

Neutrino Mass
and

Grand Unification

Lecture VII

9/11/2021

LMU

Fall 2021



SU(5) grand unified theory:

Construction

Georgi,
Glashow
'1974

$$\bar{5}_F = \left(\begin{array}{c} d^c \\ d^c \\ d^c \\ \bar{\nu} \\ e \end{array} \right)_L \quad \left. \begin{array}{l} \{ \quad \text{SU}(3)_C \\ \{ \quad \text{SU}(2)_L \end{array} \right.$$

$$\left(\begin{array}{c} u \\ d \end{array} \right)_L \quad \begin{matrix} (3) \\ (6) \end{matrix} \quad \left(u^c \right)_L, \quad \left(d^c \right)_L \quad \begin{matrix} (3) \\ (3) \end{matrix}$$

$$\left(\begin{array}{c} e \\ \nu \end{array} \right)_L \quad \begin{matrix} (2) \\ (1) \end{matrix} \quad \left(e^c \right)_L$$

$$SU(5) : \quad T_a = T_a^+, \quad T_r T_a = 0$$

$$a=1, 2, \dots, 24$$

$$\bullet \quad Q \equiv Q_{\text{em}} = \sum c_a T_a$$

$$T_V Q_{\text{em}} = 0$$

$$\Leftrightarrow \sum_{\text{repr}} q_{\text{em}} = 0$$

$$5_F = \left(\begin{array}{c} d \\ d \\ d \\ \hline e^c \\ -\nu^c \end{array} \right) \} \quad SU(3)_C$$

R

reminde : $D_2 \rightarrow U_2 D_2$ ($SU(2)$)

$$\tilde{D}_2 \equiv 1 \subseteq D^* \Leftarrow$$

$$\Rightarrow \tilde{D}_2 \rightarrow U_2 \tilde{D}_2$$

$$\ell = \begin{pmatrix} u \\ e \end{pmatrix}_L = \text{doublet}$$

$$\hat{\ell} \equiv i\tau_2 \ell^c = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} u^c \\ e^c \end{pmatrix} = \begin{pmatrix} e^c \\ -u^c \end{pmatrix}$$

assuming $\bar{q}_v = 0$
 (see below)

$$(*) Q(5) = \text{diag} \left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0 \right)$$

but: + 10 more fermions

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, (u^c)_L, (e^c)_L$$

$$10 = 5 \times 2$$

(a) why not 2 more $5_F^{'} \circ ?$

impossible \Rightarrow we would
violate $(*) !$



(b) 10_F (repr. of $SU(5)$)

Representations of $SU(N)$

$SU(2)$

$$D \Leftrightarrow \sigma = 1/2$$

$$(3) T(\sigma=1) = (D \times D)$$

Tensor product

$R \times R = \text{representation}$

But not irreducible

$$\Rightarrow R \times R = (R \times R)_S + (R \times R)_A$$

$$SU(2) : D \times D = S + A$$

(+) (s)

triplet singlet

$$SU(5) : 5 \times 5 = S + A$$

$$S : \frac{5 \cdot 6}{2} = 15$$

$$A : \frac{5 \cdot 4}{2} = 10$$

$SU(5)$

$$\begin{array}{c} \boxed{25^- = 15^- + 10^-} \\ \hline \boxed{10_{repv} = (5 \times 5)_{AS}} \end{array}$$

