Neutrino Mass

and

Grand Unification

Lecture IV

29/10/2021

LMU

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Standard Model: EW part

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv q_L \qquad u_R \,,\; d_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv l_L \qquad e_R$$

$$G_{SM} = SU(2)_L \times U_Y(1)$$

$$\Downarrow$$

$\exists \ \Phi$: Higgs doublet

$\Uparrow$ GIM = Glashow, Illiopulos, Maiani '69

GIM

$$\mathcal{L}_Y = \bar{q}_L\, y_d\, \Phi\, d_R + \bar{q}_L\, y_u\, \underbrace{i\sigma_2 \Phi^*}_{\tilde{\Phi}}\, u_R$$

$$+ \bar{l}_L\, y_e\, \Phi\, e_R \quad + h.c. \quad (\text{Weinberg})$$

$$\Updownarrow$$

$$\boxed{Y\,\Phi = +\,\Phi}$$

$$\boxed{Q = T_3 + Y/2}$$

$$\Rightarrow \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\Uparrow$$

Mass terms for f (Lorentz)

$f \equiv \psi \qquad \bar{\psi} = \psi^{+} \gamma^{0}$

(a) $\bar{\psi} \psi m_D = (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) m_D$

$\{\gamma^0, \gamma^5\} = 0$ (Dirac)

(b) $(\psi_L^T C \psi_L + h.c.) m_M$ (Majorana)

$C^T = -C, \quad C \gamma_\mu C^T = -\gamma_\mu^T$

$\Rightarrow \psi_L^T C \psi_L \xrightarrow[\text{Lorentz}]{} \psi_L^T \Lambda^T C \Lambda \psi_L = \text{inv.}$

$\Lambda = e^{i \theta_{\mu\nu} \Sigma^{\mu\nu}}$

$\Sigma^{\mu\nu} = \frac{1}{4i} [\gamma^\mu, \gamma^\nu]$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \qquad C \equiv i\gamma_2\gamma_0$$

$$\Downarrow$$

$$\psi_L^T C \psi_L = u_L^T \, i\sigma_2 \, u_L \quad = \text{INV.}$$

$$u_L \longrightarrow e^{i\vec{\sigma}/2\cdot(\underset{\uparrow}{\vec{\theta}} + i\underset{\uparrow}{\vec{\chi}})}\, u$$

ROT BOOST

Q. Can I write "Majorana" Yukawas?

$$\rightarrow \left(l_L^T C \underset{\substack{\uparrow \\ \Phi}}{\ } l_L \right) \not\ni \boxed{3 \ SU(2) \text{ doublets}}$$

ℓ = doublet

Φ = doublet

$\psi\,\psi$ $(d=3)$ $\qquad \mathcal{L}_{kin} = \bar{\psi}\,\partial\,\psi$

Φ $(d=1)$ $\quad \not\exists$ $\qquad + \frac{1}{2}(\partial_\mu \Phi)^2$

$$\Rightarrow \boxed{\begin{array}{c}\text{2 fermions, 1 scalar} \\ d=4\end{array}}$$

$$\Downarrow$$

$d(\mathcal{L}) = 4$ + Lorentz + $SU(2)$

$$\Downarrow$$

NO $\quad l\,\bar{\Phi}\,l$

$q\,\bar{\Phi}\,q$

$d_R\,\bar{\Phi}\,d_R$

s^W = "weak" spin

$$s^W(l + \bar{\Phi} + l) = \pm 1/2 \pm 1/2 \pm 1/2$$

$$\neq 0$$

. what about $d = 5$?

$l\;l\;\bar{\Phi}\;\bar{\Phi}$

$Y \quad -1 \quad -1 \quad +1 \quad +1$

Lorentz +

$SU(2) \times U(1)$ inv?

$\boxed{\text{YES}}$

$SU(2)$: $\pm 1/2 \pm 1/2 \pm 1/2 \pm 1/2 = 0 + \cdots$

Weinberg $d = 5$
effective theory
1979

However, we must stick
to $d(\mathcal{L}) = 4$

$\Updownarrow$

$$\boxed{\text{Finite physical amplitudes}}$$

Spontaneous Symmetry Breaking (SSB)

$$\mathcal{L}_{\Phi} = (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi)$$

$$\boxed{V(\Phi) = \frac{\lambda}{4} \left(\Phi^+ \Phi - v^2 \right)^2} \quad (d=4)$$

why $\Phi^T i \sigma_2 \Phi$ not ?

$r(SU(2)) = 1$ rank

rank = # of Cartan gen.

