Neutrino Mass

and

Grand Unification

Lecture III

26/10/2021

LHU
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Standard Model:

Electro - Weak part

$$SU(2)_L \times U_Y(1) \equiv G_{SM}^{ew}$$

$$Q \equiv Q_{em} = T_3 + Y/2$$

$$\boxed{Y = 2\,(Q - T_3)} \qquad (1)$$

Glashow 1961

$$J_W^\mu = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L$$

$$L = \frac{1+\gamma_5}{2} \qquad R = \frac{1-\gamma_5}{2}$$

$$\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\sigma^\mu = (1; \sigma_i)$$

$$\bar{\sigma}^\mu = (1; -\sigma_i)$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3$$

$$\{\gamma_5, \gamma_\mu\} = 0 \qquad \gamma_5^2 = +1$$

$$\Sigma_{\mu\nu} = \frac{1}{4i}\left[\gamma_\mu, \gamma_\nu\right]$$

$$\Lambda = S = e^{i\theta^{\mu\nu}\Sigma_{\mu\nu}} \leftarrow$$

Σ_{0i} — BOOST

Σ_{ij} — ROT

$$L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

$$\psi \rightarrow \Lambda \psi \quad (\text{spinor})$$

$$\psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \begin{array}{l} \leftarrow \text{LH (up)} \\ \leftarrow \text{RH (down)} \end{array}$$

$\boxed{\text{DIRAC}}$ $\quad p^\mu \gamma^\mu \psi = m\psi$

$\boxed{1928}$

- $m = 0$ $\quad p^\mu \gamma_\mu = 0$

$$\Rightarrow \begin{pmatrix} 0 & E - \vec{p}\cdot\vec{\sigma} \\ E + \vec{p}\cdot\vec{\sigma} & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix} = 0$$

$$\Rightarrow \quad E\, u_{L,R} = \mp\, \vec{p}\cdot\vec{\sigma}\, u_{L,R}$$

$|\vec{p}| = p = E$ $\qquad \vec{S} = \vec{\sigma}/2$

$$\Downarrow$$

$$h = \hat{p}\cdot\vec{S} = \frac{\vec{p}\cdot\vec{S}}{E} = \frac{\vec{p}\cdot\vec{\sigma}}{2E}$$

$$\boxed{h\, u_{L,R} = \mp\, u_{L,R}}$$

$\Downarrow$ $\cancel{P}$ = maximal

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R,\ d_R$$

$$l_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R,\ \cancel{\nu_R}$$

P = good

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R .$$

$\exists \, \nu_R \Leftrightarrow m_\nu = 0 \, ?$

$$D_\mu = \partial_\mu \underbrace{- i g A_\mu^a T_a}_{SU(2)} \underbrace{- i g' \frac{Y}{2} B_\mu}_{U(1)}$$

$$\boxed{T_a f_L = \frac{\sigma_a}{2} f_L} \qquad \boxed{T_a f_R = 0}$$

$$U = e^{i \theta_a T_a} \equiv SU(2)$$

singlet = trivial $\Leftrightarrow U = 1$

$$\mathcal{L}_f^D = i\bar{f}\gamma^\mu D_\mu f - m_f \bar{f} f$$

$$f \equiv \psi_f \Rightarrow \bar{f} f = \bar{f}_L f_R + \bar{f}_R f_L$$

$$\bar{f} = f^\dagger \gamma^0 \qquad \{\gamma^0, \gamma_5\} = 0$$

$$\boxed{m_f = 0 \quad \text{by} \quad SU(2)_L}$$

· DIRAC mass

$$m_D \bar{f} f = \left(\bar{f}_L f_R + \bar{f}_R f_L\right) m_D$$

$$= \left(u_L^\dagger u_R + u_R^\dagger u_L\right) m_D$$

- MAJORANA mass $\boxed{1938}$

$$m_M \, f_L^T C f_L + h.c.$$

(def.) $C^T = -C$, $C \gamma_\mu C^T = -\gamma_\mu^T$

$$C^+ = -C$$

$$\boxed{C = i \gamma_2 \gamma_0} \quad \text{(choice)}$$

$$\Rightarrow \quad m_M \, u_L^T i \sigma_2 u_L + h.c.$$

$\boxed{\text{Lorentz}}$ $\quad u_L \to e^{i \vec{\sigma}/2 \cdot (\vec{\theta} + i \vec{\chi})}$