$$\boxed{Q(10) = ?}$$

$$10_{ij} = 5_i \cdot 5_j$$

$$\rightarrow V_{iu} V_{je} 5_u 5_e =$$

$$V_{iu} V_{je} 10_{ue}$$

$$10_{ij} \rightarrow V_{iu} 10_{ue} {V_{ej}}^T$$

$$\boxed{10 \rightarrow U 10 U^T}$$

$$U = e^{i\Theta^a T} \quad (\text{summed})$$

$$U^T = e^{i\Theta^a T^T}$$

$\frac{1}{T_a}$ = generator of $SU(5)$ on 10

$$\boxed{T_a \ 10 = T_a \ 10 + 10 \ T_a^T}$$

$$(U = 1 + i\theta_a T_a + \dots)$$

$$Q = \sum c_a T_a$$

$$\Rightarrow \boxed{Q \ 10 = Q \ 10 + 10 \ Q^T \\ = Q \ 10 + 10 \ Q}$$

$$(Q \in \text{Cartan})$$

$$1O_{ij} = 5_i \cdot 5_j$$

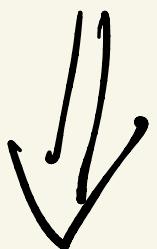
$Q = \text{sum of charges } 1O_{ij}$

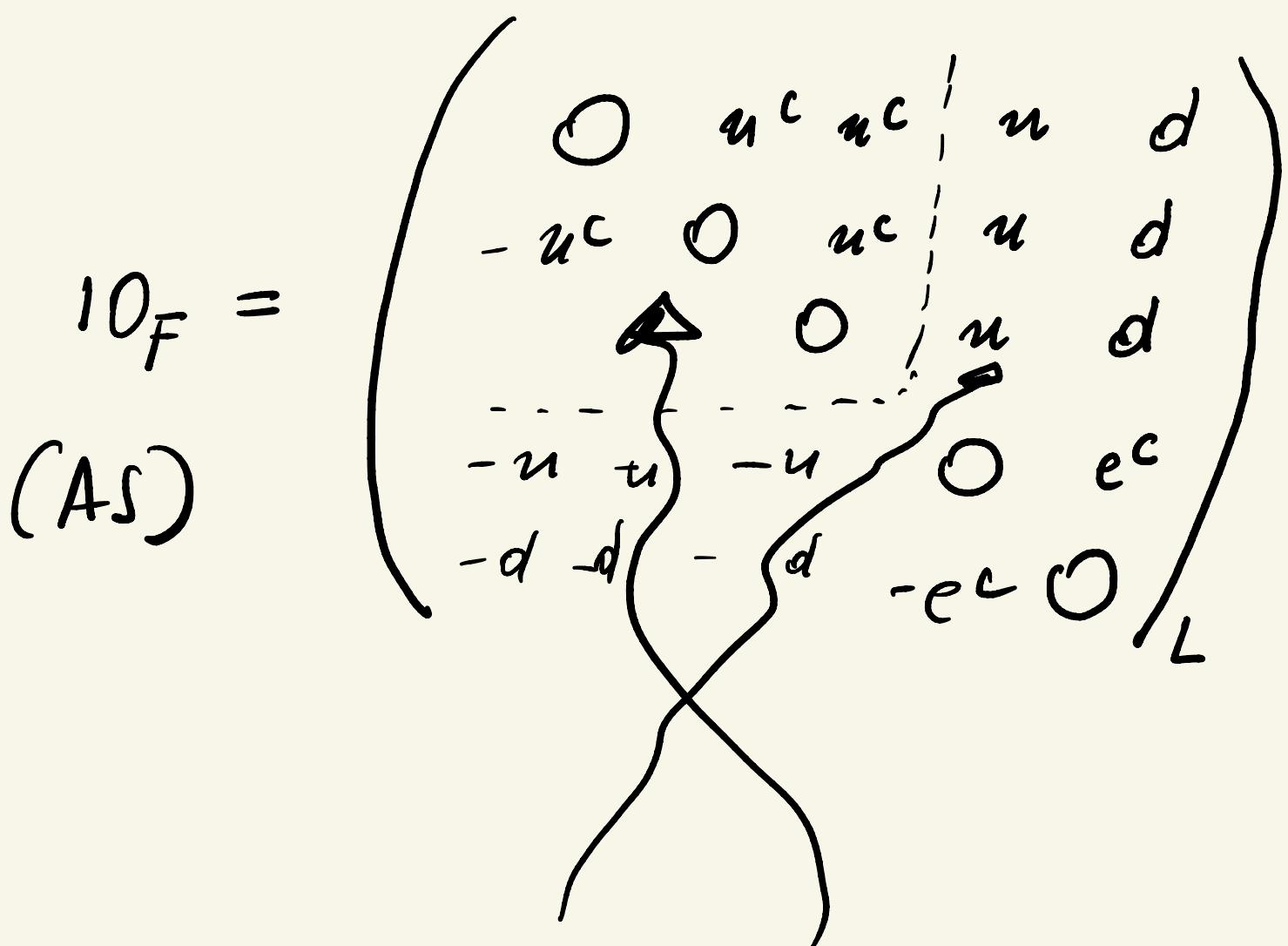
** $Q(1O_{ij}) = Q(5_i) + Q(5_j)$

\Rightarrow True of all
"charges"

$$\bar{T}_{3W}(1O_{ij}) = \bar{T}_{3W}(5_i) + \bar{T}_{3W}(5_j)$$