{commuting gen.}

$\Rightarrow$ one invariant $\Phi^+\Phi$

$$\Phi \longrightarrow U\Phi = \begin{pmatrix} 0 \\ \phi \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} \Rightarrow U = ?$$

$\Rightarrow$ $\boxed{\text{one invariant}}$

$$\Phi = \begin{pmatrix} R_1 + iR_2 \\ R_3 + iR_4 \end{pmatrix} \Rightarrow$$

$$\boxed{\Phi^{+}\Phi = \sum_{i=1}^{4} R_i^{2}} \leftarrow SO(4) \text{ accidental}$$

<u>Minimum</u> $\quad \Phi_0^{+}\Phi_0 = v^2$

$$\sum_{i=0}^{4} (R_i^{0})^2 = v^2$$

$$\mathcal{M}_0 = \{ \Phi_0 : V = V_{min} \}$$

vacuum manifold

$$\Rightarrow \boxed{\mathcal{M}_0 = S_3} \quad \text{3-dim sphere}$$

$$\Phi_0' = \begin{pmatrix} v_1 e^{i\alpha} \\ v_2 e^{i\beta} \end{pmatrix} \xrightarrow[\text{symmetry}]{} U \Phi_0' = \begin{pmatrix} 0 \\ v \end{pmatrix} \equiv \Phi_0$$

$$U^+ \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} v_1 e^{i\alpha} \\ v_2 e^{i\beta} \end{pmatrix}$$

$\boxed{\text{Find } U}$ *

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

vacuum (ground state)

vacuum expectation value (vev)

$$\Phi_0 \xrightarrow{SU(2)} U\Phi_0 = e^{i\theta_a T_a}\,\Phi_0 \overset{?}{=} \Phi_0$$

$$\Phi_0 \xrightarrow[U(1)]{} e^{i\alpha Y/2}\,\Phi_0 \overset{?}{=} \Phi_0$$

NO

$$Y\,\Phi_0 \neq 0$$

$$T_a\,\Phi_0 \neq 0 \qquad \left(T_a = \sigma_a/2\right)$$

$\uparrow$

gen. break symmetry

$\Downarrow$

$SU(2) \times U(1)$ is "broken"

$$T_1 \Phi_0 = \frac{\sigma_1}{2} \Phi_0 \neq 0$$

$$T_2 \Phi_0 = \sigma_2/2 \, \Phi_0 \neq 0$$

$$\begin{cases} T_3 \Phi_0 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \Phi_0 = \begin{pmatrix} 0 \\ -1/2 \, v \end{pmatrix} \\ \frac{Y}{2} \Phi_0 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \Phi_0 = \begin{pmatrix} 0 \\ +1/2 \, v \end{pmatrix} \end{cases}$$

$$\left(T_3 + \frac{Y}{2}\right) \Phi_0 = 0$$

$$\boxed{Q_{em} \Phi_0 = 0}$$

$$\boxed{\begin{array}{l} SU(2)_L \times U_Y(1) \\ \Phi_0 \\ \quad \hookrightarrow \quad U(1)_{em} \end{array}}$$

Φ = SU(2) doublet

$$= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \phi^+, \phi_0 = \text{complex}$$

Higgs boson = neutral

$$\boxed{\text{Who is Higgs?}}$$

$$\Phi = \begin{pmatrix} \overbrace{G_2/2 - i\, G_1/2}^{\phi^+} \\ \underbrace{v + h - i\, G_3/2}_{\phi_0} \end{pmatrix} \frac{1}{\sqrt{2}} + \cdots$$

$$\Phi \longrightarrow U\Phi = \Phi'$$

$$A_\mu \to U A_\mu U^+ + {}^{i}\!/_{g} (\partial_\mu U) U^+ = A_\mu'$$

$$\boxed{U = e^{i G_i T_i / 2}} \qquad T_i \equiv \sigma_i / 2$$

$$\Downarrow \qquad \boxed{A_\mu \equiv A_\mu^a T_a}$$

$$\boxed{\begin{aligned} \Phi' &= \begin{pmatrix} 0 \\ v+h \end{pmatrix} \frac{1}{\sqrt{2}} \\ A_\mu' &= \cdots \end{aligned}}$$

"unitary gauge"

From now on: $\boxed{\Phi', A' \longrightarrow \Phi, A}$

<u>work with:</u>

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}, \quad A_\mu$$

h = Higgs boson

$$D_\mu \Phi = \left(\partial_\mu - i g T_a A_\mu^a - i g' \tfrac{1}{2} B_\mu\right) \Phi$$

$$= \underbrace{\partial_\mu \Phi}_{\text{kinetic}} - i \underbrace{\left(g T_a A_\mu^a + g' \tfrac{1}{2} B_\mu\right) \Phi}_{\text{mass, interaction?}}$$

$$\Downarrow$$

$$(D_\mu \Phi)^2 \xrightarrow[\text{int}]{} \left[\left(g T_a A_\mu^a + g'/_2 B_\mu\right) \Phi\right]^2$$

$$\parallel$$

$$\frac{1}{2} \begin{pmatrix} g A_3 + g' B & g(A_1 - i A_2) \\ g(A_1 + i A_2) & (-g A_3 + g' B) \end{pmatrix}_\mu \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} g\,(A_1 - i A_2)_\mu \\ (-g A_3 + g' B)_\mu \end{pmatrix} (v+h) \frac{1}{\sqrt{2}}$$

$$\Downarrow$$

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{1}{2} \cdot \frac{1}{4} (v+h)^2 \times \left[g^2 (A_1^2 + A_2^2) + (g^2 + g'^2) \left(\frac{g A_3 - g' B}{\sqrt{g^2 + g'^2}} \right)^2 \right]$$