$\uparrow$ ROT $\quad \uparrow$ BOOST

$$u_R \to e^{i\,\vec{\sigma}/2\,(\vec{\theta} - i\,\vec{x})}$$

$$\Downarrow$$

$$m_M \, \underbrace{u_L^T \, i\sigma_2 \, u_L}_{\text{Anti-symmetric}} \qquad (I=0,\ \text{invariant})$$

$$\downarrow$$

$$l_L^T \, C \, i\sigma_2 \, l_L$$

Lorentz invariance

SU(2) invariance

$$\downarrow$$

$D \to U D$ reminder

$$D^T i\sigma_2 D \to D^T U^T i\sigma_2 U D$$

$$= D^T i\sigma_2 U^+ U D = D^T i\sigma_2 D \quad \checkmark$$

- $\ell^T i\sigma_2 C \ell = 0$

$$= -\ell^T i\sigma_2^T C^T \ell$$

$$= -\ell^T i\sigma_2 C \ell$$

- scalar $D = \Phi \quad (\Phi \to U\Phi)$

$$\Phi^T i\sigma_2 \Phi = 0$$

$$\bullet \quad e_R^T C e_R = -e_R^T C^T e_R$$

$$= e_R^T C e_R$$

$$Q e_R = Y/2 \, e_R = -e_R$$

$$\Rightarrow \quad Y e_R = -2 e_R$$

$$\Downarrow$$

$$i \bar{f}_L \gamma^\mu D_\mu f_L \quad =$$

$$= i \bar{f}_L \gamma^\mu \partial_\mu f_L \quad +$$

$$\bar{f}_L \gamma^\mu \left(g \frac{\sigma_a}{2} A_\mu^a + g' \frac{Y}{2} B_\mu \right) f_L$$

$\underline{\cdot 1, 2}$

$$= (\bar{u}\ \bar{d})_L\ \gamma^\mu\ g \begin{pmatrix} 0 & A_1 - iA_2 \\ A_1 + iA_2 & 0 \end{pmatrix}_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$= \frac{g}{\sqrt{2}}\ \bar{u}_L\ \gamma^\mu\ \underbrace{\frac{A_1 - iA_2}{\sqrt{2}}}_{W_\mu^+}\ d_L + h.c.$$

$$\Downarrow$$

$$\boxed{\mathcal{L}_{Wu} = \frac{g}{\sqrt{2}}\ \bar{u}_L\ \gamma^\mu d_L\ W_\mu^+ + h.c.}$$

$$\Downarrow$$

• neutral piece

$$\bar{f}\gamma^\mu \left(g T_3 A_3 + g' \frac{Y}{2} B\right)_\mu f$$

$$T_3 f_L = \frac{\sigma_3}{2} f_L$$

$$T_3 f_R = 0$$

$$\Downarrow$$

$$Y/2 = Q - T_3$$

$$\Downarrow$$

$$\bar{f}\gamma^\mu \left[T_3 \left(g A_3 - g' B\right)_\mu + g' Q B_\mu\right] f$$

Neutral gauge int.