- - - -



$SU(3)_C$ $SU(2)_L$ 

need : $\begin{pmatrix} u \\ d \end{pmatrix}_L, (u^c)_L, (e^c)_L$

(color) (color)

 (Dirac)

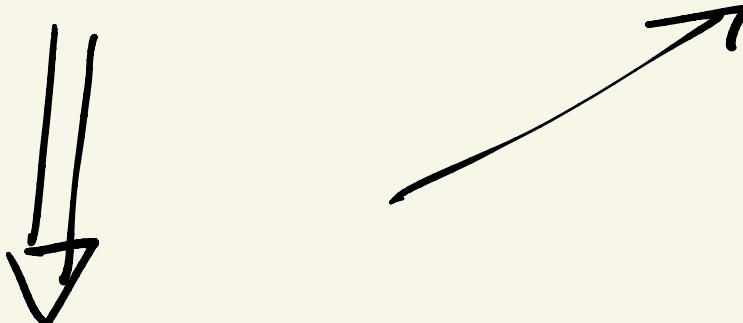
$$Q(u^c) = -Q(u)$$

 (QED) 

$$\sum Q(10) = (3Q(u^c) + 3Q(u)) = 0$$

//

$$0 \Rightarrow [+ 3Q(d) + Q(e^c)] = 0$$



$$3Q_d + Qe^c = 0$$

charge quantisation

↓ step back

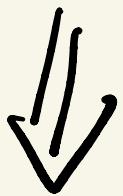
$$Q(\bar{5}_F) = 0$$

$$\bar{S}_F = \begin{pmatrix} d^c \\ d^c \\ d^c \\ v \\ e \end{pmatrix} \Leftrightarrow S_F = \begin{pmatrix} d \\ d \\ d \\ e^c \\ -vc \end{pmatrix}_R$$

$$\{Q(s) = 3Q(d) + Q(e^c) + Q(v) = 0,$$

H

O



$$Q(v) = 0$$

Reminder

$$Q(v) = Q(e) + 1$$

$$Q = T_3 + \gamma_L$$

$$Q(v) = 1_L + \gamma_L$$

$$Q(e) = -1_L + \gamma_L$$



$$Q(v) - Q(e) = 1$$

\Leftrightarrow Minimal $SU(5)$ "predicts"

