

Neutrino Mass

end

Grand Unification

Lecture III

26/10 /2021

LMU
Fall 2021



Standard Model:

Electro - Weak part

$$SU(2)_L \times U(1) \equiv G_{SM}^{ew}$$

$$Q \equiv Q_{em} = T_3 + Y_2$$

$$\boxed{Y = 2(Q - T_3)} \quad (1)$$

Glashow 1961

$$J^\mu_w = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L$$

$$L = \frac{1 + \gamma_5}{2} \quad R = \frac{1 - \gamma_5}{2}$$

$$\{ \gamma_\mu, \gamma_\nu \} = 2 g_{\mu\nu}$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\sigma^\mu = (1; \sigma_i)$$

$$\bar{\sigma}^\mu = (1; -\sigma_i)$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3$$

$$\{ \gamma_5, \gamma_\mu \} = 0 \quad \gamma_5^2 = +1$$

$$\Sigma_{\mu\nu} = \frac{i}{4\pi} [\gamma_\mu, \gamma_\nu]$$

$$\lambda = \xi = e^{i\theta^{\mu\nu}\Sigma_{\mu\nu}} +$$

Σ_{0i} - BOOST

Σ_{ij} - ROT

$$L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

$$\psi \rightarrow \lambda \psi \quad (\text{spinor})$$

C

$$\psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \leftarrow RH(\text{darm})$$

DIRAC

$$p^\mu \gamma^\mu \psi = m \psi$$

1928

$$\bullet m = 0 \quad p^\mu \partial_\mu = 0$$

$$\Rightarrow \begin{pmatrix} 0 & E - \vec{p} \cdot \vec{\sigma} \\ E + \vec{p} \cdot \vec{\sigma} & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix} = 0$$

$$\Rightarrow E u_{L,R} = \mp \vec{p} \cdot \vec{\sigma} u_{L,R}$$

$$|\vec{p}| = p = E \quad \vec{s} = \vec{\sigma}/2$$



$$h = \vec{p} \cdot \vec{s} = \frac{\vec{p} \cdot \vec{s}}{E} = \frac{\vec{p} \cdot \vec{\sigma}}{2E}$$

$$h u_{L,R} = \mp u_{L,R}$$



$\rho = \text{maximal}$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow$$

u_R, d_R &

$$l_L = \begin{pmatrix} v \\ e \end{pmatrix}_L \rightarrow$$

$e_R, \cancel{\nu_R}$

$P = \underline{\text{good}}$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$- - \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$- \begin{pmatrix} v \\ e \end{pmatrix}_L$$

$$- \begin{pmatrix} v \\ e \end{pmatrix}_R$$

\downarrow
 $\exists \underline{v}_R \Leftrightarrow \underline{m}_v = 0 ?$

$$D_\mu = \partial_\mu - i g A_\mu^a T_a - i g' \frac{Y}{2} B_\mu$$

$\underbrace{\hspace{10em}}$ $\underbrace{\hspace{10em}}$
 $SU(2)$ $U(1)$

$$\boxed{T_a f_L = \frac{\sigma_a}{2} f_L} \quad \boxed{T_a f_R = 0}$$

$V = e^{i \theta_a T_a} = SU(2)$

singlet = trivial $\Leftrightarrow V=1$

$$\mathcal{L}_f = i \bar{f} \gamma^\mu \partial_\mu f - m_f \bar{f} f$$

$$f \equiv \gamma_f \Rightarrow \bar{f} f = \bar{f}_L f_L + \bar{f}_R f_R +$$

$$\bar{f} = f^+ \gamma^0 \quad \{ \gamma^0, \gamma_5 \} = 0$$

$m_f = 0 \quad \text{by } SU(2)_L$

DIRAC mass

$$m_f \bar{f} f = (\bar{f}_L f_R + \bar{f}_R f_L) m_D$$

$$= (u_L^+ u_R + u_R^+ u_L) m_D$$

• MAJORANA mass

1938

$$m_\mu f_L^\top C f_L + h.c.$$

$$(\text{def.}) \quad C^\top = -C, \quad C \gamma_\mu C^\top = -\gamma_\mu^\top$$

$$C^\dagger = -C$$

$$\boxed{C = i \gamma_2 \gamma_0} \quad (\text{choice})$$

$$\Rightarrow m_\mu u_L^\top i \sigma_2 u_L + h.c.$$

Lorentz

$$; \vec{\sigma}_L \cdot (\vec{\theta} + i \vec{x})$$

$$u_L \rightarrow e \quad \uparrow \quad \uparrow$$

ROT

BOOST

$$u_R \rightarrow e^{i \vec{\sigma}_L (\vec{\theta} - \vec{x})}$$



$$\underbrace{u_M u_L^T i \sigma_2 u_L}_{\text{Anti-symmetric}} \quad (j=0, \text{ invariant})$$