- <u>v</u> piece $\qquad W^\pm_\mu = \dfrac{A_1 \mp i A_2}{\sqrt{2}}$

$$\Downarrow$$

$$\boxed{M_W^2 = \frac{1}{4} g^2 v^2}$$

$$Z_\mu \equiv \frac{(g A_3 - g' B)_\mu}{\sqrt{g^2+g'^2}}$$

$$\boxed{\tan\theta_W = g'/g}$$

$$\Downarrow$$

$$\boxed{M_Z^2 = \frac{1}{4}(g^2+g'^2)v^2}$$

$$A_\mu \equiv \frac{(g' A_3 + g B)_\mu}{\sqrt{g^2+g'^2}} = \sin\theta_W A_\mu^3 + \cos\theta_W B_\mu$$

$$\Rightarrow \boxed{M_A = 0}$$

In summary

$$\boxed{M_{W^+} = M_{W^-} = \frac{g}{2} v \qquad M_Z = \frac{M_W}{\cos\theta_W}}$$

$$\boxed{e = g \sin\theta_W}$$

θ_W: determined at LEP

Large Electron - Positron

$l = 27$ km

$$\boxed{10^9 \quad W, Z}$$

$$M_W = 80\ \text{GeV} \qquad \theta_W \simeq 30^\circ$$

$$M_Z = 90\ \text{GeV}$$

$$\sin^2\theta_W = 0.23$$

- $W^\pm$ "ate" ϕ^+, ϕ^-
- Z "ate" G_3

$$\Downarrow$$

$$\frac{1}{2}(D_\mu \Phi)^+(D^\mu \Phi) = \text{kinetic}$$

$$+ M_W^2\, W_\mu^+ W^- \left(1 + \frac{h}{v}\right)^2$$

$$+ \frac{1}{2} M_Z^2\, Z_\mu Z^\mu \left(-\shortparallel- \right)$$

$\Rightarrow$ $\boxed{\text{Z and } W^{\pm} \text{ are Proca fields}}$

$\Downarrow$

$$\Delta_{\mu\nu}(z) = -\frac{1}{q^2 - M_Z^2}\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right]$$

NOT good

What to do?

$\boxed{U(1)}$ Higgs $\boxed{1964}$

$$\phi \to e^{i\alpha}\phi \quad (Q\phi = 1)$$

$$\mathcal{L}_h = (D_\mu\phi)^*(D^\mu\phi) - V(\phi)$$

$$V(\phi) = \frac{1}{4}\left(|\phi|^2 - v^2\right)^2$$

$$\mathcal{M}_0 = \left\{ |\phi_0|^2 = v^2 \right\} = \left\{ (R_1^0)^2 + (R_2^0)^2 = S_1 \right\}$$

$$\phi = R_1 + i R_2$$

$$|\phi|^2 = R_1^2 + R_2^2$$

$\|$

circle

$\Downarrow$

$$\phi = e^{i\theta/v}\,(v+h)\,\frac{1}{\sqrt{2}}$$

$$\phi' = e^{-i\theta/v}\,(v+h) = (v+h)\,\frac{1}{\sqrt{2}}$$

$$A_\mu' = A_\mu + \frac{i}{g}\,\partial_\mu \theta$$

$$\Downarrow$$

unitary gauge

$$\phi_v = \frac{1}{\sqrt{2}}\,(v+h)$$

$$A_\mu$$

$$\boxed{m_A = gv}$$

$\Longrightarrow$ Proca

NOT good for computations

$$\Downarrow$$

$\boxed{\text{different gauge}}$ 't Hooft 1971 -

$\Leftrightarrow$ QED $\quad A_\mu$ + gauge fixing

$\Longrightarrow \boxed{\text{keep } G}$

$$\boxed{\phi_R = \frac{1}{\sqrt{2}}(v + h + iG)}$$

$\Downarrow$ $\boxed{\text{rest is history}}$

$\Downarrow$

$$\Delta \mathcal{L}_{g.f} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu + \xi v G\right)^2$$

one gets

$$D^R(G) = \frac{i}{q^2 - \underbrace{\xi m_A^2}_{\text{mass of } G \text{ depends on } \xi}}$$

$\Longleftrightarrow$ G is not physical

$$\Delta^R_{\mu\nu}(A) = \frac{-i}{q^2 - m_A^2}\left[g_{\mu\nu} + (\xi - 1)\frac{q_\mu q_\nu}{q^2 - \xi m_A^2}\right]$$

for any finite $\xi \Rightarrow$ good propagator !!!

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unitary gauge: $\xi \to \infty$

$$\boxed{\text{G = would have been (Nambu) Goldstone}}$$

Nambu - Goldstone mode:

<u>global</u> symmetry $\Rightarrow$ G is

a physical particle