Neutrino Mass and Grand Unification

Lecture VII

9/11/2021

LMU
Fall 2021

SU(5) grand unified theory:

Construction

Georgi, Glashow '1974

$$\bar{5}_F = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e \end{pmatrix}_L \begin{matrix} \left.\vphantom{\begin{matrix} d^c \\ d^c \\ d^c \end{matrix}}\right\} SU(3)_c \\ \left.\vphantom{\begin{matrix} \nu \\ e \end{matrix}}\right\} SU(2)_L \end{matrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L (6) \qquad \overset{(3)}{(u^c)_L}, \quad \overset{(3)}{(d^c)_L} \checkmark$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \checkmark \; (2) \qquad (e^c)_L \; (1)$$

$$SU(5): \qquad T_a = T_a^\dagger, \quad \mathrm{Tr}\, T_a = 0$$

$$a = 1, 2, \cdots, 24$$

- $Q \equiv Q_{em} = \sum c_a T_a$

$$\mathrm{Tr}\, Q_{em} = 0$$

$$\Longleftrightarrow \sum_{repr} q_{em} = 0$$

$$5_F = \begin{pmatrix} d \\ d \\ d \\ \hdashline e^c \\ -\nu^c \end{pmatrix}_R \quad \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} SU(3)_c$$

reminder : $D_2 \longrightarrow U_2 D_2 \quad (SU(2))$

$$\tilde{D}_2 \equiv i \sigma_2 D^* \Longleftarrow$$

$$\Longrightarrow \tilde{D}_2 \longrightarrow U_2 \tilde{D}_2$$

$l \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ = doublet

$$\tilde{l} \equiv i\sigma_2\, l^c = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \nu^c \\ e^c \end{pmatrix} = \begin{pmatrix} e^c \\ -\nu^c \end{pmatrix}$$

$\Downarrow$ assuming $q_\nu = 0$ (see below)

(*) $Q(5) = \mathrm{diag}\left(-1/3, -1/3, -1/3, 1, 0\right)$

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but: + 10 more fermions

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad (u^c)_L, \quad (e^c)_L$$

$10 = 5 \times 2$

(a) why not 2 more 5_F's?

Impossible $\Rightarrow$ we would violate (*)!

$\Downarrow$

(b) 10_F (repr. of $SU(5)$)

Representations of SU(N)

SU(2) $\quad D \Leftrightarrow s = 1/2$

(3) $T\ (s=1) = (D \times D)$

Tensor product

$R \times R$ = representation
but not irreducible

$$\Rightarrow R \times R = (R \times R)_S + (R \times R)_A$$

$SU(2)$: $D \times D = \underset{\substack{(T) \\ \text{triplet}}}{S} + \underset{\substack{(S) \\ \text{singlet}}}{A}$

$SU(5)$: $5 \times 5 = S + A$

S: $\frac{5 \cdot 6}{2} = 15$ $\quad$ A: $\frac{5 \cdot 4}{2} = 10$

$$\boxed{25 = 15 + 10}$$

$SU(5)$ $$\boxed{10_{repr} = (5 \times 5)_{AS}}$$

$$\boxed{Q(10) = ?}$$

$$10_{ij} = 5_i 5_j$$

$$\rightarrow \quad V_{ik} V_{je} 5_k 5_e =$$

$$V_{ik} V_{je} 10_{ke}$$

$$10_{ij} \rightarrow V_{ik} 10_{ke} V_{ej}^T$$

$$\boxed{10 \rightarrow U 10 U^T}$$

$$U = e^{i\Theta T} \quad \text{(summed)}$$

$$U^T = e^{i\Theta T^T}$$

$\hat{T}_a$ = generator of $SU(5)$ on 10

$$\boxed{T_a 10 = T_a 10 + 10\, T_a^T}$$

$$(U = 1 + i\theta_a T_a + \cdots)$$

$$Q = \Sigma c_a T_a$$

$$\Rightarrow \boxed{\begin{aligned} Q\,10 &= Q\,10 + 10\, Q^T \\ &= Q\,10 + 10\, Q \\ &\quad (Q \in \text{Cartan}) \end{aligned}}$$