Anti-symmetric

$$l_L^T G i \sigma_2 l_L$$



Lorentz
invariance

$SU(2)$

invariance



$$D \rightarrow U D$$

remind

$$D^T i \sigma_2 D \rightarrow D^T U^T i \sigma_2 U D$$

$$= D^T i \sigma_2 U^+ U D = D^T i \sigma_2 D$$

✓

• $\ell^T i \sigma_2 G \ell = 0$



$$= -\ell^T i \sigma_2^T C^T \ell$$

$$= -\ell^T i \sigma_2 C \ell$$

• scalar $D = \bar{\Phi}$ ($\bar{\Phi} \rightarrow U \bar{\Phi}$)

$$\bar{\Phi}^T i \sigma_2 \bar{\Phi} = 0$$

$$\begin{aligned} \cdot e_R^T C e_R &= -e_R^T C^T e_R \\ &= e_R^T C e_R \end{aligned}$$

$$Q e_R = \gamma_2 e_R = -e_R$$

$$\Rightarrow \gamma e_R = -2 e_R$$



$$i \overline{f_L} \partial^\mu \not{D}_\mu f_L =$$

$$= i \overline{f_L} \gamma^\mu \not{\partial}_\mu f_L +$$

$$\overline{f_L} \partial^\mu \left(g \frac{\sigma_a}{2} A_\mu{}^a + g' \frac{\psi}{2} B_\mu \right) f_L$$

1,2

$$= (\bar{u} \sigma^i)_L \gamma^\mu g \begin{pmatrix} 0 & A_1 - i A_2 \\ A_1 + i A_2 & 0 \end{pmatrix}_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$= \frac{g}{\sqrt{2}} \bar{u}_L \gamma^\mu \frac{A_1 - i A_2}{\sqrt{2}} d_L + h.c.$$

$\underbrace{}$

$\bar{W}_\mu^- +$



$$\cancel{\gamma_{W_L}} = \frac{g}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L W_\mu^+ + h.c.$$



• neutral piece

$$\int \bar{f} \delta^{\mu} (g T_3 A_3 + g' \frac{Y}{2} B)_{\mu} f$$

↑

$$T_3 f_L = \frac{\alpha_3}{2} f_L$$

$$T_3 f_R = 0$$



$$Y_2 = Q - T_3$$



$$\int \bar{f} \delta^{\mu} [T_3 (g A_3 - g' B)_{\mu} + g' Q B_{\mu}] f$$

| Neutral gauge wf.

$$= \{ f_{\text{photon}} : e A_\mu \bar{f} Q \partial^\mu f \}$$

$$= \bar{f} \partial^\mu \left[e Q_{cm} A_\mu + g_z Q_z \bar{\chi}_\mu \right] f$$



$$A_\mu = \sin \theta_W A_\mu^3 + \cos \theta_W B_\mu$$

$$\bar{\chi}_\mu = \cos \theta_W A_\mu^3 - \sin \theta_W B_\mu$$

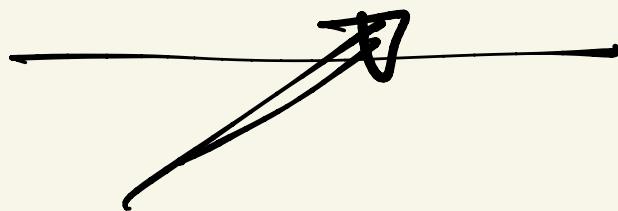
$$\tan \theta_W = g'/g$$



Show

$$g_z = \frac{g}{\cos \theta_W}$$

$$Q_2 = T_3 - Q_{ew} \sin^2 \theta_W$$



$$\theta_W \approx 30^\circ \quad (\sin^2 \theta_W \approx 0.23)$$

$$e = g \sin \theta_W \quad *$$

exp.