that there is no v_R

generations = xerox
copying

↓

other generations = same
quantum numbers

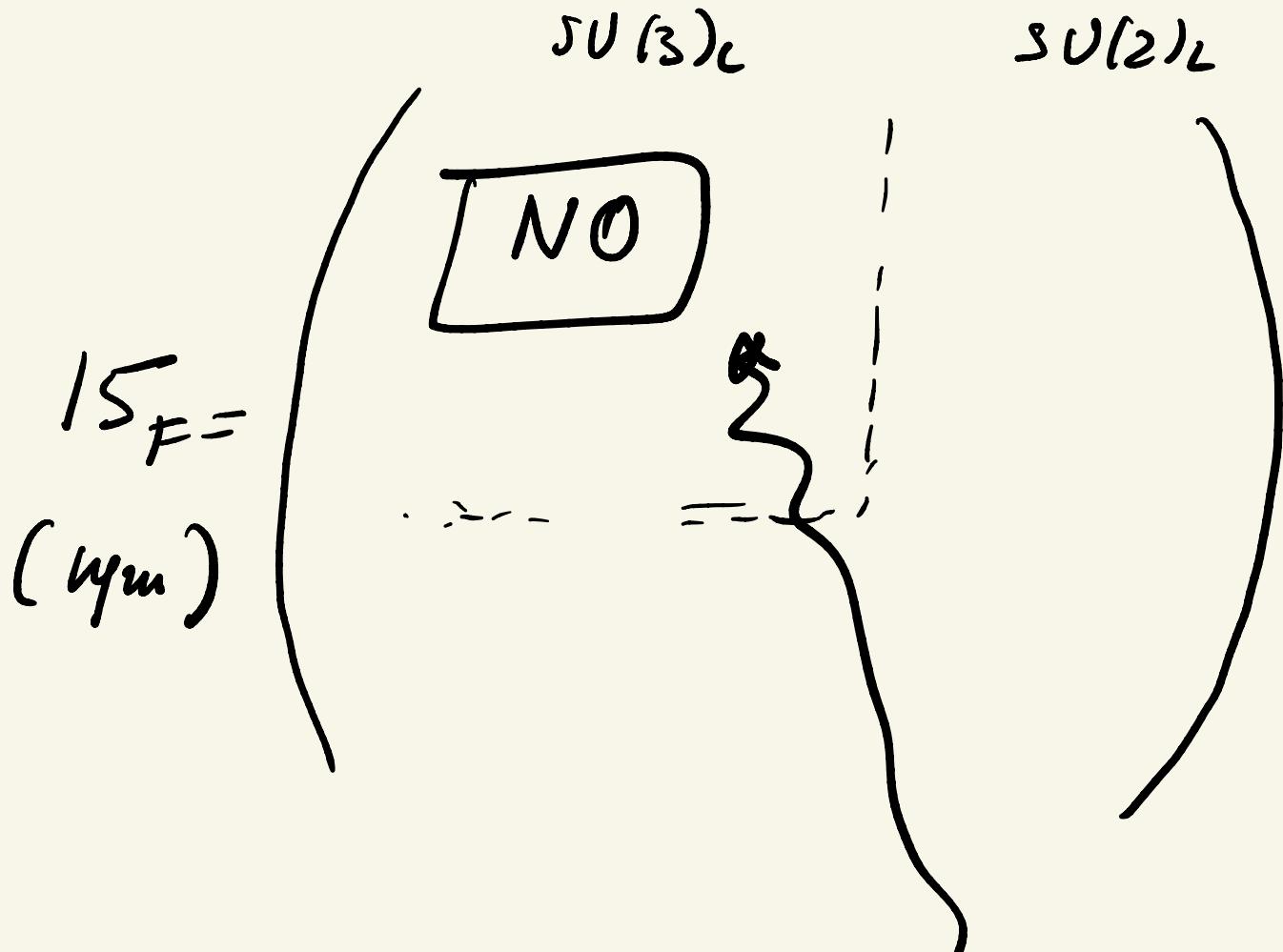
$$\text{Fermions} = \left[\overline{5}_F + 10_F \right]_L$$

$\underbrace{\hspace{10em}}$

15 fermions (SM)

- Why not 15_F ?

$$15_F = (5 \times 5)_{\text{sym}}$$



$$(3_c \times 3_c)_{14\mu m} = \textcircled{6} + 3^*$$

$$6 = \frac{3 \cdot 4}{2} \longrightarrow (14\mu m) \quad (\text{anti-}14\mu m)$$

$$3^* = \frac{3 \cdot 2}{2}$$



$$\underline{F_{\text{enriched}}} = \underbrace{(\bar{5}_F + 10_F)}_L$$

$$\bar{5}_F = \begin{pmatrix} d^c & \xrightarrow{\gamma} \text{glue } y \\ v & w \end{pmatrix} \times$$

$$10_F = \begin{pmatrix} u^c & u^d \\ - & - \end{pmatrix} \quad e^c$$



new gauge bosons:

$$x, y = \boxed{\begin{array}{l} \text{colorful} \\ \text{flavorful} \end{array}}$$

$$X_\mu^\alpha \left[\bar{d}_L^c \gamma^\mu e_L + \bar{u}_L \gamma^\mu u_L^c + \bar{e}_L \gamma^\mu d_L \right]$$

$$Y_\mu^\alpha \left[\bar{d}_L^c \gamma^\mu v_L + \bar{d}_L \gamma^\mu u_L^c + \bar{e}_L \gamma^\mu u_L \right]$$