$$10_{ij} = 5_i 5_j$$

$$\boxed{Q = \text{sum of charges in } 5 + 5}$$

$$\boxed{** \quad Q(10_{ij}) = Q(5_i) + Q(5_j)}$$

$\Rightarrow$ True of all "charges"

$$T_{3w}(10_{ij}) = T_{3w}(5_i) + T_{3w}(5_j)$$

- - - -

$\Downarrow$

$$
\begin{array}{cc}
 & \begin{array}{cc} SU(3)_c & SU(2)_L \end{array} \\
10_F = & \left(\begin{array}{ccc|cc}
0 & u^c & u^c & u & d \\
-u^c & 0 & u^c & u & d \\
 & & 0 & u & d \\
\hline
-u & -u & -u & 0 & e^c \\
-d & -d & -d & -e^c & 0
\end{array}\right)_L
\end{array}
$$

(AS)

need : $\begin{pmatrix} u \\ d \end{pmatrix}_L$, $(u^c)_L$, $(e^c)_L$

(colr) (colr)

(Dirac) $\boxed{Q(u^c) = -Q(u)}$ (QED)

$\Downarrow$

$$\Sigma Q(10) = \left(3\,Q(u^c) + 3Q(u)\right) = 0$$

$$\parallel$$

$$0 \Rightarrow \left[+ 3\,Q(d) + Q(e^c) \right] = 0$$

$$\Downarrow$$

$$\boxed{3\,Q_d + Q_{e^c} = 0}$$

$$\boxed{\text{charge quantisation}}$$

$\Downarrow$ step back

$$Q(\bar{5}_F) = 0$$

$$\bar{5}_F = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e \end{pmatrix}_L \Leftrightarrow 5_F = \begin{pmatrix} d \\ d \\ d \\ e^c \\ -\nu^c \end{pmatrix}_R$$

$$\sum Q(5) = \underbrace{3\,Q(d) + Q(e^c) + Q(\nu) = 0}_{= 0}$$

$$\Downarrow$$

$$\boxed{Q(\nu) = 0}$$

<u>reminder</u> $Q(\nu) = Q(e) + 1$

$$Q = T_3 + Y/2$$

$$Q(\nu) = 1/2 + Y/2$$

$$Q(e) = -1/2 + Y/2$$

$$\Downarrow$$

$$Q(\nu) - Q(e) = 1$$

$\Leftrightarrow$ Minimal $SU(5)$ "predicts" that there is <u>no</u> ν_R

<u>generations</u> = xerox copying

$\Downarrow$

$$\boxed{\text{other generations} = \text{same quantum numbers}}$$

$$\text{Fermions} = \underbrace{\left[\bar{5}_F + 10_F\right]_L}_{15 \text{ fermions (SM)}}$$

- why not 15_F ?

$$15_F = (5 \times 5)_{sym}$$

$$\begin{array}{ccc} & SU(3)_c & SU(2)_L \\ \underset{(\text{sym})}{15_F =} & \multicolumn{2}{c}{\begin{pmatrix} \boxed{NO} & \\ & \end{pmatrix}} \end{array}$$

$$(3_c \times 3_c)_{sym} = 6 + 3^*$$

(sym) (antisym)

$$6 = \frac{3 \cdot 4}{2}$$

$$3^* = \frac{3 \cdot 2}{2}$$

$$\Downarrow$$

$$\underline{\text{Fermions}} = \underbrace{\left(\bar 5_F + 10_F\right)_L}$$

$$\bar 5_F = \begin{pmatrix} d^c \\ \nu \\ e \end{pmatrix}_L \quad \text{glue} \quad Y \quad X$$

$$10_F = \begin{pmatrix} u^c & u \quad d \\ & e^c \end{pmatrix}_L$$

$$\Downarrow$$

new gauge bosons!