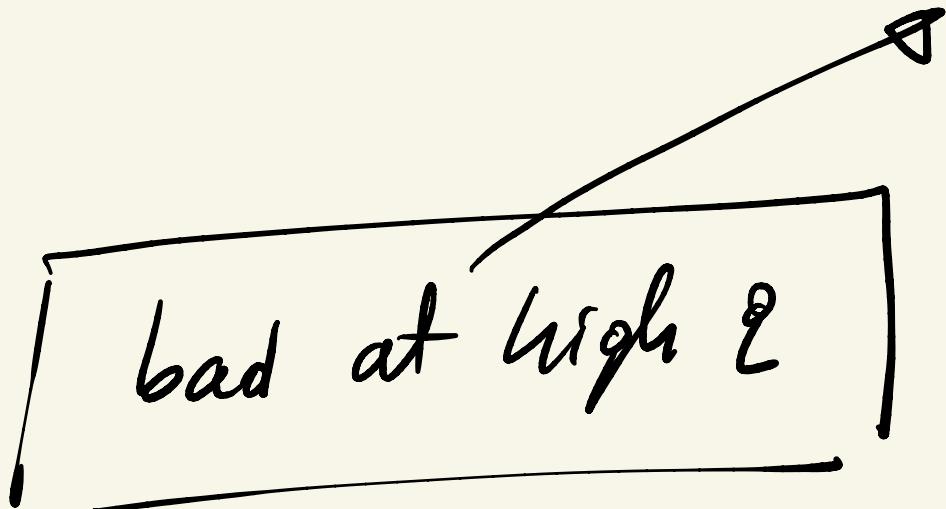
$$M_W = 80 \text{ GeV}$$

$$M_T = 90 \text{ GeV}$$

$\bar{w}, w^+, \bar{w}^+ \leftarrow \underline{\text{massive gauge boson}}$

$$\Delta_{\mu\nu}^P(A) = \frac{-i}{q^2 - m_A^2} \left[g_{\mu\nu} - \frac{g_\mu g_\nu}{m_A^2} \right]$$

(m_A)



\cancel{X} f: Weinberg 1967

$(u)_L; u_R \bar{d}_R$ "V-A was the leg"

$(v)_L; v_R \bar{e}_R$ Weinberg '69

Higgs

$SU(2)_L \times U(1)_Y$

$$\mathcal{L}_f = i \overline{f} \gamma^\mu D_\mu f$$

$$\mathcal{L}_{A,B} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon_{abc} A_\mu^b A_\nu^c \quad | \quad B_\mu = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$\mathcal{L}_{\text{Kiges}}$

$$\begin{aligned} q_L &\rightarrow U q_L \\ \Rightarrow \bar{\Phi} &\rightarrow U \bar{\Phi} \end{aligned}$$

$$\mathcal{L}_Y = \bar{q}_L y_d \bar{\Phi} d_R + \bar{l}_L y_e \bar{\Phi} e_R$$

\circlearrowleft \checkmark

\mathcal{G}/M weak key

$$SU(2) \text{ inv. } \bar{\Phi} \rightarrow U \bar{\Phi}$$

$$U(1) \text{ inv. } : \boxed{Y_{\bar{\Phi}} = +1}$$

$$Y d_R = 2 Q d_R = -2/3 \quad \leftarrow$$

$$Y q_L = 2(Q - T_3) q_L = +1/3 q_L$$

$$\downarrow + \bar{q}_L g_n i\tau_2 \bar{\Phi}^* u_R$$

$$Y u_R = 2 Q u_R = \frac{4}{3} u_R \leftarrow$$

SU(2): $\bar{q}_L - \bar{\Phi}^* u_R \rightarrow$

$\bar{q}_L \underbrace{U^+ U^*}_{-1} \bar{\Phi}^* u_R$

ded: $\bar{q}_L i\tau_2 \bar{\Phi}^* u_R \rightarrow$

$$\bar{q}_L U^+ i\tau_2 U^* \bar{\Phi}^* u_R$$

$$= \bar{q}_L \underbrace{U^+ U}_{1} i\tau_2 \bar{\Phi}^* u_R \swarrow$$



$$\mathcal{L}_{\Phi} = (\partial_\mu \Phi)^+ (\partial^\mu \Phi) - V(\Phi)$$

$$-V(\Phi) = \frac{\lambda}{4} (\Phi^+ \Phi - v^2)^2$$

$$= \frac{\lambda}{4} \Phi^+ \Phi - \frac{\lambda}{2} v^2 \Phi^+ \Phi + \frac{\lambda}{9} v^4$$



$$\Phi_0^+ \Phi_0 = v^2 \rightarrow \text{the minimum}$$

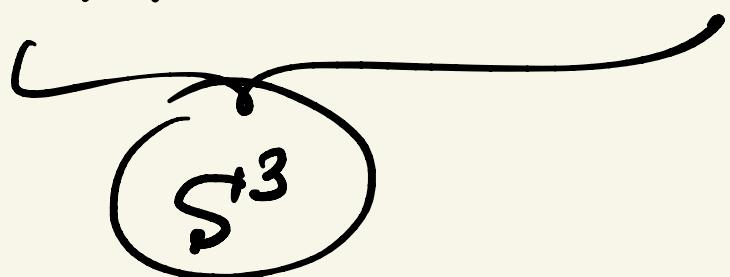
vacuum manifold

$$M_0 = \{ \Phi_0 : V = V_{\min} = 0 \}$$

$$= \{ \Phi_0^+ \Phi_0 = v^2 \} =$$

$$\Phi = \begin{pmatrix} R_1 + iR_2 \\ R_3 + iR_4 \end{pmatrix}$$

$$\Phi^+ \Phi_0 = \sum_{i=1}^4 (R_i^0)^2 = \alpha^2$$



Crucial $\rho = \text{maximal!!}$

* imagine $P = \text{good}$

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L ; \quad \begin{pmatrix} u \\ d \end{pmatrix}_R \equiv e_R$$