$\alpha = r, y, b$

$-1 + \frac{2}{3} = -\frac{1}{3}$

$$Q(x) = \frac{4}{3} \quad Q(y) = \frac{1}{3} \quad \left. \begin{array}{l} Q(x) = Q(y) + 1 \\ \text{doublet} = \begin{pmatrix} x \\ y \end{pmatrix} \end{array} \right\}$$

$x^a, y^a = 3$ of color

$X^\alpha \bar{u}^a u^c \epsilon_{abc}$ color singlet anti symmetric

SU(3) $T^\alpha = 3$ of color

$\sum_{\alpha\beta\gamma} T^\alpha T^\beta \gamma^\delta = \text{color singlet}$



$\sum_{\alpha\beta\gamma} U_{\alpha\alpha'} U_{\beta\beta'} U_{\gamma\gamma'} T^{\alpha'} T^{\beta'} T^{\gamma'}$

" " "

$$= \sum_{\alpha'\beta'\gamma'} \underbrace{\det U}_{1} T^{\alpha'} T^{\beta'} T^{\gamma'}$$

$$= \sum_{\alpha\beta\gamma} T^\alpha T^\beta T^\gamma$$

Q.E.D.

$$\downarrow \quad \text{to be fixed}$$

$$X [\bar{d}^c e + \bar{u} u^c + \bar{e}^c d]$$

$$y [\bar{d}^c \nu + \bar{u} u^c + \bar{e}^c u]$$

$X = \{ \text{lepto-quarks} \}$
 $y \{ \text{diri-quarks} \}$

Can B (Baryon number)

be conserved ?

$L = \text{lepton number}$

SM

$$A \text{ [} \bar{e}e \text{]} \Rightarrow B(A) = 0$$

$$w \text{ [} \bar{u}d \text{]} \Rightarrow B(w) = 0$$

$$\Rightarrow \boxed{\Delta B = \Delta L = 0}$$

$$x \text{ } \bar{d}^c e \Rightarrow B(x) = -\frac{1}{3}$$

$$L(x) = -1$$

but

$$x \bar{u} u^c \Rightarrow B(x) = \cancel{\frac{2}{3}}^1 !$$

$\Rightarrow \boxed{\text{breaks } B !}$

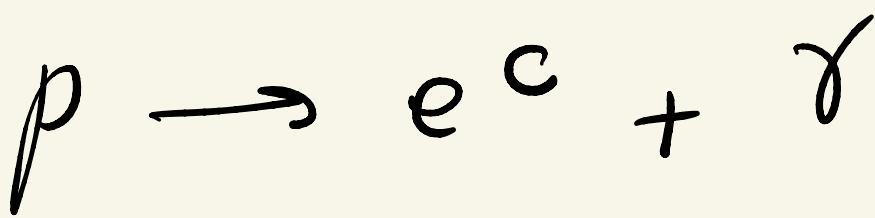
$\dashv - L !$

\Downarrow

B, L are broken!

\Downarrow

$\rho \neq$ not stable \therefore



$$X^\alpha, \bar{X}^\alpha = 3+3 \quad \left. \right\} \textcircled{12}$$

$$\psi^\alpha, \bar{\psi}^\alpha = 3+3$$

S.M.: gluons, W^+, W^-, Z, γ

$$8 + 3 + 1 = 12$$

$$\boxed{12 + 12 = 24}$$

~~24~~
gauge bosons
of $SU(5)$