$$= \{ \exists \text{ photon}: \; e A_\mu \bar{f} Q \gamma^\mu f \}$$

$$= \bar{f} \gamma^\mu \left[e Q_{em} A_\mu + g_Z Q_Z Z_\mu \right] f$$

$$\Downarrow$$

$$\boxed{\begin{aligned} A_\mu &= \sin\theta_W A_\mu^3 + \cos\theta_W B_\mu \\ Z_\mu &= \cos\theta_W A_\mu^3 - \sin\theta_W B_\mu \\ \tan\theta_W &= g'/g \end{aligned}}$$

$$\Downarrow \qquad \boxed{\text{SHOW}}$$

$$g_Z = \frac{g}{\cos\theta_W}$$

$$Q_Z = T_3 - Q_{em}\sin^2\theta_W$$

$$\theta_W \simeq 30^\circ \quad (\sin^2\theta_W \simeq 0.23)$$

$$\boxed{e = g\sin\theta_W \quad *}$$

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exp.

$$\boxed{\begin{aligned} M_W &= 80\ \text{GeV} \\ M_Z &= 90\ \text{GeV} \end{aligned}}$$

$W^-, W^+, Z \leftarrow$ massive gauge boson

$$\Delta^P_{\mu\nu}(A) = \frac{-i}{q^2 - m_A^2}\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{m_A^2}\right]$$

(m_A)

bad at high q

$\Downarrow$ Higgs mechanism

$\cancel{P}$ f :

$\boxed{\text{Weinberg 1967}}$

$\begin{pmatrix} u \\ d \end{pmatrix}_L$; u_R d_R

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$; e_R

"V-A was the key"

Weinberg '09

$\boxed{\text{Higgs}}$ $\boxed{SU(2)_L \times U(1)_Y}$

$$\mathcal{L}_f = i \bar{f} \gamma^\mu D_\mu f$$

$$\mathcal{L}_{A,B} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon_{abc} A_\mu^b A_\nu^c$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$\mathcal{L}_{Higgs}$

$$\boxed{\begin{array}{l} q_L \to U q_L, \\ \Rightarrow \Phi \to U \Phi \end{array}}$$

$$\mathcal{L}_Y = \bar{q}_L y_d \Phi d_R + \boxed{\bar{l}_L y_e \Phi e_R}$$

G/H

Weinberg

SU(2) inv. $\quad \Phi \to U \Phi$

$U_Y(1)$ inv. ! $\boxed{Y_\Phi = +1}$

$$Y d_R = 2 Q d_R = -2/3$$

$$Y q_L = 2(Q - T_3) q_L = +1/3 \, q_L$$

$$\downarrow \quad + \; \bar{q}_L \, g_u \, i\sigma_2 \, \Phi^* u_R$$

$$Y_{u_R} = 2Q \, u_R = 4/3 \, u_R \quad \leftarrow$$

SU(2): $\bar{q}_L \; \Phi^* \; u_R \longrightarrow$

$$\bar{q}_L \underbrace{U^+ U^*}_{\neq 1} \Phi^* u_R$$

check: $\bar{q}_L \; i\sigma_2 \; \Phi^* \; u_R \longrightarrow$

$$\bar{q}_L \, U^+ i\sigma_2 U^* \; \Phi^* \, u_R$$

$$= \bar{q}_L \underbrace{U^+ U}_{1} i\sigma_2 \; \Phi^* u_R \quad \checkmark$$

$$\Downarrow$$

$$\mathcal{L}_\Phi = (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^+\Phi - v^2\right)^2$$

$$= \frac{\lambda}{4}\,\Phi^+\Phi - \frac{\lambda}{2} v^2 \Phi^+\Phi + \frac{\lambda}{4} v^4$$

$$\Downarrow$$

$\Phi_0^+ \Phi_0 = v^2$ is the minimum

<u>vacuum manifold</u>

$$\mathcal{M}_0 = \{ \Phi_0 \therefore V = V_{min} = 0 \}$$

$$= \{ \Phi_0^+ \Phi_0 = v^2 \} =$$

$$\Phi = \begin{pmatrix} R_1 + i R_2 \\ R_3 + i R_4 \end{pmatrix}$$

$$\underbrace{\Phi_0^{+} \Phi_0 = \sum_{i=1}^{4} (R_i^0)^2 = v^2}_{S^3}$$

$$\boxed{\text{crucial } \cancel{P} = \text{maximal!!}}$$

· imagine P = good !