$X, Y = \boxed{\text{colorful}}$ $\boxed{\text{flavorful}}$

$$X_\mu^\alpha \left[\overline{d^c_L}^{\,\alpha} \gamma^\mu e_L + \bar{u}_L^{\beta} \gamma^\mu u_L^{c\,\gamma} + \overline{e^c_L} \gamma^\mu d_L \right]$$

$$Y_\mu^\alpha \left[\overline{d^c_L}^{\,\alpha} \gamma^\mu \nu_L + \bar{d}_L^{\alpha} \gamma^\mu u^c_L + \overline{e^c_L} \gamma^\mu u_L \right]$$

$-1 + 2/3 = -1/3$

$$\boxed{\alpha = r, y, b}$$

$$\left.\begin{aligned} Q(x) &= 4/3 \\ Q(y) &= 1/3 \end{aligned}\right\} \underbrace{Q(x) = Q(y) + 1}_{\text{doublet} = \begin{pmatrix} X \\ Y \end{pmatrix}}$$

$$\boxed{X^\alpha, Y^\alpha = 3 \text{ of color}}$$

$$\underbrace{X^\alpha \bar{u}^\beta u^{c\,\gamma}}_{\text{color singlet}} \; \epsilon_{\alpha\beta\gamma}$$

antisymmetric

SU(3) $\quad T^\alpha$ = 3 of color

$$\varepsilon_{\alpha\beta\gamma}\, T^\alpha T^\beta T^\gamma = \text{color singlet}$$

$$\downarrow$$

$$\underbrace{\varepsilon_{\alpha\beta\gamma}\, U_{\alpha\alpha'}\, U_{\beta\beta'}\, U_{\gamma\gamma'}}_{\parallel}\, T^{\alpha'} T^{\beta'} T^{\gamma'}$$

$$= \varepsilon_{\alpha'\beta'\gamma'}\, \underbrace{\det U}_{1}\, T^{\alpha'} T^{\beta'} T^{\gamma'}$$

$$= \varepsilon_{\alpha\beta\gamma}\, T^\alpha T^\beta T^\gamma$$

Q.E.D.

$\Downarrow$ to be fixed

$$X\left[\,\bar{d^c}\, e + \bar{u}\, u^c + \bar{e^c}\, d\,\right]$$

$$y\left[\,\bar{d^c}\, \nu + \bar{d}\, u^c + \bar{e^c}\, u\,\right]$$

$$\left.\begin{matrix} X \\ y \end{matrix}\right\} = \left\{ \begin{matrix} \text{lepto-quarks} \\ \text{di-quarks} \end{matrix} \right\}$$

$$\boxed{\text{Can } B \text{ (Baryon number) be conserved?}}$$

L = lepton number

__SM__

$$A \; [\bar{e}\, e] \Rightarrow B(A) = 0$$

$$W \; [\bar{u}\, d] \Rightarrow B(W) = 0$$

$$\Rightarrow \boxed{\Delta B = \Delta L = 0}$$

$$X \quad \bar{d}^c\, e \Rightarrow B(X) = -1/3$$

$$L(X) = -1$$

but

$$X \quad \bar{u}\, u^c \Rightarrow B(X) = 2/3 \;!$$

⇒ breaks B!
  —"— L!

$\Downarrow$

$$\boxed{B, L \text{ are broken!}}$$

$\Downarrow$

$p \neq$ not stable :(

$$p \to e^c + \gamma$$

$$e^c + \pi^0$$

$$\nu + \pi^+$$

?

$$\left.\begin{array}{ll} X^\alpha, \ \bar{X}^\alpha & = 3+3 \\ Y^\alpha, \ \bar{Y}^\alpha & = 3+3 \end{array}\right\} 12$$

SM: $\underbrace{\text{gluons}, \ W^+, W^-, Z, \gamma}_{8+3+1=12}$

$$\boxed{12 + 12 = 24}$$

gauge bosons of $SU(5)$