$$\downarrow \quad Y q_L = Y e_R = 1/3$$

$$q_L \rightarrow U e_L \quad \bar{e}_R \rightarrow U \bar{e}_R$$

$$m_q (\bar{q}_L q_R + \bar{e}_R e_L) = m_\rho \bar{\ell} \ell$$

$$\rightarrow m_\rho (\bar{q}_L \underbrace{U^\dagger U}_{1} q_R + h.c.)$$



$$m_u = m_d \equiv m_\rho$$

\Rightarrow need a Higgs field

coll Σ

$$\mathcal{L}'_{\Sigma} = \bar{q}_L \gamma_5 \sum q_k + h.c.$$

$$\rightarrow \bar{q}_L \gamma_5 U \underbrace{\Sigma' U^+}_{\Sigma} q_L + h.c.$$

$$\Sigma \rightarrow \Sigma' = U \Sigma U^+$$

Adjoint

$$\Sigma = \Sigma^+, \quad T_V \Sigma = 0$$

$$\Rightarrow \boxed{3 \text{ real elements}}$$

$$\tilde{\mathcal{L}}_Y = \overline{g_L} \mu_Q q_R + \overline{\overline{g_L} g_S} \sum q_R + h.c.$$

$\Sigma \rightarrow \Sigma^0 \Leftrightarrow \text{minimum of } V_\Sigma$

$$\boxed{\Sigma^0 \rightarrow U \Sigma^0 U^\dagger} \quad \Sigma^0 = \#$$

$$\Sigma^0 = \Sigma^{0+} \Rightarrow \boxed{\begin{aligned} \Sigma^0 &= \text{diag } (\vartheta, -\vartheta) \\ &= \vartheta \sigma_3 \end{aligned}}$$

$$\boxed{Y_\Sigma = 0}$$

$$\Sigma^0 \rightarrow U \Sigma_0 U^\dagger$$

$$\vartheta \sigma_3 \rightarrow U \vartheta \sigma_3 U^\dagger = \vartheta \sigma_3$$

if

$$U = U_3 = e^{i\theta \sigma_3}$$

$$\Sigma_0 : \quad SU(2) \times U(1)$$



$$U(1) \times U(1)$$

\Rightarrow $m_z = 0$ show



more Higgs !

doublet ?

Werry !

BUT

$P = \text{maximal}$



$\Phi = \text{sufficient}$



SM = theory of origin
of mass

sha: $m_p = 0$

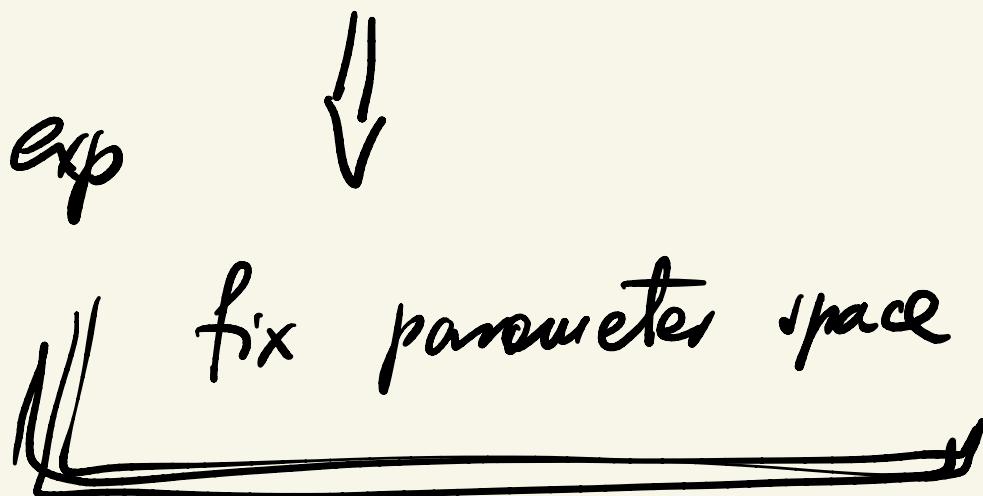
$\Rightarrow m_u = -m_d$



Bottom line:

(most) general \mathcal{L} based on

symmetry (principle, idea)



- Georgi + Glashow:

$$SU(2) = SO(3)$$

Theory ???

$$SU(2) \times U(1) \not\subset \left(\begin{matrix} e \\ e \end{matrix}\right), \left(\begin{matrix} 4 \\ 0 \end{matrix}\right)$$

$(\begin{matrix} e^+ \\ \nu \\ e \end{matrix})$

extremely ugly $SO(3)$
model of leptons

no 2 brns!