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \; ; \quad \begin{pmatrix} u \\ d \end{pmatrix}_R \equiv q_R$$

$$\Downarrow \quad Y_{q_L} = Y_{q_R} = 1/3$$

$$q_L \to U q_L \qquad q_R \to U q_R$$

$$m_q(\bar{q}_L q_R + \bar{q}_R q_L) = m_q \bar{q} q$$

$$\to m_q(\bar{q}_L \underbrace{U^+ U}_{1} q_R + h.c.)$$

$$\Downarrow$$

$$\boxed{m_u = m_d \equiv m_q}$$

$\Rightarrow$ need a Higgs field

call Σ

$$\mathcal{L}^{Y}_{\Sigma} = \bar{q}_L Y_\Sigma \Sigma q_R + h.c.$$

$$\to \bar{q}_L Y_\Sigma \underbrace{U^\dagger \Sigma' U q_R}_{\Sigma} + h.c.$$

$$\boxed{\Sigma \to \Sigma' = U \Sigma U^\dagger}$$

Adjoint

$$\Sigma = \Sigma^\dagger, \quad \mathrm{Tr}\,\Sigma = 0$$

$\Rightarrow$ 3 real elements

$\Downarrow$

$$\mathcal{L}_Y = \bar{q}_L m_q q_R + \boxed{\bar{q}_L y_\Sigma \Sigma q_R} + h.c.$$

$$\Sigma \longrightarrow \Sigma^0 \Leftrightarrow \text{minimum of } V_\Sigma$$

$$\boxed{\Sigma^0 \Rightarrow U \Sigma^0 U^\dagger} \qquad \Sigma^0 = \#$$

$$\Sigma^0 = \Sigma^{0\dagger} \Rightarrow \boxed{\begin{aligned}\Sigma^0 &= \mathrm{diag}\,(v, -v) \\ &= v\,\sigma_3\end{aligned}}$$

$$\mathrm{Tr}\,\Sigma^0 = 0$$

$$\boxed{Y_\Sigma = 0}$$

$$\Sigma^0 \longrightarrow U \Sigma_0 U^\dagger$$

$$v\,\sigma_3 \longrightarrow U v\,\sigma_3 U^\dagger = v\,\sigma_3$$

iff

$$U = U_3 = e^{i\theta\sigma_3}$$

Σ_0 ! $\quad SU(2) \times U_Y(1)$

$$\downarrow$$

$$U(1) \times U(1)$$

$\Rightarrow$ $\boxed{m_Z = 0 \quad \text{show}}$

$$\Downarrow$$

more Higgs!

doublet?

$\boxed{\text{Messy!}}$

$$\boxed{\begin{array}{c}\text{BUT} \\ \not{P} = \text{maximal}\end{array}}$$

$$\Downarrow$$

$$\boxed{\Phi = \text{sufficient}}$$

$$\Downarrow$$

$$\boxed{\begin{array}{c}\text{SM} = \text{theory of origin} \\ \text{of mass}\end{array}}$$

<u>Sha!</u> $m_e = 0$

$$\Rightarrow \boxed{m_u = -m_d}$$

$$\Downarrow$$

Bottom line:

(most) general $\mathcal{L}$ based on symmetry (principle, idea)

$\Downarrow$

exp

fix parameter space

- Georgi + Glashow:

$SU(2) = SO(3)$

theory ???

$SU(2) \times U(1) \not\leftarrow \begin{pmatrix} \nu \\ e \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix}$

$$\begin{pmatrix} E^+ \\ \nu \\ e \end{pmatrix}$$

extremely ugly SO(3) model of leptons

<u>no</u> Z boson!