{u}^c d + \bar{u}^c e^- \right]$$


$$t \quad S_H = \begin{pmatrix} T^a \\ \Phi \end{pmatrix} \quad \begin{matrix} \leftarrow & \text{color} \\ & \text{triplet} \end{matrix}$$

Higgs

$\mathcal{B} - \mathcal{L}$ = probe of $SU(5)$?

U

1979

Effective

B L

Weinberg

A stylized, symmetrical logo consisting of two curved lines forming a heart-like shape above a vertical line.

learning from Fermi

$$\underline{\text{Fermi}} \quad t_{\text{eff}} = 6_F \bar{p} u \bar{e} v$$

$$\text{weak int.} = \frac{1}{\lambda_F^2} \bar{p} \mu \bar{e} v$$

Weinberg \mathcal{S}, \mathcal{L} :

$$\frac{1}{\Lambda_B^2} q \bar{q} \bar{q} l \quad (d=6)$$

↑
[4 fermions]

- Assume that $\Lambda_B (\equiv$
 \equiv scale of new \mathcal{S}, \mathcal{L}) $\gg M_W$

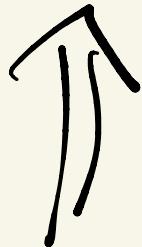
\Rightarrow [valid in GUT]



$SU(3) \times SU(2) \times U(1)$



symmetric



effective int. ($d=6$)

precise notation

$$\underline{q}_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\underline{l}_L = \begin{pmatrix} v \\ e \end{pmatrix}_L$$

u_R, d_R, e_R

- Lorentz inv.

$$\underline{q}_L^T C \underline{q}_L, \quad \underline{l}_L^T C \underline{l}_L$$

$$\underline{q}_L^T C \underline{l}_L, \quad \underline{u}_R^T C \underline{u}_R,$$

$$\underline{u}_R^T C \underline{d}_R, \quad \underline{u}_R^T C \underline{e}_R \quad \dots$$

$\Downarrow \frac{1}{\lambda_B^2}$ out

$$Q_1 \stackrel{?}{=} (\bar{q}_L^T C q_L) (\bar{e}_L^T C \bar{e}_L) (?)$$

}

L currents inv.

one quark,

one lepton

but $SU(2)_L$ inv. ! ?

\Rightarrow NO !

$$\bar{q}_L^T C i\Gamma_2 q_L \quad \Leftarrow$$

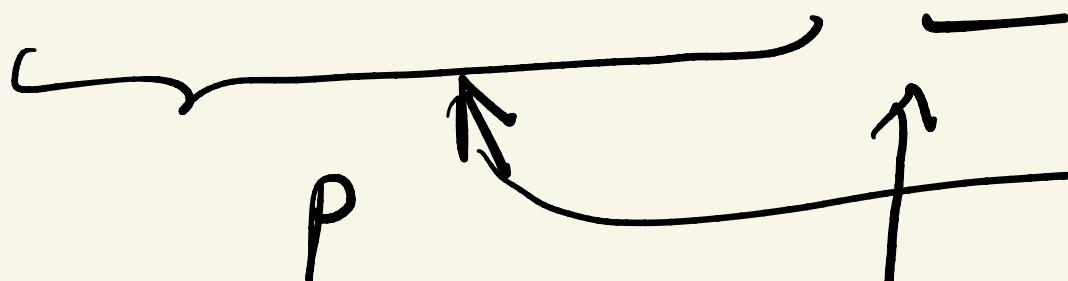
$SU(2)_L$ inv.



$$Q = \left(\underline{q}_L^T C i \sigma_2 \underline{q}_L \right) \left(\underline{q}_L^T C i \sigma_2 l_L \right)$$

Σ_{approx} $d, \beta, \delta = r, y, b$

$$= \left(\underline{u}_L^T C d_L \right) \left(\underline{u}_L^T C e_L - d_L^T v_L \right)$$



 P

$$O_2 = \left(\underline{q}_L^T C i \sigma_2 \underline{q}_L \right) \left(\underline{u}_R^T C e_R \right)$$

$$O_3 = \left(\underline{q}_L^T C i \sigma_2 l_L \right) \left(\underline{u}_R^T C d_R \right)$$

$$O_4 = \left(\underline{u}_A^T C e_R \right) \left(\underline{u}_R^T C d_R \right)$$

Only 4!

Abbott, Wise
'84



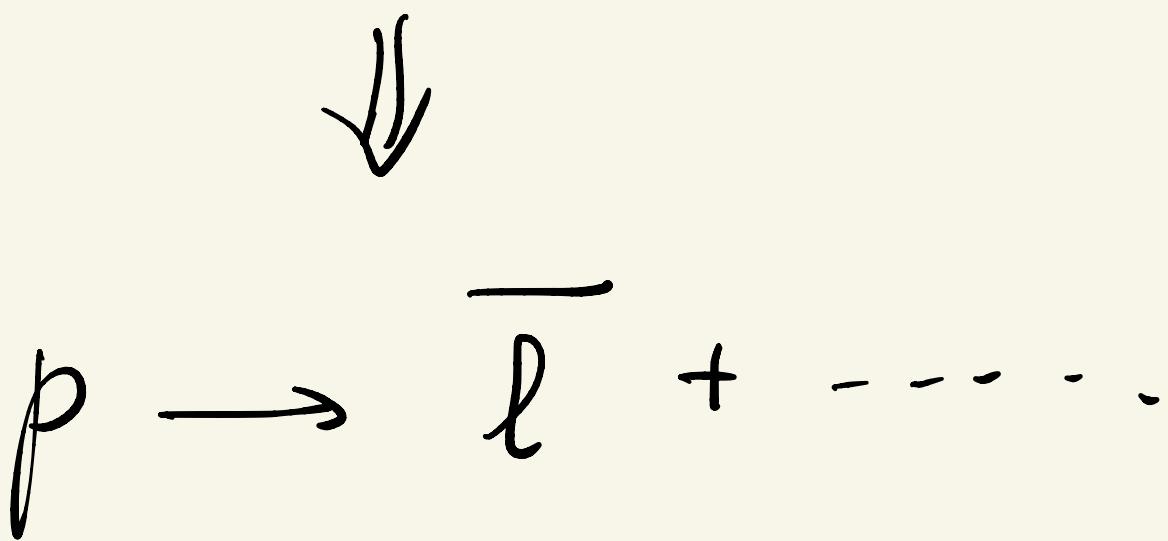
$$\Omega_{1,2,3,4} = 129 \ell$$

(all)

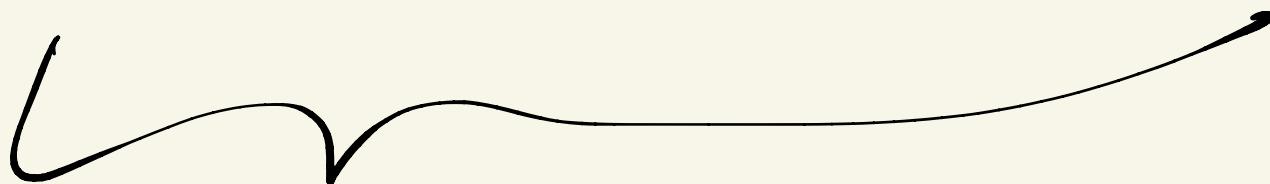


lepton

NOT $\bar{\ell}$



l goes out



$$\cancel{B-L}$$

Q. Why not $p \rightarrow e + \pi^+ + \pi^+$?

A. ~~B-L~~ \Rightarrow impossible

